$\lim_{\ell \to \infty} \left(\mathrm{AdS}_3/\mathrm{CFT}_2\right)$
Flat Space Holography

Daniel Grumiller

Institute for Theoretical Physics
TU Wien

All about AdS3
ETH Zurich, November 2015

Some of our papers on flat space holography

- A. Bagchi, D. Grumiller and W. Merbis,
  "Stress tensor correlators in three-dimensional gravity,"
  arXiv:1507.05620.
- A. Bagchi, R. Basu, D. Grumiller and M. Riegler,
  "Entanglement entropy in Galilean conformal field theories and flat holography,"
  Phys. Rev. Lett. **114** (2015) 11, 111602 [arXiv:1410.4089].
- H. Afshar, A. Bagchi, R. Fareghbal, D. Grumiller and J. Rosseel,
  "Spin-3 Gravity in Three-Dimensional Flat Space,"
  Phys. Rev. Lett. **111** (2013) 12, 121603 [arXiv:1307.4768].
- A. Bagchi, S. Detournay, D. Grumiller and J. Simon,
  "Cosmic Evolution from Phase Transition of Three-Dimensional Flat Space,"
  Phys. Rev. Lett. **111** (2013) 18, 181301 [arXiv:1305.2919].
- A. Bagchi, S. Detournay and D. Grumiller,
  "Flat-Space Chiral Gravity,"
  Phys. Rev. Lett. **109** (2012) 151301 [arXiv:1208.1658].

Outline

Outline

Motivations

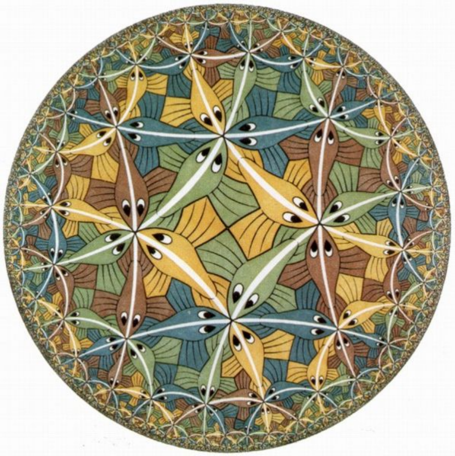
Flat space holography basics

Recent results

Generalizations & open issues

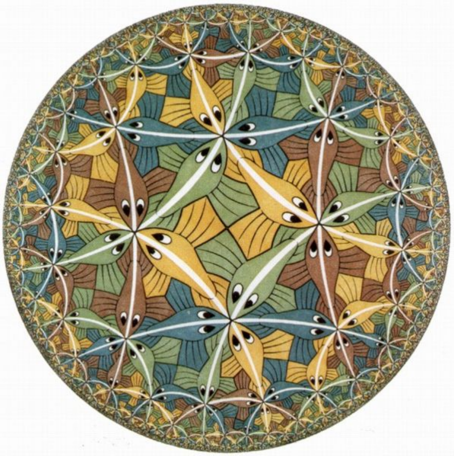
Holography beyond AdS/CFT?

This talk focuses on holography (in the quantum gravity sense).



Holography beyond AdS/CFT?

This talk focuses on holography (in the quantum gravity sense).



Main question: how general is holography?

Testing the holographic principle

How general is holography?

- To what extent do (previous) lessons rely on the particular constructions used to date?
- Are they tied to stringy effects and to string theory in particular, or are they general lessons for quantum gravity?

see numerous talks at KITP workshop “Bits, Branes, Black Holes” 2012

and at ESI workshop “Higher Spin Gravity” 2012

Testing the holographic principle

How general is holography?

- To what extent do (previous) lessons rely on the particular constructions used to date?
- Are they tied to stringy effects and to string theory in particular, or are they general lessons for quantum gravity?
- Does holography apply only to unitary theories?

- originally holography motivated by unitarity

Testing the holographic principle

How general is holography?

- To what extent do (previous) lessons rely on the particular constructions used to date?
- Are they tied to stringy effects and to string theory in particular, or are they general lessons for quantum gravity?
- Does holography apply only to unitary theories?

- originally holography motivated by unitarity
- plausible AdS/CFT-like correspondence could work non-unitarily
- AdS_3/log CFT_2 first example of non-unitary holography DG, (Jackiw), Johansson '08; Skenderis, Taylor, van Rees '09; Henneaux, Martinez, Troncoso '09; Maloney, Song, Strominger '09; DG, Sachs/Hohm '09; Gaberdiel, DG, Vassilevich '10; ... DG, Riedler, Rosseel, Zojer '13
- recent proposal by Vafa '14

Testing the holographic principle

How general is holography?

- To what extent do (previous) lessons rely on the particular constructions used to date?
- Are they tied to stringy effects and to string theory in particular, or are they general lessons for quantum gravity?
- Does holography apply only to unitary theories?
- Can we establish a flat space holographic dictionary?

the answer appears to be yes — see my current talk and recent papers by Bagchi et al., Barnich et al., Strominger et al., '12-'15

Testing the holographic principle

How general is holography?

- To what extent do (previous) lessons rely on the particular constructions used to date?
- Are they tied to stringy effects and to string theory in particular, or are they general lessons for quantum gravity?
- Does holography apply only to unitary theories?
- Can we establish a flat space holographic dictionary?
- Generic non-AdS holography/higher spin holography?

non-trivial hints that it might work
Gary, DG Rashkov '12; Afshar et al '12; Gutperle et al '14-'15; Gary, DG, Prohazka, Rey '14; Lei, Ross '15; Lei, Peng '15; Breunhölder, Gary, DG, Prohazka '15; ...

Testing the holographic principle

How general is holography?

- To what extent do (previous) lessons rely on the particular constructions used to date?
- Are they tied to stringy effects and to string theory in particular, or are they general lessons for quantum gravity?
- Does holography apply only to unitary theories?
- Can we establish a flat space holographic dictionary?
- Generic non-AdS holography/higher spin holography?

- Address questions above in simple class of 3D toy models
- Exploit gauge theoretic Chern–Simons formulation
- Restrict to kinematic questions, like (asymptotic) symmetries

Outline

Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...)

if holography is true $\Rightarrow$ must work in flat space

Just take large AdS radius limit of 10^4 AdS/CFT papers?

Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...)

if holography is true ⇒ must work in flat space

> Just take large AdS radius limit of 10^4 AdS/CFT papers?

- Works straightforwardly sometimes, otherwise not

Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...)

if holography is true $\Rightarrow$ must work in flat space

Just take large AdS radius limit of 10^4 AdS/CFT papers?

- Works straightforwardly sometimes, otherwise not
- Example where it works nicely: asymptotic symmetry algebra

Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...)

if holography is true ⇒ must work in flat space

> Just take large AdS radius limit of 10^4 AdS/CFT papers?

- Works straightforwardly sometimes, otherwise not
- Example where it works nicely: asymptotic symmetry algebra
- Take linear combinations of Virasoro generators $\mathcal{L}_n$, $\bar{\mathcal{L}}_n$

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n} \qquad M_n = \frac{1}{\ell}\left(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}\right)$$

Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...)

if holography is true $\Rightarrow$ must work in flat space

Just take large AdS radius limit of 10^4 AdS/CFT papers?

- Works straightforwardly sometimes, otherwise not
- Example where it works nicely: asymptotic symmetry algebra
- Take linear combinations of Virasoro generators $\mathcal{L}_n$, $\bar{\mathcal{L}}_n$

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n} \qquad M_n = \frac{1}{\ell}\left(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}\right)$$

- Make Inönü–Wigner contraction $\ell \to \infty$ on ASA

$$[L_n,\, L_m] = (n-m)\, L_{n+m} + \frac{c_L}{12}\,(n^3 - n)\,\delta_{n+m,\,0}$$
$$[L_n,\, M_m] = (n-m)\, M_{n+m} + \frac{c_M}{12}\,(n^3 - n)\,\delta_{n+m,\,0}$$
$$[M_n,\, M_m] = 0$$

Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...)

if holography is true $\Rightarrow$ must work in flat space

Just take large AdS radius limit of 10^4 AdS/CFT papers?

- Works straightforwardly sometimes, otherwise not
- Example where it works nicely: asymptotic symmetry algebra
- Take linear combinations of Virasoro generators $\mathcal{L}_n$, $\bar{\mathcal{L}}_n$

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n} \qquad M_n = \frac{1}{\ell}\left(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}\right)$$

- Make Inönü–Wigner contraction $\ell \to \infty$ on ASA

$$[L_n,\, L_m] = (n-m)\, L_{n+m} + \frac{c_L}{12}\,(n^3 - n)\,\delta_{n+m,\,0}$$
$$[L_n,\, M_m] = (n-m)\, M_{n+m} + \frac{c_M}{12}\,(n^3 - n)\,\delta_{n+m,\,0}$$
$$[M_n,\, M_m] = 0$$

- This is nothing but the BMS_3 algebra (or GCA_2, URCA_2, CCA_2)!
  Ashtekar, Bicak, Schmidt, '96; Barnich, Compere '06
  L_n: diffeos of circle, M_n: supertranslations, $c_{L/M}$: central extensions

Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...)

if holography is true $\Rightarrow$ must work in flat space

> Just take large AdS radius limit of 10^4 AdS/CFT papers?

