Hypersymmetric black holes in 2+1 gravity

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Based on joint work with A. Pérez, D. Tempo, R. Troncoso (2015)

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- Conclusions

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Note : one has also $sl(2,\mathbb{R}) \simeq sp(2,\mathbb{R}) \simeq su(1,1)$ and thus the chain of isomorphisms $so(2,1) \simeq sl(2,\mathbb{R}) \simeq sp(2,\mathbb{R}) \simeq su(1,1)$

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- The parameter k is related to the (2+1)-dimensional Newton constant G as $k = \ell/4G$, where ℓ is the AdS radius of curvature.

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in terms of which one finds indeed

$$I[e,\omega] = \frac{1}{8\pi G}\int_M d^3x \left(\frac{1}{2} eR + \frac{e}{\ell^2}\right)$$

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Note that the Chern-Simons gauge transformations enable one to go to gauges where the triad is degenerate.

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The first condition ensures that the theory contains gravity and only bosonic fields of "spins" 2 and 1 (and a single "graviton"). The second condition ensures that spinors are spin-$\frac{3}{2}$ fields.

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One then considers general (super)algebras containing the gravitational subalgebra $sl(2,\mathbb{R})$, but with their bosonic subalgebra not necessarily of the form $sl(2,\mathbb{R}) \oplus \mathcal{G}$.

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Assuming principal embedding of $sl(2,\mathbb{R})$ in $sp(4)$, one gets one spin-2 field, one spin-4 field and one spin-$\frac{5}{2}$ field.

The spin-4 field decouples in the limit of zero cosmological constant, where one gets the theory of Aragone and Deser (1984).

Some conventions

Basis of $osp(1|4)$:

$$\left[L_i, L_j\right] = (i-j)\, L_{i+j}\,,$$
$$\left[L_i, U_m\right] = (3i-m)\, U_{i+m}\,,$$
$$\left[L_i, \mathscr{S}_p\right] = \left(\frac{3}{2}i - p\right) \mathscr{S}_{i+p}\,,$$
$$\left[U_m, U_n\right] = \frac{1}{2^2 3}(m-n)\left(\left(m^2+n^2-4\right)\left(m^2+n^2-\frac{2}{3}mn-9\right)-\frac{2}{3}(mn-6)\,mn\right)L_{m+n} + \frac{1}{6}(m-n)\left(m^2-mn+n^2-7\right)U_{m+n}\,,$$
$$\left[U_m, \mathscr{S}_p\right] = \frac{1}{2^3 3}\left(2m^3-8m^2p+20mp^2+82p-23m-40p^3\right)\mathscr{S}_{i+p}\,,$$
$$\left\{\mathscr{S}_p, \mathscr{S}_q\right\} = U_{p+q} + \frac{1}{2^2 3}\left(6p^2-8pq+6q^2-9\right)L_{p+q}\,.$$

Here L_i, with $i = 0, \pm 1$, stand for the spin-2 generators that span the gravitational $sl(2,\mathbb{R})$ subalgebra, while U_m and $\mathscr{S}_p$, with $m = 0, \pm1, \pm2, \pm3$ and $p = \pm\frac{1}{2}, \pm\frac{3}{2}$, correspond to the spin-4 and fermionic spin-$\frac{5}{2}$ generators, respectively.

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$$A^{+} = A^{i}_{\mu}L_i + B^{m}_{\mu}U_m + \psi^{p}_{\mu}\mathscr{S}_p$$

and a similar expression holds for A^{-}.

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These interaction terms are not known in closed form. They can be constructed perturbatively.

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We shall come back to that question later.

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Coussaert, Henneaux, van Driel 1995

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The functions $\epsilon_\pm(\varphi)$ are arbitrary functions of φ and parametrize the asymptotic symmetries.

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Furthermore, one easily finds that the generators of the asymptotic symmetry algebra are given explictly by the $\mathscr{L}^{\pm}$'s themselves (when the constraints hold) and read explicitly

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Thus, the Virasoro algebra emerges in the reduction procedure enforced by the AdS boundary conditions.

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Sugra : Henneaux, Maoz, Schwimmer (2000) ;
Higher spins : S.-J. Rey + MH (2010) ; A. Campoleoni, S. Fredenhagen, S. Pfenninger, S. Theisen (2010)

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with $\epsilon_\pm$, $\chi_\pm$ and $\vartheta_\pm$ arbitrary functions of φ. The “charges" $\mathscr{L}^\pm(\varphi)$, $\mathscr{U}^\pm(\varphi)$ and $\psi^\pm(\varphi)$ form the $W_{(2,\frac{5}{2},4)}$-superalgebra.

