Integrability and the Conformal Field Theory of the Higgs branch

Bogdan Stefański, jr.
City University London

17 November 2015

Based on 1411.3676, JHEP **1506** (2014) 103 with O. Ohlsson Sax, A. Sfondrini

AdS_3 with $8+8$ susys and integrability

- String theory on $AdS_3 \times S^3 \times M_4$ where $M_4 = \begin{cases} T^4 \\ S^3 \times S^1 \end{cases}$

AdS_3 with $8+8$ susys and integrability

- String theory on $\mathrm{AdS}_3 \times \mathrm{S}^3 \times M_4$ where $M_4 = \begin{cases} \mathrm{T}^4 \\ \mathrm{S}^3 \times \mathrm{S}^1 \end{cases}$

  supported by R-R $\oplus$ NS-NS 3-form flux

AdS_3 with $8+8$ susys and integrability

- String theory on $AdS_3 \times S^3 \times M_4$ where $M_4 = \begin{cases} T^4 \\ S^3 \times S^1 \end{cases}$

  supported by R-R $\oplus$ NS-NS 3-form flux

- All-loop integrable wsheet 2-body S matrix known

  [Borsato, Ohlsson Sax, Lloyd, Sfondrini, BS, Torrielli]

AdS_3 with $8+8$ susys and integrability

- String theory on $AdS_3 \times S^3 \times M_4$ where $M_4 = \begin{cases} T^4 \\ S^3 \times S^1 \end{cases}$

  supported by R-R $\oplus$ NS-NS 3-form flux

- All-loop integrable wsheet 2-body S matrix known

  [Borsato, Ohlsson Sax, Lloyd, Sfondrini, BS, Torrielli]

- **expectation**: integrability solves spectral problem
  (from string side)

[Abbott, Aniceto, Babichenko, Bianchi, Beccaria, David, Dekel, Engelund, Hernandez, Hoare, Levkovich-Maslyuk, Macorini, McKeown, Nieto, Pittelli, Prinsloo, Regelskis, Roiban, Sahoo, Stepanchuk, Sundin, Tseytlin, Wolf, Wulff, Zarembo]

AdS_3 with $8+8$ susys and integrability

- String theory on $\mathrm{AdS}_3 \times \mathrm{S}^3 \times M_4$ where $M_4 = \begin{cases} \mathrm{T}^4 \\ \mathrm{S}^3 \times \mathrm{S}^1 \end{cases}$

  supported by R-R $\oplus$ NS-NS 3-form flux

- All-loop integrable wsheet 2-body S matrix known

  [Borsato, Ohlsson Sax, Lloyd, Sfondrini, BS, Torrielli]

- **expectation**: integrability solves spectral problem (from string side)

[Abbott, Aniceto, Babichenko, Bianchi, Beccaria, David, Dekel, Engelund, Hernandez, Hoare, Levkovich-Maslyuk, Macorini, McKeown, Nieto, Pittelli, Prinsloo, Regelskis, Roiban, Sahoo, Stepanchuk, Sundin, Tseytlin, Wolf, Wulff, Zarembo]

- This talk focuses on pure R-R, $M_4 = T^4$ case
  Global symmetry is $psu(1,1|2)^2$

CFT_2 integrability: expectations from AdS_3

- $\lambda \longrightarrow 0$ limit gives local spin-chain [Ohlsson Sax, BS, Torrielli '11]

CFT_2 integrability: expectations from AdS_3

- $\lambda \longrightarrow 0$ limit gives local spin-chain [Ohlsson Sax, BS, Torrielli '11]

- Sites of spin-chain $\mathrm{L} \otimes \mathrm{R}$

CFT_2 integrability: expectations from AdS_3

- $\lambda \longrightarrow 0$ limit gives local spin-chain [Ohlsson Sax, BS, Torrielli '11]

- Sites of spin-chain $\text{L} \otimes \text{R}$

$$\text{L sites} \; : \qquad \begin{cases} S_b \\ 0^{\oplus 2} \oplus S_f \end{cases}$$

S_b/S_f $\frac{1}{2}$-BPS irrep of $psu(1,1|2)$ with bos/ferm h.w. state

CFT_2 integrability: expectations from AdS_3

- $\lambda \longrightarrow 0$ limit gives local spin-chain [Ohlsson Sax, BS, Torrielli '11]

- Sites of spin-chain $\mathrm{L} \otimes \mathrm{R}$

$$\text{L sites} \; : \qquad \begin{cases} S_b \\ 0^{\oplus 2} \oplus S_f \end{cases}$$

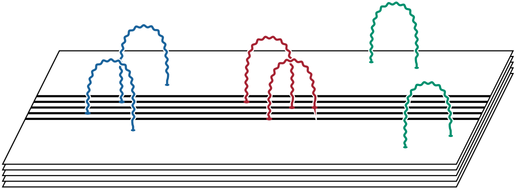
S_b/S_f $\frac{1}{2}$-BPS irrep of $psu(1,1|2)$ with bos/ferm h.w. state

- R sites same as L sites

Outline

1. UV gauge theory
   - D1/D5 system, $\mathcal{L}_{\mathrm{UV}}$
   - Coulomb and **Higgs** branches in UV and IR
2. CFT_H
   - $\mathcal{L}_{IR}$
   - ADHM σ-model and small instantons
   - origin of the Higgs branch, $\mathcal{L}_{\mathrm{eff}}$
3. Δ from $\mathcal{L}_{\mathrm{eff}}$ and spin-chains
4. Outlook and Conclusions