- Works straightforwardly sometimes, otherwise not
- Example where it works nicely: asymptotic symmetry algebra
- Take linear combinations of Virasoro generators $\mathcal{L}_n$, $\bar{\mathcal{L}}_n$

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n} \qquad M_n = \frac{1}{\ell}\left(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}\right)$$

- Make Inönü–Wigner contraction $\ell \to \infty$ on ASA

$$[L_n,\, L_m] = (n-m)\, L_{n+m} + \frac{c_L}{12}\,(n^3-n)\,\delta_{n+m,\,0}$$
$$[L_n,\, M_m] = (n-m)\, M_{n+m} + \frac{c_M}{12}\,(n^3-n)\,\delta_{n+m,\,0}$$
$$[M_n,\, M_m] = 0$$

- This is nothing but the BMS_3 algebra (or GCA_2, URCA_2, CCA_2)!
  If dual field theory exists it must be a 2D Galilean CFT!
  Bagchi et al., Barnich et al.

Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...)

if holography is true $\Rightarrow$ must work in flat space

Just take large AdS radius limit of 10^4 AdS/CFT papers?

- Works straightforwardly sometimes, otherwise not
- Example where it works nicely: asymptotic symmetry algebra
- Take linear combinations of Virasoro generators $\mathcal{L}_n$, $\bar{\mathcal{L}}_n$

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n} \qquad M_n = \frac{1}{\ell}\left(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}\right)$$

- Make Inönü–Wigner contraction $\ell \to \infty$ on ASA

$$[L_n,\, L_m] = (n-m)\, L_{n+m} + \frac{c_L}{12}\,(n^3 - n)\,\delta_{n+m,\,0}$$
$$[L_n,\, M_m] = (n-m)\, M_{n+m} + \frac{c_M}{12}\,(n^3 - n)\,\delta_{n+m,\,0}$$
$$[M_n,\, M_m] = 0$$

- This is nothing but the BMS_3 algebra (or GCA_2, URCA_2, CCA_2)!
- Example where it does not work easily: boundary conditions

Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...)

if holography is true ⇒ must work in flat space

Just take large AdS radius limit of 10^4 AdS/CFT papers?

- Works straightforwardly sometimes, otherwise not
- Example where it works nicely: asymptotic symmetry algebra
- Take linear combinations of Virasoro generators $\mathcal{L}_n$, $\bar{\mathcal{L}}_n$

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n} \qquad M_n = \frac{1}{\ell}\left(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}\right)$$

- Make Inönü–Wigner contraction $\ell \to \infty$ on ASA

$$[L_n,\, L_m] = (n-m)\, L_{n+m} + \frac{c_L}{12}\,(n^3-n)\,\delta_{n+m,\,0}$$
$$[L_n,\, M_m] = (n-m)\, M_{n+m} + \frac{c_M}{12}\,(n^3-n)\,\delta_{n+m,\,0}$$
$$[M_n,\, M_m] = 0$$

- This is nothing but the BMS_3 algebra (or GCA_2, URCA_2, CCA_2)!
- Example where it does not work easily: boundary conditions
- Example where it does not work: highest weight conditions

Flat space Einstein gravity as isl(2) Chern–Simons theory

For details, references and spin-3 generalization see Gary, DG, Riegler, Rosseel '14

Flat space Einstein gravity as isl(2) Chern–Simons theory

For details, references and spin-3 generalization see Gary, DG, Riegler, Rosseel '14

- AdS gravity in CS formulation: sl$(2)\oplus$sl(2) gauge algebra

  Achucarro, Townsend '86; Witten '88

Flat space Einstein gravity as isl(2) Chern–Simons theory

For details, references and spin-3 generalization see Gary, DG, Riegler, Rosseel '14

- AdS gravity in CS formulation: sl$(2)\oplus$sl(2) gauge algebra
- Flat space: isl(2) gauge algebra

$$S_{\text{CS}}^{\text{flat}} = \frac{k}{4\pi}\int\langle\mathcal{A}\wedge\mathrm{d}\mathcal{A}+\tfrac{2}{3}\,\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A}\rangle$$

with isl(2) connection ($a=0,\,\pm1$)

$$\mathcal{A} = e^a M_a + \omega^a L_a$$

isl(2) algebra (global part of BMS/GCA)

$$[L_a,\,L_b] = (a-b)L_{a+b}$$
$$[L_a,\,M_b] = (a-b)M_{a+b}$$
$$[M_a,\,M_b] = 0$$

Note: e^a dreibein, ω^a (dualized) spin-connection

Bulk EOM: gauge flatness $\to$ Einstein equations

$$\mathcal{F} = \mathrm{d}\mathcal{A}+\mathcal{A}\wedge\mathcal{A} = 0$$

Flat space Einstein gravity as isl(2) Chern–Simons theory

For details, references and spin-3 generalization see Gary, DG, Riegler, Rosseel '14

- AdS gravity in CS formulation: sl$(2)\oplus$sl(2) gauge algebra
- Flat space: isl(2) gauge algebra

$$S_{\text{CS}}^{\text{flat}} = \frac{k}{4\pi}\int\langle\mathcal{A}\wedge\mathrm{d}\mathcal{A}+\tfrac{2}{3}\,\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A}\rangle$$

with isl(2) connection ($a=0,\,\pm1$)

$$\mathcal{A}=e^aM_a+\omega^aL_a$$

- Boundary conditions in CS formulation:

$$\mathcal{A}(r,\,u,\,\varphi)=b^{-1}(r)\,\big(\,\mathrm{d}+a(u,\,\varphi)+o(1)\big)\,b(r)$$

Flat space Einstein gravity as isl(2) Chern–Simons theory

For details, references and spin-3 generalization see Gary, DG, Riegler, Rosseel '14

- AdS gravity in CS formulation: sl$(2)\oplus$sl(2) gauge algebra
- Flat space: isl(2) gauge algebra

$$S_{\text{CS}}^{\text{flat}} = \frac{k}{4\pi} \int \langle \mathcal{A} \wedge \mathrm{d}\mathcal{A} + \tfrac{2}{3}\,\mathcal{A} \wedge \mathcal{A} \wedge \mathcal{A} \rangle$$

with isl(2) connection ($a = 0,\, \pm 1$)

$$\mathcal{A} = e^a M_a + \omega^a L_a$$

- Boundary conditions in CS formulation:

$$\mathcal{A}(r,\, u,\, \varphi) = b^{-1}(r)\,\big(\mathrm{d} + a(u,\, \varphi) + o(1)\big)\, b(r)$$

- Flat space boundary conditions: $b(r) = \exp\big(\frac{1}{2}\, r M_{-1}\big)$ and

$$a(u,\, \varphi) = \big(M_1 - M(\varphi) M_{-1}\big)\, \mathrm{d}u + \big(L_1 - M(\varphi) L_{-1} - N(u,\, \varphi) M_{-1}\big)\, \mathrm{d}\varphi$$

with $N(u,\, \varphi) = L(\varphi) + \frac{u}{2}\, M'(\varphi)$

Flat space Einstein gravity as isl(2) Chern–Simons theory

For details, references and spin-3 generalization see Gary, DG, Riegler, Rosseel '14

- AdS gravity in CS formulation: sl$(2)\oplus$sl(2) gauge algebra
- Flat space: isl(2) gauge algebra

$$S_{\rm CS}^{\rm flat} = \frac{k}{4\pi}\int\langle\mathcal{A}\wedge\mathrm{d}\mathcal{A}+\tfrac{2}{3}\,\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A}\rangle$$

with isl(2) connection $(a=0,\,\pm1)$

$$\mathcal{A}=e^aM_a+\omega^aL_a$$

- Boundary conditions in CS formulation:

$$\mathcal{A}(r,\,u,\,\varphi)=b^{-1}(r)\,\big(\mathrm{d}+a(u,\,\varphi)+o(1)\big)\,b(r)$$

- Flat space boundary conditions: $b(r)=\exp\big(\tfrac{1}{2}\,rM_{-1}\big)$ and

$$a(u,\,\varphi)=\big(M_1-M(\varphi)M_{-1}\big)\,\mathrm{d}u+\big(L_1-M(\varphi)L_{-1}-N(u,\,\varphi)M_{-1}\big)\,\mathrm{d}\varphi$$

with $N(u,\,\varphi)=L(\varphi)+\frac{u}{2}\,M'(\varphi)$

- metric

$$g_{\mu\nu}\sim\tfrac{1}{2}\,\widetilde{\rm tr}\langle\mathcal{A}_\mu\mathcal{A}_\nu\rangle\quad\to\quad\mathrm{d}s^2=M\,\mathrm{d}u^2-2\,\mathrm{d}u\,\mathrm{d}r+2N\,\mathrm{d}u\,\mathrm{d}\varphi+r^2\,\mathrm{d}\varphi^2$$

Outline

Motivations

Flat space holography basics

Recent results

Generalizations & open issues

Correlation functions in flat space holography

AdS/CFT good tool for calculating correlators
What about flat space/Galilean CFT correspondence?

