CROSSING EQUATION FOR

THE $AdS_5 \times S^5$ S-MATRIX

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FIND ANOMALOUS DIMENSIONS OF ALL OPERATORS IN $\mathcal{N}=4$ SYM $\equiv$ FIND ENERGIES OF ALL STRING EXCITATIONS OF THE SUPERSTRING IN $AdS_5 \times S^5$

'SPIN CHAIN' 'WORLDSHEET QFT'

DISCRETENESS $\rightarrow \sqrt{1+\square \sin^2\frac{p}{2}} \leftarrow$ RELATIVISTIC IQFT + FILLED LEVELS

$\nwarrow$ GIANT MAGNONS

COHERENT STATES

CLASSICAL SOLUTIONS (SPECTRAL CURVES)

S-MATRIX (SPIN CHAIN) $\rightarrow$ $\leftarrow$ S-MATRIX (ZF ALGEBRA)

HUBBARD $\rightarrow$ WRAPPING INTERACTIONS $\leftarrow$ GENERIC VIRTUAL CORRECTIONS FOR IQFT ON A CYLINDER

CROSSING SYMMETRY

- EQUATIONS FOR STRING ENERGIES (ANOMALOUS DIMENSIONS) ARE EXPRESSED THROUGH THE BETHE ANSATZ EQUATIONS:

$$\text{"} \; e^{ip_i L} = \prod_{j \neq i} S(p_i, p_j) \; \text{"} \qquad E = \sum_i E(p_i)$$

- THERE IS A DISAGREEMENT AT THREE LOOP ORDER
  ⤳ BUT DIFFERENT REGIONS OF VALIDITY

$$\hat{S}_{\text{STRING}}(p_i, p_j) = \underbrace{S_0(p_i, p_j)}_{\text{'DRESSING FACTOR'}} \cdot \underbrace{\hat{S}(p_i, p_j)}_{\text{FIXED BY SYMMETRIES}}$$

HOPE: $S_0(p_i, p_j)$ DESCRIBES STRING RESULTS AT STRONG COUPLING

ARUTYONOV, FROLOV, STAUDACHER
HERNANDEZ, LOPEZ

$S_0(p_i, p_j) \to \underbrace{S_{BDS}(p_i, p_j)}_{\text{GAUGE THEORY}}$ AT WEAK COUPLING

BEISERT, DIPPEL, STAUDACHER

// $S_0 \equiv S_{BDS} \cdot \sigma^2$

QUESTION: HOW TO DETERMINE $S_0(p_i, p_j)$?

REQUIREMENTS FOR AN S-MATRIX

$$S : V_1 \otimes V_2 \to V_2 \otimes V_1$$

⓪ S IS INVARIANT UNDER SOME SYMMETRY (SUPER) ALGEBRA

① S SATISFIES UNITARITY

$$S(1,2)\, S(2,1) = \mathrm{id}$$

② S SATISFIES THE YANG BAXTER EQUATION

$$\underbrace{S(1,2)\, S(1,3)\, S(2,3) = S(2,3)\, S(1,3)\, S(1,2)}$$

- PURELY ALGEBRAIC CONSISTENCY CONDITIONS
- DO NOT DEPEND ON RELATIVISTIC STRUCTURE
- GENERICALLY DETERMINES $S(1,2)$ ONLY UP TO A SCALAR FACTOR

$$S(1,2) = \underbrace{S_0(1,2)}_{\text{SCALAR}} \cdot \underbrace{\hat{S}(1,2)}_{\text{MATRIX}}$$