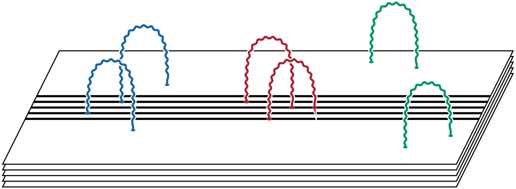
UV gauge theory: D1-D5 system



- D1-D5 branes

	0	1	2	3	4	5	6	7	8	9
$N_c \times$ D1	•	•								
$N_f \times$ D5	•	•					•	•	•	•

UV gauge theory: D1-D5 system

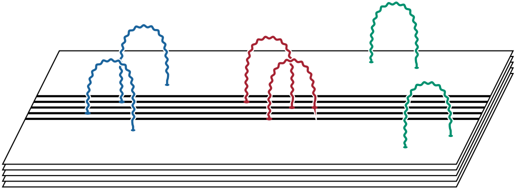


- D1-D5 branes

	0	1	2	3	4	5	6	7	8	9
$N_c \times$ D1	●	●								
$N_f \times$ D5	●	●					●	●	●	●

(columns 2–5) R-symmetry: $su(2)_L \times su(2)_R$

UV gauge theory: D1-D5 system

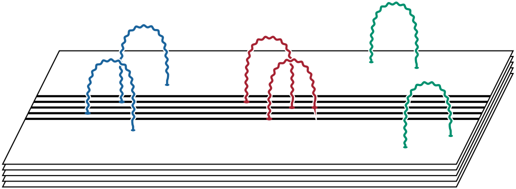


- D1-D5 branes

	0	1	2	3	4	5	6	7	8	9
$N_c \times$ D1	•	•								
$N_f \times$ D5	•	•					•	•	•	•

- D1: $(8,8)$ susy $U(N_c)$ vector mplet - dim. red. of $\mathcal{N}=4$ SYM

UV gauge theory: D1-D5 system



- D1-D5 branes

	0	1	2	3	4	5	6	7	8	9
$N_c \times$ D1	•	•								
$N_f \times$ D5	•	•					•	•	•	•

- D1: $(8, 8)$ susy $U(N_c)$ vector mplet - dim. red. of $\mathcal{N} = 4$ SYM
- D5: break susy to $(4, 4)$

UV gauge theory: $(4,4)$ susy 2d QCD

Open-string low-energy dofs are gluons A_μ, quarks λ and (4,4) susy

UV gauge theory: $(4,4)$ susy 2d QCD

Open-string low-energy dofs are gluons A_μ, quarks λ and (4,4) susy

- D1-D1 strings $\longleftrightarrow$ $(8,8)$ $U(N_c)$ vector-multiplet:

  $(4,4)$ vector Φ: $\quad \phi^i\,,\ \psi\,,\ A_\mu,\ D$

  $(4,4)$ hyper T: $\quad t^a,\ \chi$

UV gauge theory: $(4,4)$ susy 2d QCD

Open-string low-energy dofs are gluons A_μ, quarks λ and (4,4) susy

- D1-D1 strings $\longleftrightarrow$ $(8,8)$ $U(N_c)$ vector-multiplet:

  $(4,4)$ vector Φ: ϕ^i , ψ , A_μ, D

  $(4,4)$ hyper T: t^a, χ

- D1-D5 strings $\longleftrightarrow$ $(4,4)$ $U(N_c) \times U(N_f)$ hyper-multiplets:

  $(4,4)$ hyper H: h^a, λ

UV gauge theory: $(4,4)$ susy 2d QCD

Open-string low-energy dofs are gluons A_μ, quarks λ and (4,4) susy

- D1-D1 strings $\longleftrightarrow$ $(8,8)$ $U(N_c)$ vector-multiplet:

  $(4,4)$ vector Φ: ϕ^i , ψ , A_μ, D

  $(4,4)$ hyper T: t^a, χ

- D1-D5 strings $\longleftrightarrow$ $(4,4)$ $U(N_c) \times U(N_f)$ hyper-multiplets:

  $(4,4)$ hyper H: h^a, λ

- D5-D5 strings decouple: suppressed by large V_{6789}

UV gauge theory: $\mathcal{L}_{\mathsf{UV}}$ fixed by susy

$$\mathcal{L}_{\mathsf{UV}} = \frac{1}{g_{\mathsf{YM}}^2}\mathcal{L}_{\Phi}(\Phi) + \mathcal{L}_T(T,\Phi) + \mathcal{L}_H(H,\Phi)$$

UV gauge theory: $\mathcal{L}_{\mathsf{UV}}$ fixed by susy

$$\mathcal{L}_{\mathsf{UV}} = \underbrace{\frac{1}{g_{\mathsf{YM}}^2}\mathcal{L}_{\Phi}(\Phi) + \mathcal{L}_T(T,\Phi)}_{\text{dim. red. of } \mathcal{L}_{\mathcal{N}=4\ \mathsf{SYM}}} + \mathcal{L}_H(H,\Phi)$$