Correlation functions in flat space holography

AdS/CFT good tool for calculating correlators
What about flat space/Galilean CFT correspondence?

- What is flat space analogue of

$$\langle T(z_1)T(z_2)\dots T(z_{42})\rangle_{\text{CFT}} \sim \frac{\delta^{42}}{\delta g^{42}}\Gamma_{\text{EH-AdS}}\Big|_{\text{EOM}}$$

?

Correlation functions in flat space holography

AdS/CFT good tool for calculating correlators
What about flat space/Galilean CFT correspondence?

- What is flat space analogue of

$$\langle T(z_1)T(z_2)\dots T(z_{42})\rangle_{\text{CFT}} \sim \frac{\delta^{42}}{\delta g^{42}}\Gamma_{\text{EH-AdS}}\Big|_{\text{EOM}}$$

  ?
- Does it work?

AdS/CFT good tool for calculating correlators
What about flat space/Galilean CFT correspondence?

- What is flat space analogue of

$$\langle T(z_1)T(z_2)\ldots T(z_{42})\rangle_{\text{CFT}} \sim \frac{\delta^{42}}{\delta g^{42}}\Gamma_{\text{EH-AdS}}\Big|_{\text{EOM}}$$

?
- Does it work?
- What is the left hand side in a Galilean CFT?

Correlation functions in flat space holography

AdS/CFT good tool for calculating correlators
What about flat space/Galilean CFT correspondence?

- What is flat space analogue of

$$\langle T(z_1)T(z_2)\dots T(z_{42})\rangle_{\text{CFT}} \sim \frac{\delta^{42}}{\delta g^{42}}\Gamma_{\text{EH-AdS}}\Big|_{\text{EOM}}$$

?

- Does it work?
- What is the left hand side in a Galilean CFT?
- Shortcut to right hand side other than varying EH-action 42 times?

AdS/CFT good tool for calculating correlators
What about flat space/Galilean CFT correspondence?

- What is flat space analogue of

$$\langle T(z_1)T(z_2)\dots T(z_{42})\rangle_{\text{CFT}} \sim \frac{\delta^{42}}{\delta g^{42}}\Gamma_{\text{EH-AdS}}\Big|_{\text{EOM}}$$

?
- Does it work?
- What is the left hand side in a Galilean CFT?
- Shortcut to right hand side other than varying EH-action 42 times?

Start slowly with 0-point function

0-point function (on-shell action)

Not check of flat space holography but interesting in its own right

- Calculate the full on-shell action Γ

0-point function (on-shell action)

Not check of flat space holography but interesting in its own right

- Calculate the full on-shell action Γ
- Variational principle?

$$\Gamma = -\frac{1}{16\pi G_N}\int \mathrm{d}^3x\sqrt{g}\,R - \frac{1}{8\pi G_N}\int \mathrm{d}^2x\sqrt{\gamma}\,K - I_{\text{counter-term}}$$

with $I_{\text{counter-term}}$ chosen such that

$$\delta\Gamma\big|_{\text{EOM}} = 0$$

for all δg that preserve flat space bc's

0-point function (on-shell action)

Not check of flat space holography but interesting in its own right

- Calculate the full on-shell action Γ
- Variational principle?

$$\Gamma = -\frac{1}{16\pi G_N}\int \mathrm{d}^3x\sqrt{g}\,R - \frac{1}{8\pi G_N}\int \mathrm{d}^2x\sqrt{\gamma}\,K - I_{\text{counter-term}}$$

with $I_{\text{counter-term}}$ chosen such that

$$\delta\Gamma\big|_{\text{EOM}} = 0$$

for all δg that preserve flat space bc's

Result (Detournay, DG, Schöller, Simon '14):

$$\Gamma = -\frac{1}{16\pi G_N}\int \mathrm{d}^3x\sqrt{g}\,R - \underbrace{\frac{1}{16\pi G_N}}_{\frac{1}{2}\text{GHY!}}\int \mathrm{d}^2x\sqrt{\gamma}\,K$$

follows also as limit from AdS using Mora, Olea, Troncoso, Zanelli '04
independently confirmed by Barnich, Gonzalez, Maloney, Oblak '15

0-point function (on-shell action)

Not check of flat space holography but interesting in its own right

- Calculate the full on-shell action Γ
- Variational principle?
- Phase transitions?

  Standard procedure (Gibbons, Hawking '77; Hawking, Page '83)

  Evaluate Euclidean partition function in semi-classical limit

  $$Z(T,\,\Omega) = \int \mathcal{D}g\, e^{-\Gamma[g]} = \sum_{g_c} e^{-\Gamma[g_c(T,\Omega)]} \times Z_{\text{fluct.}}$$

  path integral bc's specified by temperature T and angular velocity Ω

  Two Euclidean saddle points in same ensemble if
  - same temperature $T = 1/\beta$ and angular velocity Ω
  - obey flat space boundary conditions
  - solutions without conical singularities

  Periodicities fixed:

  $$(\tau_E,\,\varphi) \sim (\tau_E + \beta,\,\varphi + \beta\Omega) \sim (\tau_E,\,\varphi + 2\pi)$$

0-point function (on-shell action)

Not check of flat space holography but interesting in its own right

- Calculate the full on-shell action Γ
- Variational principle?
- Phase transitions?
  3D Euclidean Einstein gravity: for each T, Ω two saddle points:
  - Hot flat space
  $$\mathrm{d}s^2 = \mathrm{d}\tau_E^2 + \mathrm{d}r^2 + r^2\,\mathrm{d}\varphi^2$$
  - Flat space cosmology
  $$\mathrm{d}s^2 = r_+^2\left(1 - \frac{r_0^2}{r^2}\right)\mathrm{d}\tau_E^2 + \frac{r^2\,\mathrm{d}r^2}{r_+^2\left(r^2 - r_0^2\right)} + r^2\left(\mathrm{d}\varphi - \frac{r_+ r_0}{r^2}\,\mathrm{d}\tau_E\right)^2$$
  shifted-boost orbifold, see Cornalba, Costa '02

0-point function (on-shell action)

Not check of flat space holography but interesting in its own right

- Calculate the full on-shell action Γ
- Variational principle?
- Phase transitions?
- Plug two Euclidean saddles in on-shell action and compare free energies

$$F_{\text{HFS}} = -\frac{1}{8G_N} \qquad F_{\text{FSC}} = -\frac{r_+}{8G_N}$$

0-point function (on-shell action)

Not check of flat space holography but interesting in its own right

- Calculate the full on-shell action Γ
- Variational principle?
- Phase transitions?
- Plug two Euclidean saddles in on-shell action and compare free energies

$$F_{\text{HFS}} = -\frac{1}{8G_N} \qquad F_{\text{FSC}} = -\frac{r_+}{8G_N}$$

- Result of this comparison
  - $r_+ > 1$: FSC dominant saddle
  - $r_+ < 1$: HFS dominant saddle

  Critical temperature:

$$T_c = \frac{1}{2\pi r_0} = \frac{\Omega}{2\pi}$$

  HFS "melts" into FSC at $T > T_c$

  Bagchi, Detournay, DG, Simon '13

1-point functions (conserved charges)

First check of entries in holographic dictionary: identification of sources and vevs

In AdS_3:

$$\delta\Gamma\big|_{\text{EOM}} \sim \int_{\partial\mathcal{M}} \text{vev} \times \delta\,\text{source} \sim \int_{\partial\mathcal{M}} T^{\mu\nu}_{\text{BY}} \times \delta g^{\text{NN}}_{\mu\nu}$$

Note that $T^{\mu\nu}_{\text{BY}}$ follows from canonical analysis as well (conserved charges)

1-point functions (conserved charges)

First check of entries in holographic dictionary: identification of sources and vevs

In AdS_3:

$$\delta\Gamma\big|_{\mathrm{EOM}} \sim \int_{\partial\mathcal{M}} \mathrm{vev}\times\delta\,\mathrm{source} \sim \int_{\partial\mathcal{M}} T^{\mu\nu}_{\mathrm{BY}}\times\delta g^{\mathrm{NN}}_{\mu\nu}$$