$\longrightarrow$ CROSSING SYMMETRY

• RAPIDITY PARAMETRIZATION

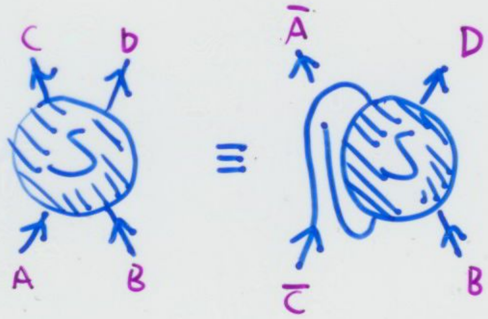
$$E = m \cosh\theta \qquad p = m \sinh\theta$$

$$S(p_1, p_2) \equiv S(\theta_1 - \theta_2)$$

• UNITARITY

$$(*) \qquad S(\theta)\, S(-\theta) = \mathrm{id}$$

• CROSSING



$$(**) \qquad S_{\bar{k} j}^{l \bar{i}}(i\pi - \theta) = S_{ij}^{kl}(\theta)$$

• (*) AND (**) FIX THE SCALAR FACTOR $s_0(\theta)$

WITH MINIMAL # OF SINGULARITIES

FURTHER SINGULARITIES → 'CDD FACTORS'

$$\prod \frac{\sinh\theta + i \sin\alpha_k}{\sinh\theta - i\sin\alpha_k} \qquad f(\theta) f(-\theta) = 1$$

$$f(i\pi - \theta) = f(\theta)$$

QUESTION:

IS IT POSSIBLE TO FORMULATE CROSSING CONDITION FOR THE $AdS_5 \times S^5$ WORLDSHEET THEORY ?

DIFFICULTIES:

- NONSTANDARD DISPERSION RELATION

$$E = \sqrt{1 + 8 g^2 \sin^2 \frac{p}{2}}$$

- THE S-MATRIX DOES NOT DEPEND ONLY ON THE DIFFERENCE OF TWO 'RAPIDITIES'
- WHAT WOULD BE THE COUNTERPART OF

$$\theta \longrightarrow \theta + i\pi \qquad // \quad S(i\pi - \theta)^{cross} = S(\theta)$$

$\rightsquigarrow$ LOOK FOR ALGEBRAIC REFORMULATION OF CROSSING...

HOPF ALGEBRA APPROACH

SYMMETRY ALGEBRA $\mathcal{A}$ ← CONTAINS LOCAL AND NONLOCAL SYMMETRY CHARGES

EXAMPLES: YANGIANS, QUANTUM AFFINE ALGEBRAS

- COPRODUCT

$$\Delta : \mathcal{A} \to \mathcal{A} \otimes \mathcal{A}$$

← HOW THE SYMMETRY CHARGES ACT ON TWO PARTICLE STATES

LOCAL $\Delta(J_i) = J_i \otimes 1 + 1 \otimes J_i$

NONLOCAL (YANGIAN)

$$\Delta(\hat{J}_i) = \hat{J}_i \otimes 1 + 1 \otimes \hat{J}_i + \alpha f_{ijk} J_j \otimes J_k$$

- FUNDAMENTAL OBJECT R MATRIX

$$R = PS \quad : \mathcal{A} \otimes \mathcal{A} \to \mathcal{A} \otimes \mathcal{A}$$

(GRADED) PERMUTATION

- SATISFIES YBE, UNITARITY

- CONVENIENT REFORMULATION OF CROSSING

$$S(i\pi - (\theta_1 - \theta_2))^{cross} = S(\theta_1 - \theta_2)$$

$$\Big\downarrow \quad \theta_1 \to -\theta_1 \quad \theta_2 \to -\theta_2$$

$$S(\theta_1 + i\pi - \theta_2)^{cross} = S(-(\theta_1 - \theta_2))$$

$$\Big\downarrow \quad \text{UNITARITY}$$

$$S(\underline{\theta_1 + i\pi} - \theta_2)^{cross} = S(\theta_1 - \theta_2)^{-1}$$

$$\Big\uparrow$$

$\theta_1 \to \theta_1 + i\pi$

'PARTICLE → ANTIPARTICLE' TRANSFORMATION

$p_1 \to -p_1$

$E_1 \to -E_1$

· ANOTHER INGREDIENT : THE ANTIPODE

$$S : A \to A$$

ANTIHOMOMORPHISM

$$S(ab) = (-1)^{d(a)d(b)} S(b)\, S(a)$$

• THE R · MATRIX IN A HOPF ALGEBRA SATISFIES

$$(S \otimes 1)\, R = R^{-1} \qquad (1 \otimes S^{-1})\, R = R^{-1}$$

• PASS TO SPECIFIC REPRESENTATION

$$\pi(S(a)) = C^{-1}\, \bar{\pi}(a)^{s.t.}\, C$$

CHARGE CONJUGATION MATRIX

SUPER TRANSPOSE

BERNARD
LE CLAIR

DELIUS

BASSI, LE CLAIR

• S ACTING ON NONLOCAL CHARGES OF A YANGIAN GIVES RISE TO A SHIFT OF THE RAPIDITY $\theta \to \theta + i\pi$

CROSSING CONDITION IN
A RELATIVISTIC INTEGRABLE
QFT
(WHEN A IS E.G. YANGIAN)