UV gauge theory: $\mathcal{L}_{\mathrm{UV}}$ fixed by susy

$$\mathcal{L}_{\mathrm{UV}} = \frac{1}{g_{\mathrm{YM}}^2}\mathcal{L}_{\Phi}(\Phi) + \mathcal{L}_T(T,\Phi) + \mathcal{L}_H(H,\Phi)$$

where

$$\mathcal{L}_{\Phi}(\Phi) = \mathrm{tr}\left[F^2 + (\nabla\phi)^2 + i\bar{\psi}\nabla\psi + D^2 + \ldots\right],$$

$$\mathcal{L}_T(T,\Phi) = \mathrm{tr}\left[\nabla t^2 + i\bar{\chi}\nabla\chi + \ldots\right],$$

$$\mathcal{L}_H(H,\Phi) = \nabla h^2 + i\bar{\lambda}\nabla\lambda + h^a\phi^i\phi^i h^a + \bar{\lambda}\Gamma^i\phi^i\lambda + \ldots,$$

UV gauge theory: $\mathcal{L}_{\mathrm{UV}}$ fixed by susy

$$\mathcal{L}_{\mathrm{UV}} = \frac{1}{g_{\mathrm{YM}}^2}\mathcal{L}_{\Phi}(\Phi) + \mathcal{L}_T(T,\Phi) + \mathcal{L}_H(H,\Phi)$$

- No $H - T$ couplings

UV gauge theory: $\mathcal{L}_{\text{UV}}$ fixed by susy

$$\mathcal{L}_{\text{UV}} = \frac{1}{g_{\text{YM}}^2}\mathcal{L}_{\Phi}(\Phi) + \mathcal{L}_T(T,\Phi) + \mathcal{L}_H(H,\Phi)$$

- No $H - T$ couplings
- $\mathcal{L}_{\text{UV}}$ has two branches of susy vacua:
  - Coulomb branch: D1 move away from D5
  - Higgs branch: D1 move/dissolve inside D5

UV gauge theory: $\mathcal{L}_{\text{UV}}$ fixed by susy

$$\mathcal{L}_{\text{UV}} = \frac{1}{g_{\text{YM}}^2}\mathcal{L}_\Phi(\Phi) + \mathcal{L}_T(T,\Phi) + \mathcal{L}_H(H,\Phi)$$

- No $H - T$ couplings
- $\mathcal{L}_{\text{UV}}$ has two branches of susy vacua:
  - Coulomb branch: D1 move away from D5
  - Higgs branch: D1 move/dissolve inside D5
- g_{YM} dimensionful: theory flows to CFT in IR

UV gauge theory: $\mathcal{L}_{\mathsf{UV}}$ fixed by susy

$$\mathcal{L}_{\mathsf{UV}} = \frac{1}{g_{\mathsf{YM}}^2}\mathcal{L}_{\Phi}(\Phi) + \mathcal{L}_T(T,\Phi) + \mathcal{L}_H(H,\Phi)$$

- No $H - T$ couplings
- $\mathcal{L}_{\mathsf{UV}}$ has two branches of susy vacua:
  - Coulomb branch: D1 move away from D5
  - Higgs branch: D1 move/dissolve inside D5
- g_{YM} dimensionful: theory flows to CFT in IR
- IR CFT $=$ CFT$_{\mathsf{C}} \oplus$ CFT$_{\mathsf{H}}$ [Witten '95, '97]

UV gauge theory: $\mathcal{L}_{\text{UV}}$ fixed by susy

$$\mathcal{L}_{\text{UV}} = \frac{1}{g_{\text{YM}}^2}\mathcal{L}_\Phi(\Phi) + \mathcal{L}_T(T,\Phi) + \mathcal{L}_H(H,\Phi)$$

- No $H - T$ couplings
- $\mathcal{L}_{\text{UV}}$ has two branches of susy vacua:
  - Coulomb branch: D1 move away from D5
  - Higgs branch: D1 move/dissolve inside D5
- g_{YM} dimensionful: theory flows to CFT in IR
- IR CFT $= \text{CFT}_\text{C} \oplus \text{CFT}_\text{H}$ [Witten '95, '97]
- CFT_H dual to AdS_3 [Maldacena '97]

CFT$_\mathrm{H}$: $\mathcal{L}_\mathrm{IR}$

- In IR $g_\mathrm{YM} \to \infty$ so $\mathcal{L}_\Phi$ irrelevant and can be dropped [Witten '97]

$$\mathcal{L}_\mathrm{IR}(\Phi, T, H) = \mathcal{L}_T(\Phi, T) + \mathcal{L}_H(\Phi, H)$$

CFT$_\mathsf{H}$: $\mathcal{L}_{\mathrm{IR}}$

- In IR $g_{\mathrm{YM}} \to \infty$ so $\mathcal{L}_{\Phi}$ irrelevant and can be dropped [Witten '97]

$$\mathcal{L}_{\mathrm{IR}}(\Phi, T, H) = \mathcal{L}_T(\Phi, T) + \mathcal{L}_H(\Phi, H)$$

- $\mathcal{L}_{\mathrm{IR}}$ marginal if Φ has **geometric** dimensions

$$[A_\mu] = 1, \qquad [\phi^i] = 1, \qquad [\Psi] = 3/2, \qquad [D] = 2$$

while H and T have canonical scaling dimensions

$$[h^a] = [t^a] = 0, \qquad [\chi] = [\lambda] = 1/2$$

CFT_H: $\mathcal{L}_{\text{IR}}$

- In IR $g_{\text{YM}} \to \infty$ so $\mathcal{L}_\Phi$ irrelevant and can be dropped [Witten '97]

$$\mathcal{L}_{\text{IR}}(\Phi, T, H) = \mathcal{L}_T(\Phi, T) + \mathcal{L}_H(\Phi, H)$$