Note that $T^{\mu\nu}_{\mathrm{BY}}$ follows from canonical analysis as well (conserved charges)

In flat space:

- non-normalizable solutions to linearized EOM?
- analogue of Brown–York stress tensor?
- comparison with canonical results

1-point functions (conserved charges)

First check of entries in holographic dictionary: identification of sources and vevs

In AdS_3:

$$\delta\Gamma\big|_{\mathrm{EOM}} \sim \int_{\partial\mathcal{M}} \mathrm{vev} \times \delta\,\mathrm{source} \sim \int_{\partial\mathcal{M}} T^{\mu\nu}_{\mathrm{BY}} \times \delta g^{\mathrm{NN}}_{\mu\nu}$$

Note that $T^{\mu\nu}_{\mathrm{BY}}$ follows from canonical analysis as well (conserved charges)

In flat space:

- non-normalizable solutions to linearized EOM?
- analogue of Brown–York stress tensor?
- comparison with canonical results

everything works (Detournay, DG, Schöller, Simon, '14)

mass and angular momentum:

$$M = \frac{g_{tt}}{8G} \qquad N = \frac{g_{t\varphi}}{4G}$$

full tower of canonical charges: see Barnich, Compere '06

2-point functions (anomalous terms)

First check sensitive to central charges in symmetry algebra

Galilean CFT on cylinder ($\varphi \sim \varphi + 2\pi$):

$$\langle M(u_1,\varphi_1)\, M(u_2,\varphi_2)\rangle = 0$$

$$\langle M(u_1,\varphi_1)\, N(u_2,\varphi_2)\rangle = \frac{c_M}{2s_{12}^4}$$

$$\langle N(u_1,\varphi_1)\, N(u_2,\varphi_2)\rangle = \frac{c_L - 2c_M\tau_{12}}{2s_{12}^4}$$

with $s_{ij} = 2\sin[(\varphi_i - \varphi_j)/2]$, $\tau_{ij} = (u_i - u_j)\,\cot[(\varphi_i - \varphi_j)/2]$

Fourier modes of Galilean CFT stress tensor on cylinder:

$$M := \sum_n M_n e^{-in\varphi} - \frac{c_M}{24}$$

$$N := \sum_n \big(L_n - inuM_n\big) e^{-in\varphi} - \frac{c_L}{24}$$

Conservation equations: $\partial_u M = 0$, $\partial_u N = \partial_\varphi M$

2-point functions (anomalous terms)

First check sensitive to central charges in symmetry algebra

Galilean CFT on cylinder ($\varphi \sim \varphi + 2\pi$):

$$\langle M(u_1, \varphi_1)\, M(u_2, \varphi_2)\rangle = 0$$
$$\langle M(u_1, \varphi_1)\, N(u_2, \varphi_2)\rangle = \frac{c_M}{2s_{12}^4}$$
$$\langle N(u_1, \varphi_1)\, N(u_2, \varphi_2)\rangle = \frac{c_L - 2c_M\tau_{12}}{2s_{12}^4}$$

with $s_{ij} = 2\sin[(\varphi_i - \varphi_j)/2]$, $\tau_{ij} = (u_i - u_j)\,\cot[(\varphi_i - \varphi_j)/2]$

Short-cut on gravity side:

- Do not calculate second variation of action

2-point functions (anomalous terms)

First check sensitive to central charges in symmetry algebra

Galilean CFT on cylinder ($\varphi \sim \varphi + 2\pi$):

$$\langle M(u_1, \varphi_1)\, M(u_2, \varphi_2)\rangle = 0$$
$$\langle M(u_1, \varphi_1)\, N(u_2, \varphi_2)\rangle = \frac{c_M}{2s_{12}^4}$$
$$\langle N(u_1, \varphi_1)\, N(u_2, \varphi_2)\rangle = \frac{c_L - 2c_M\tau_{12}}{2s_{12}^4}$$

with $s_{ij} = 2\sin[(\varphi_i - \varphi_j)/2]$, $\tau_{ij} = (u_i - u_j)\, \cot[(\varphi_i - \varphi_j)/2]$
Short-cut on gravity side:

- Do not calculate second variation of action
- Calculate first variation of action on non-trivial background

2-point functions (anomalous terms)

First check sensitive to central charges in symmetry algebra

Galilean CFT on cylinder ($\varphi \sim \varphi + 2\pi$):

$$\langle M(u_1,\varphi_1)\, M(u_2,\varphi_2)\rangle = 0$$
$$\langle M(u_1,\varphi_1)\, N(u_2,\varphi_2)\rangle = \frac{c_M}{2s_{12}^4}$$
$$\langle N(u_1,\varphi_1)\, N(u_2,\varphi_2)\rangle = \frac{c_L - 2c_M\tau_{12}}{2s_{12}^4}$$

with $s_{ij} = 2\sin[(\varphi_i - \varphi_j)/2]$, $\tau_{ij} = (u_i - u_j)\,\cot[(\varphi_i - \varphi_j)/2]$

Short-cut on gravity side:

- Do not calculate second variation of action
- Calculate first variation of action on non-trivial background
- Can iterate this procedure to higher n-point functions

2-point functions (anomalous terms)

First check sensitive to central charges in symmetry algebra

Galilean CFT on cylinder ($\varphi \sim \varphi + 2\pi$):

$$\langle M(u_1,\varphi_1)\,M(u_2,\varphi_2)\rangle = 0$$
$$\langle M(u_1,\varphi_1)\,N(u_2,\varphi_2)\rangle = \frac{c_M}{2s_{12}^4}$$
$$\langle N(u_1,\varphi_1)\,N(u_2,\varphi_2)\rangle = \frac{c_L - 2c_M\tau_{12}}{2s_{12}^4}$$

with $s_{ij} = 2\sin[(\varphi_i-\varphi_j)/2]$, $\tau_{ij} = (u_i - u_j)\,\cot[(\varphi_i-\varphi_j)/2]$
Short-cut on gravity side:

- Do not calculate second variation of action
- Calculate first variation of action on non-trivial background
- Can iterate this procedure to higher n-point functions

Summarize first how this works in the AdS case

2-point functions (anomalous terms)

Illustrate shortcut in $\mathrm{AdS}_3/\mathrm{CFT}_2$ (restrict to one holomorphic sector)

- On CFT side deform free action S_0 by source term μ for stress tensor

$$S_\mu = S_0 + \int \mathrm{d}^2 z \, \mu(z, \bar{z}) T(z)$$

2-point functions (anomalous terms)

Illustrate shortcut in $\mathrm{AdS}_3/\mathrm{CFT}_2$ (restrict to one holomorphic sector)

- On CFT side deform free action S_0 by source term μ for stress tensor

$$S_\mu = S_0 + \int \mathrm{d}^2 z \, \mu(z, \bar{z}) T(z)$$

- Localize source

$$\mu(z, \bar{z}) = \epsilon \, \delta^{(2)}(z - z_2, \bar{z} - \bar{z}_2)$$

2-point functions (anomalous terms)

Illustrate shortcut in $\mathrm{AdS}_3/\mathrm{CFT}_2$ (restrict to one holomorphic sector)

- On CFT side deform free action S_0 by source term μ for stress tensor

$$S_\mu = S_0 + \int \mathrm{d}^2 z \, \mu(z, \bar{z}) T(z)$$

- Localize source

$$\mu(z, \bar{z}) = \epsilon \, \delta^{(2)}(z - z_2, \bar{z} - \bar{z}_2)$$

- 1-point function in μ-vacuum $\to$ 2-point function in 0-vacuum

$$\langle T^1 \rangle_\mu = \langle T^1 \rangle_0 + \epsilon \, \langle T^1 \, T^2 \rangle_0 + \mathcal{O}(\epsilon^2)$$

2-point functions (anomalous terms)

Illustrate shortcut in AdS_3/CFT_2 (restrict to one holomorphic sector)

- On CFT side deform free action S_0 by source term μ for stress tensor

$$S_\mu = S_0 + \int \mathrm{d}^2 z\, \mu(z,\bar{z}) T(z)$$

- Localize source

$$\mu(z,\bar{z}) = \epsilon\, \delta^{(2)}(z - z_2, \bar{z} - \bar{z}_2)$$

- 1-point function in μ-vacuum $\to$ 2-point function in 0-vacuum

$$\langle T^1 \rangle_\mu = \langle T^1 \rangle_0 + \epsilon\, \langle T^1\, T^2 \rangle_0 + \mathcal{O}(\epsilon^2)$$

- On gravity side exploit sl(2) CS formulation with chemical potentials

$$A = b^{-1}(\mathrm{d} + a)b \qquad\qquad b = e^{\rho L_0}$$

$$a_z = L_+ - \frac{\mathcal{L}}{k} L_- \qquad\qquad a_{\bar{z}} = \mu L_+ + \dots$$

