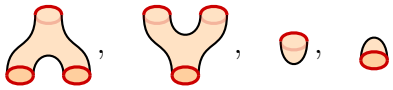
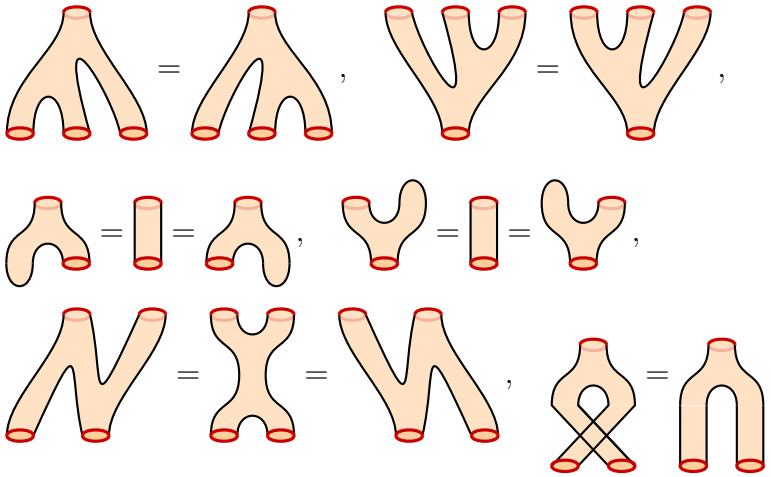
Handout on open-closed 2-dimensional bordisms

The category Bord_2 of *closed bordisms* is freely generated as a symmetric monoidal category[1] by generators $\mathcal{G}^{\mathrm{c}}$ and relations $\mathcal{R}^{\mathrm{c}}$:

$\mathcal{G}^{\mathrm{c}}$:

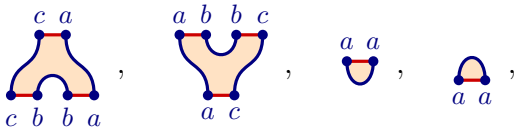


$\mathcal{R}^{\mathrm{c}}$:



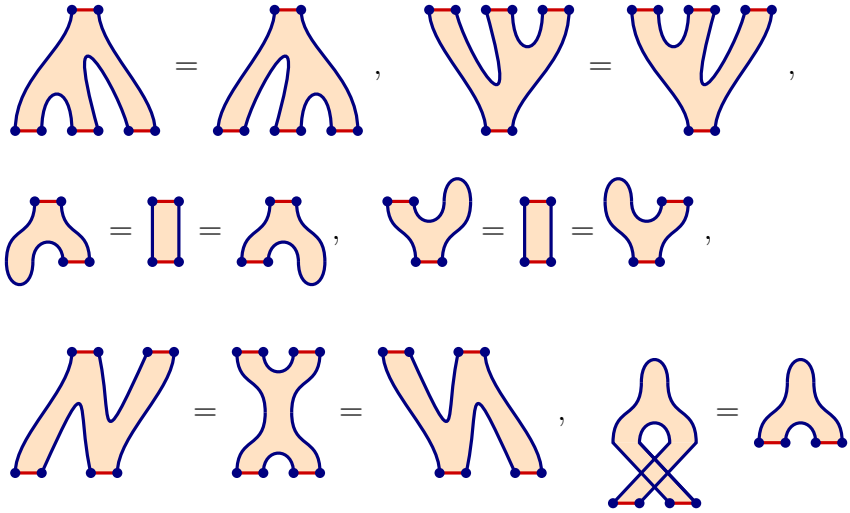
The category $\mathrm{Bord}_2^{\mathrm{o}}(B)$ of *open bordisms with boundary conditions B* is freely generated as a symmetric monoidal category by generators $\mathcal{G}^{\mathrm{o}}$ and relations $\mathcal{R}^{\mathrm{o}}$:

$\mathcal{G}^{\mathrm{o}}$:



for all $a, b, c \in B$

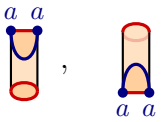
$\mathcal{R}^{\mathrm{o}}$:



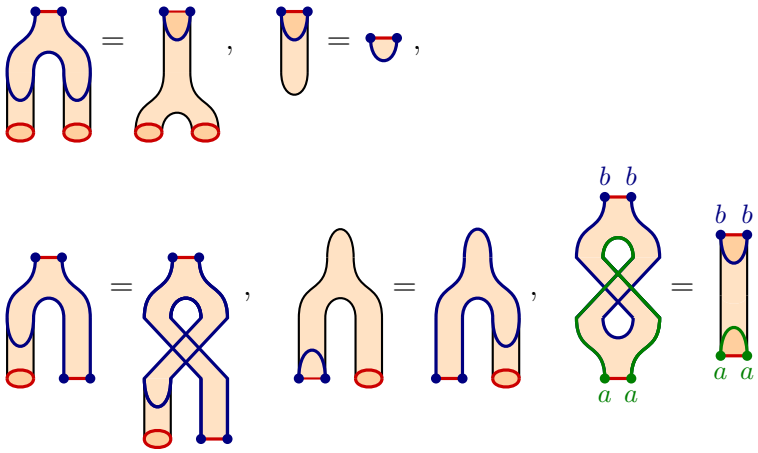
[1] This means that, up to the identities in the relations, every bordism can be obtained by vertically and horizontally composing a finite number of generators.

The category $\mathrm{Bord}_2^{\mathrm{oc}}(B)$ of *open-closed bordisms with boundary conditions* B is freely generated as a symmetric monoidal category by generators $\mathcal{G}^{\mathrm{c}}, \mathcal{G}^{\mathrm{o}}, \mathcal{G}^{\mathrm{oc}}$ and relations $\mathcal{R}^{\mathrm{c}}, \mathcal{R}^{\mathrm{o}}, \mathcal{R}^{\mathrm{oc}}$:

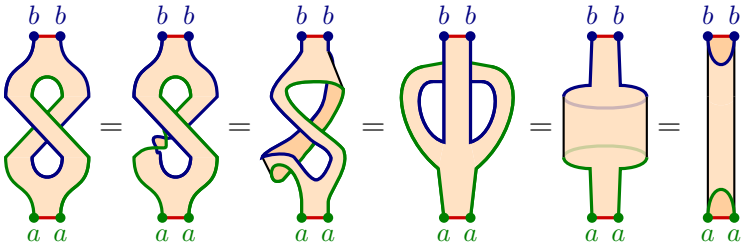
$\mathcal{G}^{\mathrm{oc}}$:



$\mathcal{R}^{\mathrm{oc}}$:



The last relation, giving rise to the *Cardy condition*, can be understood as the sequence of diffeomorphisms:



References

[Ba] J. Baez, *Some Definitions Everyone Should Know*, http://math.ucr.edu/home/baez/qg-winter2001/definitions.pdf.

[Ca] N. Carqueville, *Lecture notes on 2-dimensional defect TQFT*, Banach Center Publications **114** (2018), 49–84, [arXiv:1607.05747].

[CR] N. Carqueville and I. Runkel, *Introductory lectures on topological quantum field theory*, Banach Center Publications **114** (2018), 9–47, [arXiv:1705.05734].

[Ko] J. Kock, *Frobenius algebras and 2D topological quantum field theories*, *London Mathematical Society Student Texts* **59**, Cambridge University Press, 2003.

[La] C. I. Lazaroiu, *On the structure of open-closed topological field theory in two dimensions*, Nucl. Phys. B **603** (2001), 497–530, [hep-th/0010269].

[LP] A. D. Lauda and H. Pfeiffer, *Two-dimensional extended TQFTs and Frobenius algebras*, Topology and its Applications **155**:7 (2008), 623–666, [math.AT/0510664].

[MS] G. W. Moore and G. Segal, *D-branes and K-theory in 2D topological field theory*, [hep-th/0609042].