- $\mathcal{L}_{\text{IR}}$ marginal if Φ has **geometric** dimensions

$$[A_\mu] = 1, \qquad [\phi^i] = 1, \qquad [\Psi] = 3/2, \qquad [D] = 2$$

while H and T have canonical scaling dimensions

$$[h^a] = [t^a] = 0, \qquad [\chi] = [\lambda] = 1/2$$

- Φ enter quadratically as auxiliary fields in $\mathcal{L}_{\text{IR}}$

CFT_H: $\mathcal{L}_{\text{ADHM}}$

- Φ auxiliary: eliminate using eoms

[Witten 97] [Berkooz, Verlinde '99] [Aharony, Berkooz '99]

$$\mathcal{L}_{\text{IR}}(\Phi, T, H) \longrightarrow \mathcal{L}_{\text{ADHM}}(H, T)$$

CFT_H: $\mathcal{L}_{\text{ADHM}}$

- Φ auxiliary: eliminate using eoms

[Witten 97] [Berkooz, Verlinde '99] [Aharony, Berkooz '99]

$$\mathcal{L}_{\text{IR}}(\Phi, T, H) \longrightarrow \mathcal{L}_{\text{ADHM}}(H, T)$$

- $\mathcal{L}_{\text{ADHM}}$ is $(4,4)$ σ-model with target space

$$\mathcal{M}_{N_c, N_f}$$

the moduli space of N_c instantons in $su(N_f)$ gauge theory

CFT$_{\mathrm{H}}$: $\mathcal{L}_{\mathrm{ADHM}}$

- Φ auxiliary: eliminate using eoms

[Witten 97] [Berkooz, Verlinde '99] [Aharony, Berkooz '99]

$$\mathcal{L}_{\mathrm{IR}}(\Phi, T, H) \longrightarrow \mathcal{L}_{\mathrm{ADHM}}(H, T)$$

- $\mathcal{L}_{\mathrm{ADHM}}$ is $(4,4)$ σ-model with target space

$$\mathcal{M}_{N_c,N_f}$$

the moduli space of N_c instantons in $su(N_f)$ gauge theory

- $\mathcal{L}_{\mathrm{ADHM}}$ gives conventional picture of Higgs branch:
  D- and F-flatness conditions equivalent to ADHM construction

CFT_H: $\mathcal{L}_{\text{ADHM}}$

- Φ auxiliary: eliminate using eoms

[Witten 97] [Berkooz, Verlinde '99] [Aharony, Berkooz '99]

$$\mathcal{L}_{\text{IR}}(\Phi, T, H) \longrightarrow \mathcal{L}_{\text{ADHM}}(H, T)$$

- $\mathcal{L}_{\text{ADHM}}$ is $(4,4)$ σ-model with target space

$$\mathcal{M}_{N_c, N_f}$$

the moduli space of N_c instantons in $su(N_f)$ gauge theory

- $\mathcal{L}_{\text{ADHM}}$ gives conventional picture of Higgs branch:
  D- and F-flatness conditions equivalent to ADHM construction

- $\mathcal{L}_{\text{ADHM}}$ has **small instanton** singularity:
  Metric on $\mathcal{M}_{N_c, N_f}$ singular when instanton size goes to zero

CFT_H: states near origin

- CFT_H states localised near origin of Higgs branch (near small instanton singularity) not captured by $\mathcal{L}_{\text{ADHM}}$ σ-model

$\mathrm{CFT_H}$: states near origin

- $\mathrm{CFT_H}$ states localised near origin of Higgs branch (near small instanton singularity) not captured by $\mathcal{L}_{\mathrm{ADHM}}$ σ-model

- For such states integrate out H [Witten 97] [Aharony, Berkooz 99]

$$\mathcal{L}_{\mathrm{IR}}(\Phi, T, H) \longrightarrow N_f \mathcal{L}_{\mathrm{eff}}(\Phi) + \mathcal{L}_T(T, \Phi)$$

$\mathrm{CFT_H}$: states near origin

- $\mathrm{CFT_H}$ states localised near origin of Higgs branch (near small instanton singularity) not captured by $\mathcal{L}_{\mathrm{ADHM}}$ σ-model

- For such states integrate out H [Witten 97] [Aharony, Berkooz 99]

$$\mathcal{L}_{\mathrm{IR}}(\Phi, T, H) \longrightarrow N_f \mathcal{L}_{\mathrm{eff}}(\Phi) + \mathcal{L}_T(T, \Phi)$$

- This "re-animates" Φ

$\mathrm{CFT_H}$: states near origin

- $\mathrm{CFT_H}$ states localised near origin of Higgs branch (near small instanton singularity) not captured by $\mathcal{L}_{\mathrm{ADHM}}$ σ-model
- For such states integrate out H [Witten 97] [Aharony, Berkooz 99]

$$\mathcal{L}_{\mathrm{IR}}(\Phi, T, H) \longrightarrow N_f \mathcal{L}_{\mathrm{eff}}(\Phi) + \mathcal{L}_T(T, \Phi)$$

- This "re-animates" Φ
- N_f factor comes from N_f copies of H

CFT_H: states near origin

- CFT_H states localised near origin of Higgs branch (near small instanton singularity) not captured by $\mathcal{L}_{\text{ADHM}}$ σ-model