Drinfeld, Sokolov '84, Polyakov '87, H. Verlinde '90
Bañados, Caro '04

2-point functions (anomalous terms)

Illustrate shortcut in $\text{AdS}_3/\text{CFT}_2$ (restrict to one holomorphic sector)

- On CFT side deform free action S_0 by source term μ for stress tensor

$$S_\mu = S_0 + \int \mathrm{d}^2 z\, \mu(z, \bar{z}) T(z)$$

- Localize source

$$\mu(z, \bar{z}) = \epsilon\, \delta^{(2)}(z - z_2, \bar{z} - \bar{z}_2)$$

- 1-point function in μ-vacuum $\to$ 2-point function in 0-vacuum

$$\langle T^1 \rangle_\mu = \langle T^1 \rangle_0 + \epsilon\, \langle T^1\, T^2 \rangle_0 + \mathcal{O}(\epsilon^2)$$

- On gravity side exploit sl(2) CS formulation with chemical potentials

$$A = b^{-1}(\mathrm{d}{+}a)b \qquad\qquad b = e^{\rho L_0}$$

$$a_z = L_+ - \frac{\mathcal{L}}{k} L_- \qquad\qquad a_{\bar{z}} = \mu L_+ + \dots$$

- Expand $\mathcal{L}(z) = \mathcal{L}^{(0)}(z) + \epsilon \mathcal{L}^{(1)}(z) + \mathcal{O}(\epsilon^2)$

2-point functions (anomalous terms)

Illustrate shortcut in AdS_3/CFT_2 (restrict to one holomorphic sector)

- On gravity side exploit CS formulation with chemical potentials

$$A = b^{-1}(d+a)b \qquad\qquad b = e^{\rho L_0}$$

$$a_z = L_+ - \frac{\mathcal{L}}{k} L_- \qquad\qquad a_{\bar z} = \mu L_+ + \dots$$

- Expand $\mathcal{L}(z) = \mathcal{L}^{(0)}(z) + \epsilon \mathcal{L}^{(1)}(z) + \mathcal{O}(\epsilon^2)$
- Write EOM to first subleading order in ϵ

$$\bar\partial \mathcal{L}^{(1)}(z) = -\frac{k}{2}\,\partial^3 \delta^{(2)}(z-z_2)$$

2-point functions (anomalous terms)

Illustrate shortcut in $\mathrm{AdS}_3/\mathrm{CFT}_2$ (restrict to one holomorphic sector)

- On gravity side exploit CS formulation with chemical potentials

$$A = b^{-1}(d+a)b \qquad\qquad b = e^{\rho L_0}$$
$$a_z = L_+ - \frac{\mathcal{L}}{k} L_- \qquad\qquad a_{\bar z} = \mu L_+ + \dots$$

- Expand $\mathcal{L}(z) = \mathcal{L}^{(0)}(z) + \epsilon \mathcal{L}^{(1)}(z) + \mathcal{O}(\epsilon^2)$
- Write EOM to first subleading order in ϵ

$$\bar\partial \mathcal{L}^{(1)}(z) = -\frac{k}{2}\, \partial^3 \delta^{(2)}(z - z_2)$$

- Solve them using the Green function on the plane $G = \ln\left(z_{12} \bar z_{12}\right)$

$$\mathcal{L}^{(1)}(z) = -\frac{k}{2}\, \partial^4_{z_1} G(z_{12}) = \frac{3k}{z_{12}^4}$$

2-point functions (anomalous terms)

Illustrate shortcut in AdS_3/CFT_2 (restrict to one holomorphic sector)

- On gravity side exploit CS formulation with chemical potentials

$$A = b^{-1}(d+a)b \qquad\qquad b = e^{\rho L_0}$$

$$a_z = L_+ - \frac{\mathcal{L}}{k} L_- \qquad\qquad a_{\bar{z}} = \mu L_+ + \dots$$

- Expand $\mathcal{L}(z) = \mathcal{L}^{(0)}(z) + \epsilon \mathcal{L}^{(1)}(z) + \mathcal{O}(\epsilon^2)$
- Write EOM to first subleading order in ϵ

$$\bar{\partial}\mathcal{L}^{(1)}(z) = -\frac{k}{2}\,\partial^3 \delta^{(2)}(z - z_2)$$

- Solve them using the Green function on the plane $G = \ln\left(z_{12}\bar{z}_{12}\right)$

$$\mathcal{L}^{(1)}(z) = -\frac{k}{2}\,\partial^4_{z_1} G(z_{12}) = \frac{3k}{z_{12}^4}$$

- This is the correct CFT 2-point function on the plane with $c = 6k$

2-point functions (anomalous terms)

Illustrate shortcut in $\text{AdS}_3/\text{CFT}_2$ (restrict to one holomorphic sector)

- On gravity side exploit CS formulation with chemical potentials

$$A = b^{-1}(d+a)b \qquad b = e^{\rho L_0}$$

$$a_z = L_+ - \frac{\mathcal{L}}{k} L_- \qquad a_{\bar z} = \mu L_+ + \dots$$

- Expand $\mathcal{L}(z) = \mathcal{L}^{(0)}(z) + \epsilon \mathcal{L}^{(1)}(z) + \mathcal{O}(\epsilon^2)$
- Write EOM to first subleading order in ϵ

$$\bar\partial \mathcal{L}^{(1)}(z) = -\frac{k}{2}\, \partial^3 \delta^{(2)}(z - z_2)$$

- Solve them using the Green function on the plane $G = \ln(z_{12}\bar z_{12})$

$$\mathcal{L}^{(1)}(z) = -\frac{k}{2}\, \partial_{z_1}^4 G(z_{12}) = \frac{3k}{z_{12}^4}$$

- This is the correct CFT 2-point function on the plane with $c = 6k$
- Generalize to cylinder

2-point functions (anomalous terms)

Apply shortcut to flat space/Galilean CFT (Bagchi, DG, Merbis '15)

- Exploit results for flat space gravity in CS formulation in presence of chemical potentials (Gary, DG, Riegler, Rosseel '14)

2-point functions (anomalous terms)

Apply shortcut to flat space/Galilean CFT (Bagchi, DG, Merbis '15)

- Exploit results for flat space gravity in CS formulation in presence of chemical potentials (Gary, DG, Riegler, Rosseel '14)
- Localize chemical potentials $\mu_{M/L} = \epsilon_{M/L}\,\delta^{(2)}(u - u_2,\, \varphi - \varphi_2)$

2-point functions (anomalous terms)

Apply shortcut to flat space/Galilean CFT (Bagchi, DG, Merbis '15)

- Exploit results for flat space gravity in CS formulation in presence of chemical potentials (Gary, DG, Riegler, Rosseel '14)
- Localize chemical potentials $\mu_{M/L} = \epsilon_{M/L}\,\delta^{(2)}(u - u_2,\, \varphi - \varphi_2)$
- Expand around global Minkowski space

$$M = -k/2 + M^{(1)} \qquad N = N^{(1)}$$

2-point functions (anomalous terms)

Apply shortcut to flat space/Galilean CFT (Bagchi, DG, Merbis '15)

- Exploit results for flat space gravity in CS formulation in presence of chemical potentials (Gary, DG, Riegler, Rosseel '14)
- Localize chemical potentials $\mu_{M/L} = \epsilon_{M/L}\,\delta^{(2)}(u-u_2,\,\varphi-\varphi_2)$
- Expand around global Minkowski space

$$M = -k/2 + M^{(1)} \qquad N = N^{(1)}$$

- Write EOM to first subleading order in $\epsilon_{M/L}$

$$\partial_u M^{(1)} = -\,k\,\epsilon_L\big(\partial_\varphi^3\delta + \partial_\varphi\delta\big)$$
$$\partial_u N^{(1)} = -\,k\,\epsilon_M\big(\partial_\varphi^3\delta + \partial_\varphi\delta\big) + \partial_\varphi M^{(1)}$$

2-point functions (anomalous terms)

Apply shortcut to flat space/Galilean CFT (Bagchi, DG, Merbis '15)

- Exploit results for flat space gravity in CS formulation in presence of chemical potentials (Gary, DG, Riegler, Rosseel '14)
- Localize chemical potentials $\mu_{M/L} = \epsilon_{M/L}\,\delta^{(2)}(u-u_2,\,\varphi-\varphi_2)$
- Expand around global Minkowski space

$$M = -k/2 + M^{(1)} \qquad N = N^{(1)}$$

- Write EOM to first subleading order in $\epsilon_{M/L}$

$$\partial_u M^{(1)} = -\,k\,\epsilon_L\big(\partial_\varphi^3\delta + \partial_\varphi\delta\big)$$
$$\partial_u N^{(1)} = -\,k\,\epsilon_M\big(\partial_\varphi^3\delta + \partial_\varphi\delta\big) + \partial_\varphi M^{(1)}$$