$W_{(2,\frac{5}{2},4)}$-superalgebra

$$i[L_m, L_n]_{PB} = (m-n)\, L_{m+n} + \frac{k}{2} m^3 \delta^0_{m+n},$$

$$i[L_m, U_n]_{PB} = (3m-n)\, U_{m+n},$$

$$i[L_m, \psi_n]_{PB} = \left(\frac{3}{2}m - n\right)\psi_{m+n},$$

$$\begin{aligned} i[U_m, U_n]_{PB} = {} & \frac{1}{2^2 3^2}(m-n)\left(3m^4 - 2m^3 n + 4m^2 n^2 - 2mn^3 + 3n^4\right) L_{m+n} \\ & + \frac{1}{6}(m-n)\left(m^2 - mn + n^2\right)\mathscr{U}_{m+n} - \frac{2^3 3\pi}{k}(m-n)\,\Lambda^{(6)}_{m+n} \\ & - \frac{7^2\pi}{3^2 k}(m-n)\left(m^2 + 4mn + n^2\right)\Lambda^{(4)}_{m+n} + \frac{k}{2^3 3^2} m^7 \delta^0_{m+n}, \end{aligned}$$

$$\begin{aligned} i[U_m, \psi_n]_{PB} = {} & \frac{1}{2^2 3}\left(m^3 - 4m^2 n + 10mn^2 - 20n^3\right)\psi_{m+n} - \frac{23\pi}{3k} i\Lambda^{(11/2)}_{m+n} \\ & + \frac{\pi}{3k}(23m - 82n)\,\Lambda^{(9/2)}_{m+n}, \end{aligned}$$

$$i[\psi_m, \psi_n]_{PB} = U_{m+n} + \frac{1}{2}\left(m^2 - \frac{4}{3}mn + n^2\right) L_{m+n} + \frac{3\pi}{k}\Lambda^{(4)}_{m+n} + \frac{k}{6} m^4 \delta^0_{m+n},$$

The generators $\mathscr{U}_n$, ψ_n have respective conformal weights 4 and $\frac{5}{2}$; unchanged central charge $c = 6k = \frac{3\ell}{2G}$ (two copies).

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(The quantum $W(2,5/2,4)$-superalgebra, with the unitarity conditions $L_m^\dagger = L_{-m}$, $U_m^\dagger = U_{-m}$, $\psi_m^\dagger = \psi_{-m}$ implied by the classical reality conditions, admits arbitrarily large values of the central charge.)

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This is a nonlinear bound.

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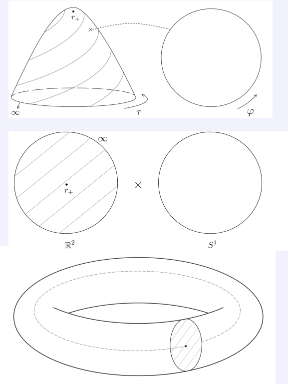

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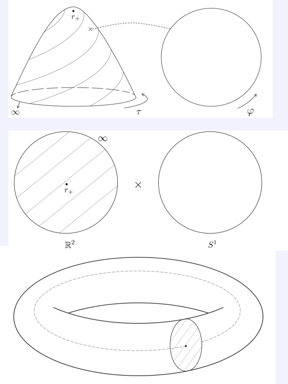

The topology of the Euclidean black hole is a solid torus, $\mathbb{R}^2 \times S^1$. The "Euclidean horizon" r_+ is the origin of a system of polar coordinates r, τ in $\mathbb{R}^2$. The Euclidean time τ is the polar angle. On the other hand, the S^1 is parametrized by the angle φ.

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This approach is crucial when higher spins are included, where there is no well-defined geometry.

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Such solutions *exist*.

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The component A_τ along Euclidean time can be determined from the equations of motion and the boundary conditions, and involve two complex functions, ξ and μ (“chemical potentials").

The regularity condition (absence of conical singularity at the origin) determines ξ and μ in terms of the charges $\mathscr{L}$, $\mathscr{U}$.

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The black holes that saturate this bound are extremal and hypersymmetric (possess Killing vector-spinors).

Conclusions and comments

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One should consider more complete models that include supersymmetry, higher spin hypersymmetry. Perhaps one must go all the way to an infinite number of spins...