THE $su(2|2)\times su(2|2)$ INVARIANT S MATRIX

- EXPLICIT (LOCAL!) $su(1|2)\times su(1|2)$ SYMMETRY

BEISERT
hep-th/0511082

- ASYMPTOTIC STATES $\equiv$ MAGNONS

$$(2B+2F)\otimes(2B+2F) = 8B+8F$$

- REPRESENTATION OF SUPERCHARGES DEFINED BY COMPLEX PARAMETERS a, b, c, d

SATISFYING: $a=1$ $\leftarrow$ NORMALIZATION

$ad-bc=1$ $\leftarrow$ $su(1|2)$

$abcd+\beta ab+\alpha cd=0$ $\leftarrow$ $su(2|2)$ AT $P_{TOTAL}=0$

$$\alpha\beta=\frac{g^2}{2}$$

- ENERGY AND MOMENTUM ENCODED IN a, b, c, d (OR $x^{\pm}$)

- S MATRIX FIXED UP TO A SCALAR FACTOR

$$S(1,2)=S_0(1,2)\cdot\left[\hat{S}_{su(2|2)}(1,2)\otimes\hat{S}_{su(2|2)}(1,2)\right]$$

- SATISFIES YBE
- SATISFIES UNITARITY

- NATURAL TO USE HOPF ALGEBRAIC FRAMEWORK

FOR CROSSING:

$$(S \otimes 1) R = R^{-1}$$

ANTIPODE

- IN THE RELATIVISTIC CASE

PARAMETER LABELING REPRESENTATION OF E.G. THE YANGIAN

$$S : \theta \longrightarrow \bar{\theta} = \theta + i\pi$$

$$E \rightarrow \bar{E} = -E$$

$$p \rightarrow \bar{p} = -p$$

- HERE ALL DYNAMICAL PROPERTIES (E, p) ARE ENCODED IN GROUP THEORETICAL PARAMETERS a, b, c, d

$$S : \underset{\substack{\cup \\ E,\, p}}{(a, b, c, d)} \rightarrow \underset{\substack{\cup \\ \bar{E},\, \bar{p}}}{(\bar{a}, \bar{b}, \bar{c}, \bar{d})}$$

- IN ADDITION $\rightsquigarrow$ CHARGE CONJUGATION MATRIX $\mathcal{C}$

THE ANTIPODE AND CROSSING

- IMPLEMENT

$$(S \otimes \mathrm{id})\, R = R^{-1} \qquad (\mathrm{id} \otimes S^{-1})\, R = R^{-1}$$

IN THE SPECIFIC REPRESENTATION $V_1 \otimes V_2$

- ACTION OF THE ANTIPODE

$$(*) \qquad \pi(S(A)) = C^{-1}\, \bar{\pi}(A)^{st}\, C$$

CHARGE CONJUGATION MATRIX (TO BE DETERMINED)

REPRESENTATION FOR FIRST PARTICLE

a_1, b_1, c_1, d_1

INVOLVES MOMENTUM!

DISTINCT REPRESENTATION FOR ANTIPARTICLE

$\bar{a}, \bar{b}, \bar{c}, \bar{d}$

(TO BE DETERMINED)

→ INVOLVES MOMENTUM!