- Solve with Green function on cylinder

$$M^{(1)} = \frac{6k\epsilon_L}{s_{12}^4} \qquad N^{(1)} = \frac{6k(\epsilon_M - 2\epsilon_L\,\tau_{12})}{s_{12}^4}$$

2-point functions (anomalous terms)

Apply shortcut to flat space/Galilean CFT (Bagchi, DG, Merbis '15)

- Exploit results for flat space gravity in CS formulation in presence of chemical potentials (Gary, DG, Riegler, Rosseel '14)
- Localize chemical potentials $\mu_{M/L} = \epsilon_{M/L}\, \delta^{(2)}(u - u_2,\, \varphi - \varphi_2)$
- Expand around global Minkowski space

$$M = -k/2 + M^{(1)} \qquad N = N^{(1)}$$

- Write EOM to first subleading order in $\epsilon_{M/L}$

$$\partial_u M^{(1)} = -\, k\, \epsilon_L \big(\partial_\varphi^3 \delta + \partial_\varphi \delta\big)$$
$$\partial_u N^{(1)} = -\, k\, \epsilon_M \big(\partial_\varphi^3 \delta + \partial_\varphi \delta\big) + \partial_\varphi M^{(1)}$$

- Solve with Green function on cylinder

$$M^{(1)} = \frac{6k\epsilon_L}{s_{12}^4} \qquad N^{(1)} = \frac{6k(\epsilon_M - 2\epsilon_L\, \tau_{12})}{s_{12}^4}$$

- Correct 2-point functions for Einstein gravity with $c_L = 0$, $c_M = 12k$

3-point functions (check of symmetries)

First non-trivial check of consistency with symmetries of dual Galilean CFT

Check of 2-point functions works nicely with shortcut; 3-point too?

3-point functions (check of symmetries)

First non-trivial check of consistency with symmetries of dual Galilean CFT

Check of 2-point functions works nicely with shortcut; 3-point too?

- Yes: same procedure, but localize chemical potentials at two points

$$\mu_{M/L}(u_1,\,\varphi_1) = \sum_{i=2}^{3} \epsilon^i_{M/L}\,\delta^{(2)}(u_1 - u_i,\,\varphi_1 - \varphi_i)$$

3-point functions (check of symmetries)

First non-trivial check of consistency with symmetries of dual Galilean CFT

Check of 2-point functions works nicely with shortcut; 3-point too?

- Yes: same procedure, but localize chemical potentials at two points

$$\mu_{M/L}(u_1,\,\varphi_1) = \sum_{i=2}^{3} \epsilon^i_{M/L}\,\delta^{(2)}(u_1-u_i,\,\varphi_1-\varphi_i)$$

- Iteratively solve EOM

$$\partial_u M = -k\partial_\varphi^3\mu_L + \mu_L\partial_\varphi M + 2M\partial_\varphi\mu_L$$
$$\partial_u N = -k\partial_\varphi^3\mu_M + (1+\mu_M)\partial_\varphi M + 2M\partial_\varphi\mu_M + \mu_L\partial_\varphi N + 2N\partial_\varphi\mu_L$$

3-point functions (check of symmetries)

First non-trivial check of consistency with symmetries of dual Galilean CFT

Check of 2-point functions works nicely with shortcut; 3-point too?

- Yes: same procedure, but localize chemical potentials at two points

$$\mu_{M/L}(u_1,\,\varphi_1) = \sum_{i=2}^{3} \epsilon^i_{M/L}\,\delta^{(2)}(u_1-u_i,\,\varphi_1-\varphi_i)$$

- Iteratively solve EOM

$$\partial_u M = -k\partial_\varphi^3\mu_L + \mu_L\partial_\varphi M + 2M\partial_\varphi\mu_L$$
$$\partial_u N = -k\partial_\varphi^3\mu_M + (1+\mu_M)\partial_\varphi M + 2M\partial_\varphi\mu_M + \mu_L\partial_\varphi N + 2N\partial_\varphi\mu_L$$

- Result on gravity side matches precisely Galilean CFT results

$$\langle M^1\,N^2\,N^3\rangle = \frac{c_M}{s_{12}^2 s_{13}^2 s_{23}^2} \qquad \langle N^1\,N^2\,N^3\rangle = \frac{c_L - c_M\,\tau_{123}}{s_{12}^2 s_{13}^2 s_{23}^2}$$

provided we choose again the Einstein values $c_L = 0$ and $c_M = 12k$

4-point functions (enter cross-ratios)

First correlators with non-universal function of cross-ratios

- Repeat this algorithm, localizing the sources at three points

4-point functions (enter cross-ratios)

First correlators with non-universal function of cross-ratios

- Repeat this algorithm, localizing the sources at three points
- Derive 4-point functions for Galilean CFTs (Bagchi, DG, Merbis '15)

$$\langle M^1 \, N^2 \, N^3 \, N^4 \rangle = \frac{2c_M \, g_4(\gamma)}{s_{14}^2 s_{23}^2 s_{12} s_{13} s_{24} s_{34}}$$

$$\langle N^1 \, N^2 \, N^3 \, N^4 \rangle = \frac{2c_L \, g_4(\gamma) + c_M \Delta_4}{s_{14}^2 s_{23}^2 s_{12} s_{13} s_{24} s_{34}}$$

with the cross-ratio function

$$g_4(\gamma) = \frac{\gamma^2 - \gamma + 1}{\gamma} \qquad \gamma = \frac{s_{12} \, s_{34}}{s_{13} \, s_{24}}$$

and

$$\Delta_4 = 4g_4'(\gamma)\eta_{1234} - (\tau_{1234} + \tau_{14} + \tau_{23}) g_4(\gamma)$$

$$\eta_{1234} = \sum (-1)^{1+i-j} (u_i - u_j) \sin(\varphi_k - \varphi_l) / (s_{13}^2 s_{24}^2)$$

4-point functions (enter cross-ratios)

First correlators with non-universal function of cross-ratios

- Repeat this algorithm, localizing the sources at three points
- Derive 4-point functions for Galilean CFTs (Bagchi, DG, Merbis '15)

$$\langle M^1\, N^2\, N^3\, N^4\rangle = \frac{2c_M\, g_4(\gamma)}{s_{14}^2 s_{23}^2 s_{12} s_{13} s_{24} s_{34}}$$

$$\langle N^1\, N^2\, N^3\, N^4\rangle = \frac{2c_L\, g_4(\gamma) + c_M \Delta_4}{s_{14}^2 s_{23}^2 s_{12} s_{13} s_{24} s_{34}}$$

with the cross-ratio function

$$g_4(\gamma) = \frac{\gamma^2 - \gamma + 1}{\gamma} \qquad \gamma = \frac{s_{12}\, s_{34}}{s_{13}\, s_{24}}$$

and

$$\Delta_4 = 4g_4'(\gamma)\eta_{1234} - (\tau_{1234} + \tau_{14} + \tau_{23})g_4(\gamma)$$

$$\eta_{1234} = \sum(-1)^{1+i-j}(u_i - u_j)\sin(\varphi_k - \varphi_l)/(s_{13}^2 s_{24}^2)$$

- Gravity side yields precisely the same result!

5-point functions (further check of consistency of flat space holography)

Last new explicit correlators I am showing to you today (I promise)

- Repeat this algorithm, localizing the sources at four points

5-point functions (further check of consistency of flat space holography)

Last new explicit correlators I am showing to you today (I promise)

- Repeat this algorithm, localizing the sources at four points
- Derive 5-point functions for Galilean CFTs (Bagchi, DG, Merbis '15)

$$\langle M^1\, N^2\, N^3\, N^4\, N^5 \rangle = \frac{4c_M\, g_5(\gamma,\,\zeta)}{\prod_{1\le i<j\le 5} s_{ij}}$$

$$\langle N^1\, N^2\, N^3\, N^4\, N^5 \rangle = \frac{4c_L\, g_5(\gamma,\,\zeta) + c_M \Delta_5}{\prod_{1\le i<j\le 5} s_{ij}}$$

with the previous definitions and ($\zeta = \frac{s_{25}\, s_{34}}{s_{35}\, s_{24}}$)

$$g_5(\gamma,\,\zeta) = \frac{\gamma+\zeta}{2(\gamma-\zeta)} - \frac{(\gamma^2-\gamma\zeta+\zeta^2)}{\gamma(\gamma-1)\zeta(\zeta-1)(\gamma-\zeta)} \times \big([\gamma(\gamma-1)+1][\zeta(\zeta-1)+1]-\gamma\zeta\big)$$

$$\Delta_5 = 4\partial_\gamma g_5(\gamma,\zeta)\eta_{1234} + 4\partial_\zeta g_5(\gamma,\zeta)\eta_{2345} - 2g_5(\gamma,\zeta)\tau_{12345}$$