- For such states integrate out H [Witten 97] [Aharony, Berkooz 99]

$$\mathcal{L}_{\text{IR}}(\Phi, T, H) \longrightarrow N_f \mathcal{L}_{\text{eff}}(\Phi) + \mathcal{L}_T(T, \Phi)$$

- This "re-animates" Φ

- N_f factor comes from N_f copies of H

- $\mathcal{L}_T$ unaffected since no $T - H$ couplings

$\mathrm{CFT_H}$: $\mathcal{L}_{\text{eff}}$

$$\int \mathcal{D}\Phi\, \mathcal{D}T\, \mathcal{D}H\ e^{i\int \mathcal{L}_{\text{IR}}} = \int \mathcal{D}\Phi\, \mathcal{D}T\ e^{i\int N_f \mathcal{L}_{\text{eff}}(\Phi) + \mathcal{L}_T(\Phi, T)}$$

$\mathrm{CFT_H}$: $\mathcal{L}_{\mathrm{eff}}$

$$\int \mathcal{D}\Phi\, \mathcal{D}T\, \mathcal{D}H\; e^{i\int \mathcal{L}_{\mathrm{IR}}} = \int \mathcal{D}\Phi\, \mathcal{D}T\; e^{i\int N_f \mathcal{L}_{\mathrm{eff}}(\Phi) + \mathcal{L}_T(\Phi, T)}$$

- Φ becomes dynamical. 2pt fn fixed by conformal invariance

$$\phi^i(x)\phi^j(0) \sim \frac{\delta^{ij}}{|x|^2}$$

CFT$_H$: $\mathcal{L}_{\rm eff}$

$$\int \mathcal{D}\Phi\, \mathcal{D}T\, \mathcal{D}H\; e^{i\int \mathcal{L}_{\rm IR}} = \int \mathcal{D}\Phi\, \mathcal{D}T\; e^{i\int N_f \mathcal{L}_{\rm eff}(\Phi)+\mathcal{L}_T(\Phi,T)}$$

- Φ becomes dynamical. 2pt fn fixed by conformal invariance

$$\phi^i(x)\phi^j(0) \sim \frac{\delta^{ij}}{|x|^2}$$

- $\mathcal{L}_{\rm eff}$ interactions follow from $\mathcal{L}_H$ interactions and integrating out

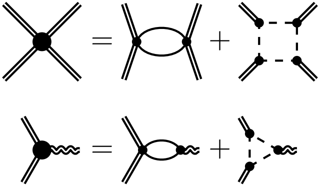
CFT$_\mathrm{H}$: $\mathcal{L}_{\text{eff}}$

$$\int \mathcal{D}\Phi\, \mathcal{D}T\, \mathcal{D}H\; e^{i\int \mathcal{L}_{\text{IR}}} = \int \mathcal{D}\Phi\, \mathcal{D}T\; e^{i\int N_f \mathcal{L}_{\text{eff}}(\Phi) + \mathcal{L}_T(\Phi, T)}$$

- Φ becomes dynamical. 2pt fn fixed by conformal invariance

$$\phi^i(x)\phi^j(0) \sim \frac{\delta^{ij}}{|x|^2}$$

- $\mathcal{L}_{\text{eff}}$ interactions follow from $\mathcal{L}_H$ interactions and integrating out



On rhs all interactions come from $\mathcal{L}_H$

$$\mathcal{L}_H \sim \nabla h^2 + i\bar{\lambda}\nabla\lambda + h^a\phi^i\phi^j h^a + \bar{\lambda}\phi^i\Gamma^i\lambda + \ldots$$

CFT$_H$: $\mathcal{L}_{\text{eff}}$

$$\int \mathcal{D}\Phi\, \mathcal{D}T\, \mathcal{D}H\; e^{i\int \mathcal{L}_{\text{IR}}} = \int \mathcal{D}\Phi\, \mathcal{D}T\; e^{i\int N_f \mathcal{L}_{\text{eff}}(\Phi) + \mathcal{L}_T(\Phi, T)}$$

- Φ becomes dynamical. 2pt fn fixed by conformal invariance

$$\phi^i(x)\phi^j(0) \sim \frac{\delta^{ij}}{|x|^2}$$

- $\mathcal{L}_{\text{eff}}$ interactions follow from $\mathcal{L}_H$ interactions and integrating out

- Rescaling

$$\Phi \longrightarrow \frac{1}{\sqrt{N_f}}\Phi$$

get $\frac{1}{N_f}$ as coupling constant.

$\mathrm{CFT_H}$: states near origin

- Gauge invariant states built from adjoint fields Φ and T

$\mathrm{CFT_H}$: states near origin

- Gauge invariant states built from adjoint fields Φ and T
- $N_c \to \infty$: perturbation series becomes 't Hooft expansion in

$$\lambda \equiv \frac{N_c}{N_f} \qquad \text{and} \qquad \frac{1}{N_c^2}$$

$\mathrm{CFT_H}$: states near origin

- Gauge invariant states built from adjoint fields Φ and T
- $N_c \to \infty$: perturbation series becomes 't Hooft expansion in

$$\lambda \equiv \frac{N_c}{N_f} \qquad \text{and} \qquad \frac{1}{N_c^2}$$

- In this limit single-trace ops dominate

$$\mathrm{tr}(\phi^{i_1}\phi^{i_2}\ldots\phi^{i_L}), \quad \mathrm{tr}(\nabla_+\psi F\phi^{i_1}\ldots\phi^{i_{L-2}}), \quad \mathrm{tr}(\chi\nabla_- t\phi^{i_1}\ldots\phi^{i_{L-2}})$$