SUPERTRANSPOSE

$$M^{st}_{ij} \equiv (-1)^{d(i)d(j)+d(j)} M_{ji}$$

⟶ FIND $\bar{\pi}$ AND C

- USE $S(A) = -A$ FOR $A \in su(1|2)$

→ CHARGE CONJUGATION MATRIX C FIXED COMPLETELY

→ $\bar{a} = 1 \quad \bar{c} = -\frac{1+bc}{\bar{b}}$

- USE $(S \otimes 1) R = R^{-1}$

FOR $R \equiv R(1,2) = S_0(1,2) \cdot \hat{R}_{su(2|2) \times su(2|2)}(1,2)$

→ $\bar{b} = \frac{-\alpha a b}{\bar{a}(\alpha + ab)}$

→ EQUATION FOR $S_0(1,2)$

$$S_0(\bar{1},2)\, S_0(1,2) = f^2(1,2)$$

EXPLICIT KNOWN FUNCTION

CROSSING TRANSFORMATION

$$\bar{b} = \frac{-\alpha b_1}{\alpha + b_1} \qquad \bar{c} = -\frac{1 + b_1 c_1}{\bar{b}}$$

- IN TERMS OF $x^{\pm}$

$$b = -\alpha\left(1 - \frac{x^-}{x^+}\right)$$

$$c = \frac{i\beta}{x^-}$$

$$\overline{x^+} = \frac{\alpha\beta}{x^+} \qquad \overline{x^-} = \frac{\alpha\beta}{x^-}$$

$p \to -p$

$E \to -E$

AS IN RELATIVISTIC CASE

- THE FUNCTION $f(1,2)$

$$f(1,2) = \frac{\frac{\alpha\beta}{x_1^+} - x_2^-}{\frac{\alpha\beta}{x_1^-} - x_2^-} \cdot \frac{x_1^+ - x_2^+}{x_1^- - x_2^+}$$

$$\alpha\beta \equiv \frac{g^2}{2}$$

- EQUATIONS FOR $S_0(1,2)$

$$S_0(1,2)\, S_0(2,1) = 1$$

$$S_0(\bar{1},2)\, S_0(1,2) = f(1,2)^2$$

(*) $$S_0(\bar{1},2)\, S_0(1,2) = f(1,2)^2$$

PROBLEM: RHS NOT INVARIANT UNDER $1 \leftrightarrow \bar{1}$

- x_i^+, x_i^- ARE NOT INDEPENDENT VARIABLES BUT SUBJECT TO A CONSTRAINT
- $f(1,2)$ HAS COMPLICATED CUT STRUCTURE

$\downarrow$

PASS TO NEW RAPIDITY VARIABLES WHICH ARE UNCONSTRAINED AND CONSIDER (*) IN THESE NEW VARIABLES

MOTIVATION:

$$E^2 - p^2 = m^2 \longrightarrow \begin{array}{l} E = m\cosh\theta \\ p = m\sinh\theta \end{array}$$

$$\begin{array}{l} E \to -E \\ p \to -p \end{array} \longrightarrow \theta \to \theta + i\pi$$

INVOLUTION

NO LONGER AN INVOLUTION

YANGIANS: $S^2 \neq id$

THE GENERALIZED RAPIDITY PLANE

- THE PARAMETERS a, b, c, d ARE CONSTRAINED BY

$$ad - bc = 1$$

$$abcd + \beta ab + \alpha cd = 0$$

- SET $a = 1$, ELIMINATE $d = 1 + bc$

- $(*)$ $$(bc^2 + c)(b + \alpha) + \beta b = 0$$

$$\downarrow$$

PASS TO THE UNIVERSAL COVERING SPACE OF THE CURVE $(*)$

$$\text{GENUS} = \underset{\substack{\shortparallel \\ 3}}{\frac{(4-1)(4-2)}{2}} - \sum_{\text{SINGULARITY}} d_i = 3 - 2 = 1$$

- UNIVERSAL COVERING SPACE $\equiv$ COMPLEX PLANE

- $(*)$ CAN BE UNIFORMIZED BY ELLIPTIC FUNCTIONS

- b AND c CAN BE EXPRESSED IN TERMS OF x, y

  ON AN ELLIPTIC CURVE

$$y^2 = 4x^3 - g_2 x - g_3$$

WHERE

$$g_2 = \frac{1}{12}\left(1 + 16\alpha\beta + 16\alpha^2\beta^2\right)$$

$$g_3 = \frac{1}{216}\left(1 + 8\alpha\beta\right)\left(-1 - 16\alpha\beta + 8\alpha^2\beta^2\right) \qquad \alpha\beta = \frac{g^2}{2}$$