5-point functions (further check of consistency of flat space holography)

Last new explicit correlators I am showing to you today (I promise)

- Repeat this algorithm, localizing the sources at four points
- Derive 5-point functions for Galilean CFTs (Bagchi, DG, Merbis '15)

$$\langle M^1\, N^2\, N^3\, N^4\, N^5\rangle = \frac{4c_M\, g_5(\gamma,\, \zeta)}{\prod_{1\le i<j\le 5} s_{ij}}$$

$$\langle N^1\, N^2\, N^3\, N^4\, N^5\rangle = \frac{4c_L\, g_5(\gamma,\, \zeta) + c_M \Delta_5}{\prod_{1\le i<j\le 5} s_{ij}}$$

with the previous definitions and ($\zeta = \frac{s_{25}\, s_{34}}{s_{35}\, s_{24}}$)

$$g_5(\gamma,\, \zeta) = \frac{\gamma+\zeta}{2(\gamma-\zeta)} - \frac{(\gamma^2-\gamma\zeta+\zeta^2)}{\gamma(\gamma-1)\zeta(\zeta-1)(\gamma-\zeta)} \times \left([\gamma(\gamma-1)+1][\zeta(\zeta-1)+1]-\gamma\zeta\right)$$

$$\Delta_5 = 4\partial_\gamma g_5(\gamma,\zeta)\eta_{1234} + 4\partial_\zeta g_5(\gamma,\zeta)\eta_{2345} - 2g_5(\gamma,\zeta)\tau_{12345}$$

- Gravity side yields precisely the same result!

n-point functions (holographic Ward identities and recursion relations)

Shortcut to 42 (Bagchi, DG, Merbis '15)

Smart check of all n-point functions?

- Idea: calculate n-point function from $(n-1)$-point function

n-point functions (holographic Ward identities and recursion relations)

Shortcut to 42 (Bagchi, DG, Merbis '15)

Smart check of all n-point functions?

- Idea: calculate n-point function from $(n-1)$-point function
- Need Galilean CFT analogue of BPZ-recursion relation

$$\langle T^1\,T^2\ldots T^n\rangle = \sum_{i=2}^{n}\Big(\frac{2}{s_{1i}^2}+\frac{c_{1i}}{2}\,\partial_{\varphi_i}\Big)\langle T^2\ldots T^n\rangle + \text{disconnected}$$

n-point functions (holographic Ward identities and recursion relations)

Shortcut to 42 (Bagchi, DG, Merbis '15)

Smart check of all n-point functions?

- Idea: calculate n-point function from $(n-1)$-point function
- Need Galilean CFT analogue of BPZ-recursion relation

$$\langle T^1\,T^2\dots T^n\rangle = \sum_{i=2}^{n}\Big(\frac{2}{s_{1i}^2}+\frac{c_{1i}}{2}\,\partial_{\varphi_i}\Big)\langle T^2\dots T^n\rangle + \text{disconnected}$$

- After small derivation we find ($c_{ij} := \cot[(\varphi_i-\varphi_j)/2]$)

$$\langle M^1\,N^2\dots N^n\rangle = \sum_{i=2}^{n}\Big(\frac{2}{s_{1i}^2}+\frac{c_{1i}}{2}\partial_{\varphi_i}\Big)\,\langle M^2\,N^3\dots N^n\rangle+\text{disconnected}$$

$$\langle N^1N^2\dots N^n\rangle = \frac{c_L}{c_M}\langle M^1N^2\dots N^n\rangle + \sum_{i=1}^{n}u_i\partial_{\varphi_i}\langle M^1N^2\dots N^n\rangle$$

n-point functions (holographic Ward identities and recursion relations)

Shortcut to 42 (Bagchi, DG, Merbis '15)

Smart check of all n-point functions?

- Idea: calculate n-point function from $(n-1)$-point function
- Need Galilean CFT analogue of BPZ-recursion relation

$$\langle T^1\, T^2 \dots T^n \rangle = \sum_{i=2}^{n} \Big(\frac{2}{s_{1i}^2} + \frac{c_{1i}}{2}\, \partial_{\varphi_i} \Big) \langle T^2 \dots T^n \rangle + \text{disconnected}$$

- After small derivation we find ($c_{ij} := \cot[(\varphi_i - \varphi_j)/2]$)

$$\langle M^1\, N^2 \dots N^n \rangle = \sum_{i=2}^{n} \Big(\frac{2}{s_{1i}^2} + \frac{c_{1i}}{2} \partial_{\varphi_i} \Big) \langle M^2\, N^3 \dots N^n \rangle + \text{disconnected}$$

$$\langle N^1 N^2 \dots N^n \rangle = \frac{c_L}{c_M} \langle M^1 N^2 \dots N^n \rangle + \sum_{i=1}^{n} u_i \partial_{\varphi_i} \langle M^1 N^2 \dots N^n \rangle$$

- We can also derive same recursion relations on gravity side!

n-point functions in flat space holography
Summary

- EH action has variational principle consistent with flat space bc's (iff we add half the GHY term!)

n-point functions in flat space holography

Summary

- EH action has variational principle consistent with flat space bc's (iff we add half the GHY term!)
- 0-point function shows phase transition exists between hot flat space and flat space cosmologies

n-point functions in flat space holography

Summary

- EH action has variational principle consistent with flat space bc's (iff we add half the GHY term!)
- 0-point function shows phase transition exists between hot flat space and flat space cosmologies
- 1-point functions show consistency with canonical charges and lead to first entries in holographic dictionary

n-point functions in flat space holography
Summary

- EH action has variational principle consistent with flat space bc's (iff we add half the GHY term!)
- 0-point function shows phase transition exists between hot flat space and flat space cosmologies
- 1-point functions show consistency with canonical charges and lead to first entries in holographic dictionary
- 2-point functions consistent with Galilean CFT for $c_L = 0$, $c_M = 12k = 3/G_N$

- EH action has variational principle consistent with flat space bc's (iff we add half the GHY term!)
- 0-point function shows phase transition exists between hot flat space and flat space cosmologies
- 1-point functions show consistency with canonical charges and lead to first entries in holographic dictionary
- 2-point functions consistent with Galilean CFT for $c_L = 0$, $c_M = 12k = 3/G_N$
- 42^{nd} variation of EH action leads to 42-point Galilean CFT correlators

n-point functions in flat space holography

Summary

- EH action has variational principle consistent with flat space bc's (iff we add half the GHY term!)
- 0-point function shows phase transition exists between hot flat space and flat space cosmologies
- 1-point functions show consistency with canonical charges and lead to first entries in holographic dictionary
- 2-point functions consistent with Galilean CFT for $c_L = 0$, $c_M = 12k = 3/G_N$
- 42^{nd} variation of EH action leads to 42-point Galilean CFT correlators
- all n-point correlators of Galilean CFT reproduced precisely on gravity side (recursion relations!)

n-point functions in flat space holography
Summary

- EH action has variational principle consistent with flat space bc's (iff we add half the GHY term!)
- 0-point function shows phase transition exists between hot flat space and flat space cosmologies
- 1-point functions show consistency with canonical charges and lead to first entries in holographic dictionary
- 2-point functions consistent with Galilean CFT for $c_L = 0$, $c_M = 12k = 3/G_N$
- 42^{nd} variation of EH action leads to 42-point Galilean CFT correlators
- all n-point correlators of Galilean CFT reproduced precisely on gravity side (recursion relations!)

Fairly non-trivial check that 3D flat space holography can work!

Other selected recent results

Some further checks that dual field theory is Galilean CFT:

Other selected recent results

Some further checks that dual field theory is Galilean CFT:

- Microstate counting?

Other selected recent results

Some further checks that dual field theory is Galilean CFT:

- Microstate counting?

  Works! (Bagchi, Detournay, Fareghbal, Simon '13, Barnich '13)

$$S_{\text{gravity}} = S_{\text{BH}} = \frac{\text{Area}}{4G_N} = 2\pi h_L \sqrt{\frac{c_M}{2h_M}} = S_{\text{GCFT}}$$

  Also as limit from Cardy formula (Riegler '14, Fareghbal, Naseh '14)

Other selected recent results

Some further checks that dual field theory is Galilean CFT:

- Microstate counting?

  Works! (Bagchi, Detournay, Fareghbal, Simon '13, Barnich '13)

$$S_{\text{gravity}} = S_{\text{BH}} = \frac{\text{Area}}{4G_N} = 2\pi h_L \sqrt{\frac{c_M}{2h_M}} = S_{\text{GCFT}}$$

  Also as limit from Cardy formula (Riegler '14, Fareghbal, Naseh '14)
- (Holographic) entanglement entropy?

Other selected recent results

Some further checks that dual field theory is Galilean CFT:

- Microstate counting?