$\mathrm{CFT_H}$: states near origin

- Gauge invariant states built from adjoint fields Φ and T
- $N_c \to \infty$: perturbation series becomes 't Hooft expansion in

$$\lambda \equiv \frac{N_c}{N_f} \qquad \text{and} \qquad \frac{1}{N_c^2}$$

- In this limit single-trace ops dominate

$$\mathrm{tr}(\phi^{i_1}\phi^{i_2}\ldots\phi^{i_L})\,, \quad \mathrm{tr}(\nabla_+\psi F\phi^{i_1}\ldots\phi^{i_{L-2}})\,, \quad \mathrm{tr}(\chi\nabla_- t\phi^{i_1}\ldots\phi^{i_{L-2}})$$

- Such operators correspond to spin-chains with sites
  - Φ fields: $(S_b)_L \otimes (S_b)_R$
  - T fields: $0^{\oplus 4} \oplus \left(0^{\oplus 2} \otimes (S_f)_L\right) \oplus \left(0^{\oplus 2} \otimes (S_f)_R\right)$

$\mathrm{CFT_H}$: states near origin

- Gauge invariant states built from adjoint fields Φ and T
- $N_c \to \infty$: perturbation series becomes 't Hooft expansion in

$$\lambda \equiv \frac{N_c}{N_f} \qquad \text{and} \qquad \frac{1}{N_c^2}$$

- In this limit single-trace ops dominate

$$\mathrm{tr}(\phi^{i_1}\phi^{i_2}\dots\phi^{i_L})\,, \quad \mathrm{tr}(\nabla_+\psi F\phi^{i_1}\dots\phi^{i_{L-2}})\,, \quad \mathrm{tr}(\chi\nabla_- t\phi^{i_1}\dots\phi^{i_{L-2}})$$

- Such operators correspond to spin-chains with sites
  - Φ fields: $(S_b)_L \otimes (S_b)_R$
  - T fields: $0^{\oplus 4} \oplus \left(0^{\oplus 2} \otimes (S_f)_L\right) \oplus \left(0^{\oplus 2} \otimes (S_f)_R\right)$
- Exact match to spin-chain from $\mathrm{AdS_3}$ [Ohlsson Sax, BS, Torrielli '11]

CFT_H: states near origin

- Gauge invariant states built from adjoint fields Φ and T
- $N_c \to \infty$: perturbation series becomes 't Hooft expansion in

$$\lambda \equiv \frac{N_c}{N_f} \qquad \text{and} \qquad \frac{1}{N_c^2}$$

- In this limit single-trace ops dominate

$$\text{tr}(\phi^{i_1}\phi^{i_2}\dots\phi^{i_L})\,, \quad \text{tr}(\nabla_+\psi F\phi^{i_1}\dots\phi^{i_{L-2}})\,, \quad \text{tr}(\chi\nabla_- t\phi^{i_1}\dots\phi^{i_{L-2}})$$

- Such operators correspond to spin-chains with sites
  - Φ fields: $(S_b)_L \otimes (S_b)_R$
  - T fields: $0^{\oplus 4} \oplus \left(0^{\oplus 2} \otimes (S_f)_L\right) \oplus \left(0^{\oplus 2} \otimes (S_f)_R\right)$
- Exact match to spin-chain from AdS_3 [Ohlsson Sax, BS, Torrielli '11]
- **Local** spin-chain appears very naturally

Δ in $\mathrm{CFT_H}$

- We restrict to $\mathcal{O}(\lambda)$ in *so*(4) subsector $\quad \mathrm{tr}\left(\phi^{i_1}\cdots\phi^{i_L}\right)$

Δ in CFT_H

- We restrict to $\mathcal{O}(\lambda)$ in $so(4)$ subsector $\quad \text{tr}\left(\phi^{i_1}\cdots\phi^{i_L}\right)$
- Ground state is 1/2 BPS: $\Delta = J$ $\quad \text{tr}\left((\phi^1 + i\phi^2)^J\right)$ protected

Δ in CFT_H

- We restrict to $\mathcal{O}(\lambda)$ in $so(4)$ subsector $\quad \text{tr}\left(\phi^{i_1}\cdots\phi^{i_L}\right)$

- Ground state is 1/2 BPS: $\Delta = J$ $\quad \text{tr}\left((\phi^1 + i\phi^2)^J\right)$ protected

- Planar gauge theory

$$\delta\mathbf{D} = \sum_{n=1}^{L} \mathcal{H}_{n,n+1}$$

Δ in $\mathrm{CFT_H}$

- We restrict to $\mathcal{O}(\lambda)$ in $so(4)$ subsector $\quad \mathrm{tr}\left(\phi^{i_1}\cdots\phi^{i_L}\right)$
- Ground state is 1/2 BPS: $\Delta = J$ $\quad \mathrm{tr}\left((\phi^1 + i\phi^2)^J\right)$ protected
- Planar gauge theory

$$\delta\mathbf{D} = \sum_{n=1}^{L} \mathcal{H}_{n,n+1}$$

- Ground state protected so

$$\delta\mathbf{D} = c_1 \sum_{n=1}^{L} \left(\mathbf{1}_{n,n+1} - \mathbf{P}_{n,n+1} + c_2\, \mathbf{K}_{n,n+1}\right)$$

Δ in CFT$_\mathrm{H}$

- We restrict to $\mathcal{O}(\lambda)$ in $so(4)$ subsector $\quad \mathrm{tr}\left(\phi^{i_1}\cdots\phi^{i_L}\right)$
- Ground state is 1/2 BPS: $\Delta = J \quad \mathrm{tr}\left((\phi^1 + i\phi^2)^J\right)$ protected
- Planar gauge theory

$$\delta\mathbf{D} = \sum_{n=1}^{L} \mathcal{H}_{n,n+1}$$

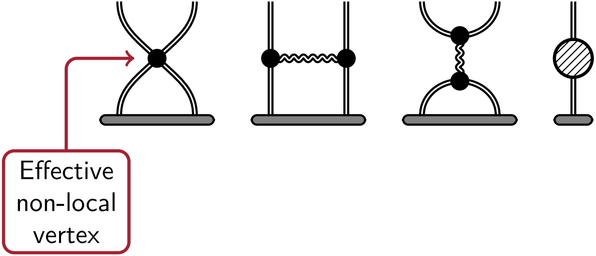
- Ground state protected so

$$\delta\mathbf{D} = c_1 \sum_{n=1}^{L} \left(\mathbf{1}_{n,n+1} - \mathbf{P}_{n,n+1} + c_2\, \mathbf{K}_{n,n+1}\right)$$

- For $so(N)$ $\delta\mathbf{D}$ is integrable if $c_2 = \frac{2}{N-2}$

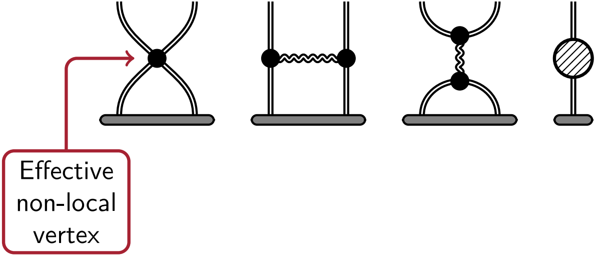
Δ in CFT_H

- Power-counting divergent leading order diagrams



Δ in CFT_H

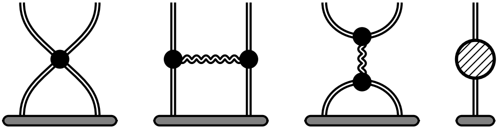
- Power-counting divergent leading order diagrams



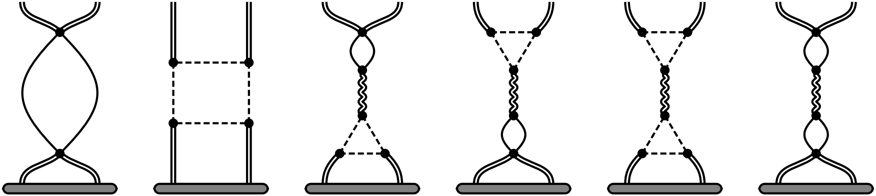
- Second and fourth diagrams give trivial $so(4)$ structure.

Δ in $\mathrm{CFT_H}$

- Power-counting divergent leading order diagrams

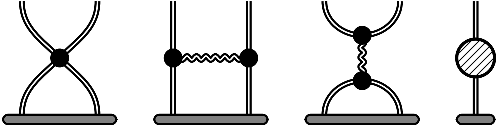


- Expanding interactions in diagrams with non-trivial *so*(4) structure

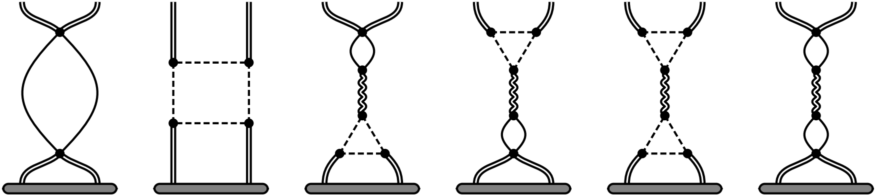


Δ in $\mathrm{CFT_H}$

- Power-counting divergent leading order diagrams



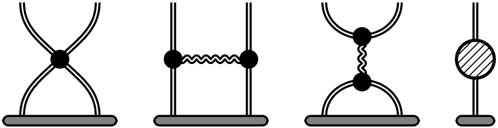
- Expanding interactions in diagrams with non-trivial $so(4)$ structure



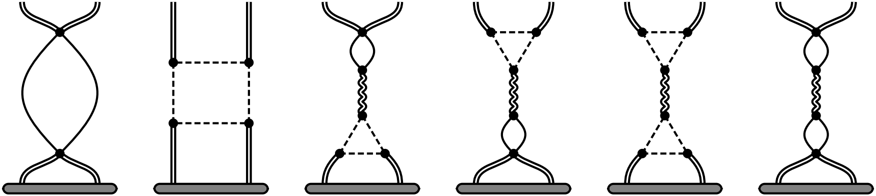
- Diagrams involving a gluon exchange vanish due to symmetry

Δ in $\mathrm{CFT_H}$

- Power-counting divergent leading order diagrams



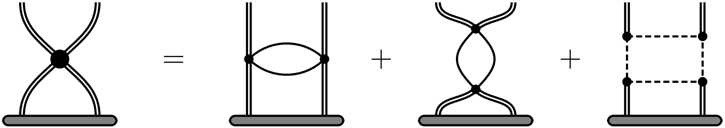
- Expanding interactions in diagrams with non-trivial $so(4)$ structure