  Works! (Bagchi, Detournay, Fareghbal, Simon '13, Barnich '13)

$$S_{\text{gravity}} = S_{\text{BH}} = \frac{\text{Area}}{4G_N} = 2\pi h_L \sqrt{\frac{c_M}{2h_M}} = S_{\text{GCFT}}$$

  Also as limit from Cardy formula (Riegler '14, Fareghbal, Naseh '14)

- (Holographic) entanglement entropy?

  Works! (Bagchi, Basu, DG, Riegler '14)

$$S_{\text{EE}}^{\text{GCFT}} = \underbrace{\frac{c_L}{6}\,\ln\frac{\ell_x}{a}}_{\text{like CFT}} + \underbrace{\frac{c_M}{6}\,\frac{\ell_y}{\ell_x}}_{\text{like grav anomaly}}$$

  Calculation on gravity side confirms result above
  (using Wilson lines in CS formulation)

Outline

Recent generalizations:

- adding chemical potentials

  Works! (Gary, DG, Riegler, Rosseel '14)

  In CS formulation:

$$A_0 \to A_0 + \mu$$

Generalizations & open issues

Recent generalizations:

- adding chemical potentials
- 3-derivative theory: flat space chiral gravity (Bagchi, Detournay, DG '12)

Conformal CS gravity at level $k = 1$ with flat space boundary conditions conjectured to be dual to chiral half of monster CFT.
Action (gravity side):

$$I_{\text{CSG}} = \frac{k}{4\pi} \int \mathrm{d}^3x \sqrt{-g}\, \varepsilon^{\lambda\mu\nu}\, \Gamma^\rho{}_{\lambda\sigma}\big(\partial_\mu \Gamma^\sigma{}_{\nu\rho} + \frac{2}{3}\, \Gamma^\sigma{}_{\mu\tau}\Gamma^\tau{}_{\nu\rho}\big)$$

Partition function (field theory side, see Witten '07):

$$Z(q) = J(q) = \frac{1}{q} + 196884\, q + \mathcal{O}(q^2)$$

Note: $\ln 196883 \approx 12.2 = 4\pi +$quantum corrections

Recent generalizations:

- adding chemical potentials
- 3-derivative theory: flat space chiral gravity (Bagchi, Detournay, DG '12)
- generalization to supergravity

  Works! (Barnich, Donnay, Matulich, Troncoso '14)

  Asymptotic symmetry algebra = super-BMS$_3$

Generalizations & open issues

Recent generalizations:

- adding chemical potentials
- 3-derivative theory: flat space chiral gravity (Bagchi, Detournay, DG '12)
- generalization to supergravity
- flat space higher spin gravity

  Remarkably it exists! (Afshar, Bagchi, Fareghbal, DG, Rosseel '13; Gonzalez, Matulich, Pino, Troncoso '13)

  New type of algebra: W-like BMS ("BMW")

$$[U_n,\, U_m] = (n-m)(2n^2+2m^2-nm-8)L_{n+m} + \frac{192}{c_M}(n-m)\Lambda_{n+m} - \frac{96\big(c_L+\frac{44}{5}\big)}{c_M^2}(n-m)\Theta_{n+m} + \frac{c_L}{12}\,n(n^2-1)(n^2-4)\,\delta_{n+m,\,0}$$

$$[U_n,\, V_m] = (n-m)(2n^2+2m^2-nm-8)M_{n+m} + \frac{96}{c_M}(n-m)\Theta_{n+m} + \frac{c_M}{12}\,n(n^2-1)(n^2-4)\,\delta_{n+m,\,0}$$

$[L, L]$, $[L, M]$, $[M, M]$ as in BMS_3 $[L, U]$, $[L, V]$, $[M, U]$, $[M, V]$ as in isl(3)

Generalizations & open issues

Recent generalizations:

- adding chemical potentials
- 3-derivative theory: flat space chiral gravity (Bagchi, Detournay, DG '12)
- generalization to supergravity
- flat space higher spin gravity

Some open issues:

- Further checks in 3D (n-point correlators, partition function, ...)

  Barnich, Gonzalez, Maloney, Oblak '15: 1-loop partition function matches BMS_3 character

Generalizations & open issues

Recent generalizations:

- adding chemical potentials
- 3-derivative theory: flat space chiral gravity (Bagchi, Detournay, DG '12)
- generalization to supergravity
- flat space higher spin gravity

Some open issues:

- Further checks in 3D (n-point correlators, partition function, ...)
- Further generalizations in 3D (massive gravity, adding matter, ...)

Recent generalizations:

- adding chemical potentials
- 3-derivative theory: flat space chiral gravity (Bagchi, Detournay, DG '12)
- generalization to supergravity
- flat space higher spin gravity

Some open issues:

- Further checks in 3D (n-point correlators, partition function, ...)
- Further generalizations in 3D (massive gravity, adding matter, ...)
- Generalization to 4D? (Barnich et al, Strominger et al)

Generalizations & open issues

Recent generalizations:

- adding chemical potentials
- 3-derivative theory: flat space chiral gravity (Bagchi, Detournay, DG '12)
- generalization to supergravity
- flat space higher spin gravity

Some open issues:

- Further checks in 3D (n-point correlators, partition function, ...)
- Further generalizations in 3D (massive gravity, adding matter, ...)
- Generalization to 4D? (Barnich et al, Strominger et al)
- Flat space limit of usual $\mathrm{AdS}_5/\mathrm{CFT}_4$ correspondence?

Generalizations & open issues

Recent generalizations:

- adding chemical potentials
- 3-derivative theory: flat space chiral gravity (Bagchi, Detournay, DG '12)
- generalization to supergravity
- flat space higher spin gravity

Some open issues:

- Further checks in 3D (n-point correlators, partition function, ...)
- Further generalizations in 3D (massive gravity, adding matter, ...)
- Generalization to 4D? (Barnich et al, Strominger et al)
- Flat space limit of usual $\mathrm{AdS}_5/\mathrm{CFT}_4$ correspondence?

- holography seems to work in flat space

Generalizations & open issues

Recent generalizations:

- adding chemical potentials
- 3-derivative theory: flat space chiral gravity (Bagchi, Detournay, DG '12)
- generalization to supergravity
- flat space higher spin gravity

Some open issues:

- Further checks in 3D (n-point correlators, partition function, ...)
- Further generalizations in 3D (massive gravity, adding matter, ...)
- Generalization to 4D? (Barnich et al, Strominger et al)
- Flat space limit of usual $\text{AdS}_5/\text{CFT}_4$ correspondence?

> - holography seems to work in flat space
> - holography more general than AdS/CFT

Generalizations & open issues

Recent generalizations:

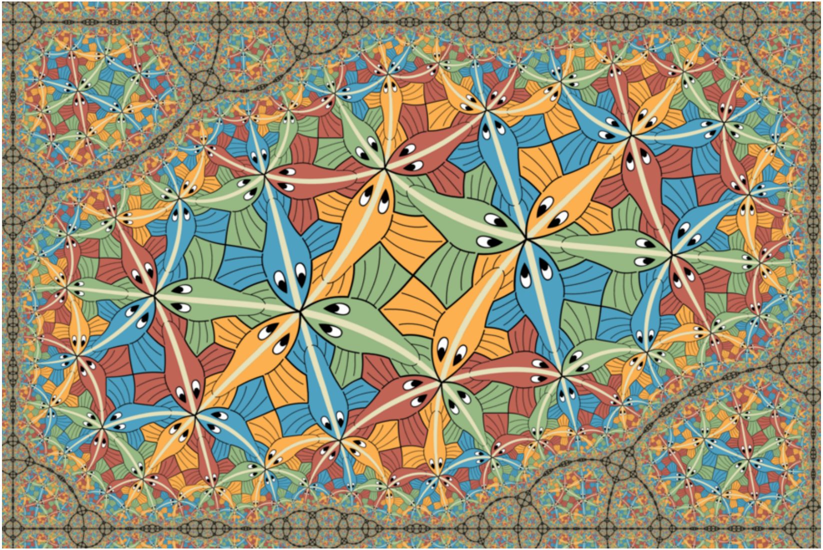
- adding chemical potentials
- 3-derivative theory: flat space chiral gravity (Bagchi, Detournay, DG '12)
- generalization to supergravity
- flat space higher spin gravity

Some open issues:

- Further checks in 3D (n-point correlators, partition function, ...)
- Further generalizations in 3D (massive gravity, adding matter, ...)
- Generalization to 4D? (Barnich et al, Strominger et al)
- Flat space limit of usual $\text{AdS}_5/\text{CFT}_4$ correspondence?

- holography seems to work in flat space
- holography more general than AdS/CFT
- (when) does it work even more generally?

Thanks for your attention!



Vladimir Bulatov, M.C.Escher Circle Limit III in a rectangle