- Diagrams involving a gluon exchange vanish due to symmetry
- Only "ϕ^4" diagram is divergent and has non-trivial $so(4)$ structure

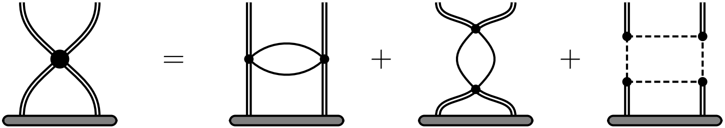
Δ in $\mathrm{CFT_H}$

- Expanding the "ϕ^4" diagram



Δ in $\mathrm{CFT_H}$

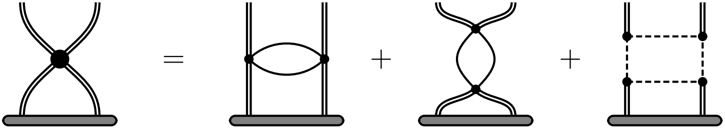
- Expanding the "ϕ^4" diagram



- First diagram has trivial $so(4)$ structure.

Δ in $\mathrm{CFT_H}$

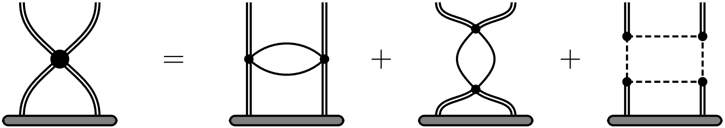
- Expanding the "ϕ^4" diagram



- First diagram has trivial $so(4)$ structure.
- Second diagram is UV finite

Δ in $\mathrm{CFT_H}$

- Expanding the "ϕ^4" diagram



- First diagram has trivial $so(4)$ structure.
- Second diagram is UV finite
- Only third diagram UV divergent and $so(4)$ non-trivial

Δ in $\mathrm{CFT_H}$

- Compute diagram, find $c_2 = 1$ and hence $so(4)$ dilatation operator

$$\delta \mathbf{D} \propto \frac{N_c}{N_f} \sum_{n=1}^{L} \left(\mathbf{1}_{n,n+1} - \mathbf{P}_{n,n+1} + \mathbf{K}_{n,n+1} \right)$$

Δ in $\mathrm{CFT_H}$

- Compute diagram, find $c_2 = 1$ and hence $so(4)$ dilatation operator

$$\delta \mathbf{D} \propto \frac{N_c}{N_f} \sum_{n=1}^{L} \left(\mathbf{1}_{n,n+1} - \mathbf{P}_{n,n+1} + \mathbf{K}_{n,n+1} \right)$$

Integrable $so(4)$ spin-chain Hamiltonian

Δ in $\mathrm{CFT_H}$

One-loop dilatation operator in $so(4)$ sector $\longleftrightarrow$ Hamiltonian of integrable $so(4)$ spin-chain

Δ in $\mathrm{CFT_H}$

One-loop dilatation operator in $so(4)$ sector	$\longleftrightarrow$	Hamiltonian of integrable $so(4)$ spin-chain

- Perturbative calculation in larger sector?

Δ in CFT_H

One-loop dilatation operator in $so(4)$ sector	$\longleftrightarrow$	Hamiltonian of integrable $so(4)$ spin-chain

- Perturbative calculation in larger sector?
- Dilatation operator from symmetries?

Conclusions and Outlook

Conclusions and Outlook

- AdS_3/CFT_2 likely to be an example of holographic integrability. Evidence of integrability found on both sides of duality.

Conclusions and Outlook

- AdS_3/CFT_2 likely to be an example of holographic integrability.
Evidence of integrability found on both sides of duality.

- I focused on D1-D5 pure R-R flux.
What happens on CFT side with mixed R-R and NS-NS flux?

Conclusions and Outlook

- AdS_3/CFT_2 likely to be an example of holographic integrability. Evidence of integrability found on both sides of duality.

- I focused on D1-D5 pure R-R flux. What happens on CFT side with mixed R-R and NS-NS flux?

- What is the connection to other points in the moduli space, such as WZW point or $\mathrm{Sym}^N(\mathrm{T}^4)$ point and to higher-spin limit?[Gopakumar, Gaberdiel,. . .]

Conclusions and Outlook

- AdS_3/CFT_2 likely to be an example of holographic integrability.
Evidence of integrability found on both sides of duality.

- I focused on D1-D5 pure R-R flux.
What happens on CFT side with mixed R-R and NS-NS flux?

- What is the connection to other points in the moduli space, such as WZW point or $\mathrm{Sym}^N(T^4)$ point and to higher-spin limit?[Gopakumar, Gaberdiel,. . .]

- What about D1-D5-D5' and its CFT_2 [Tong]

Conclusions and Outlook

- AdS_3/CFT_2 likely to be an example of holographic integrability. Evidence of integrability found on both sides of duality.

- I focused on D1-D5 pure R-R flux. What happens on CFT side with mixed R-R and NS-NS flux?

- What is the connection to other points in the moduli space, such as WZW point or $\text{Sym}^N(T^4)$ point and to higher-spin limit?[Gopakumar, Gaberdiel,. . .]

- What about D1-D5-D5' and its CFT_2 [Tong]

- Integrability in AdS_3/CFT_2 has a rich structure that needs to be investigated more fully: large space of parameters, massless modes, TBA, Quantum Spectral Curve

Thank you!