SO THE UNIFORMIZATION IS

$$y(z) = \wp'(z; g_2, g_3) \qquad x(z) = \wp(z; g_2, g_3)$$

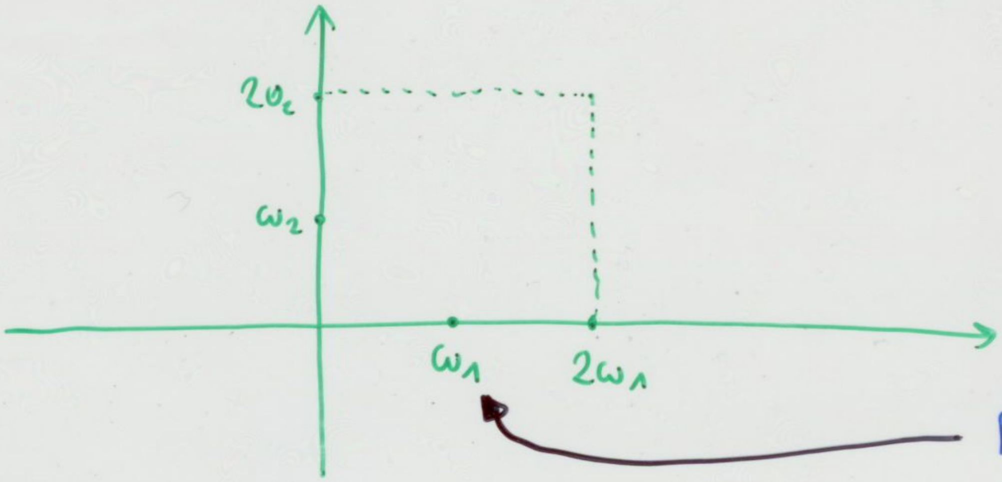
EXPLICITLY

$$b(z) = -\frac{1}{\beta}\left(\wp(z) + \frac{4\alpha\beta - 1}{12}\right)$$

$$c(z) = \frac{\wp'(z) - \beta\, b(z) - \alpha\beta}{2\beta\, b(z)\left(b(z) + \alpha\right)}$$

GENERALIZED RAPIDITIES

- $b(z), c(z), x^{\pm}(z)$ BECOME ELLIPTIC FUNCTIONS ON THE COMPLEX PLANE $\equiv$ PERIODIC W.R.T $2\omega_1, 2\omega_2$



$$P(\omega_1) = \frac{1+8\alpha\beta}{12}$$

- CROSSING TRANSFORMATION HAS A VERY SIMPLE FORM:

$$b(z+\omega_1) = \bar{b}(z) \qquad x^+(z+\omega_1) = \frac{\alpha\beta}{x^+(z)}$$

$$c(z+\omega_1) = \bar{c}(z) \qquad x^-(z+\omega_1) = \frac{\alpha\beta}{x^-(z)}$$

I.E.

$$z \longrightarrow z + \omega_1$$

// ANALOG OF RELATIVISTIC

$$\theta \longrightarrow \theta + i\pi$$

EQUATIONS FOR $S_0(z_1, z_2)$

- UNITARITY

$$S_0(z_1, z_2)\, S_0(z_2, z_1) = 1$$

- CROSSING

$$S_0(z_1 + \omega_1, z_2)\, S_0(z_1, z_2) = f(z_1, z_2)^2$$

$$S_0(z_1, z_2 - \omega_1)\, S_0(z_1, z_2) = f(z_1, z_2)^2$$

AS IN RELATIVISTIC CASE
COMES FROM

$$(id \otimes S^{-1}) R = R^{-1}$$

- IN ORDER FOR THESE EQUATIONS TO BE CONSISTENT

$f(z_1, z_2)$ HAS TO SATISFY

$$f(z_1 - \omega_1, z_2) = \frac{1}{f(z_2, z_1)}$$

SATISFIED BY OUR FUNCTION $f(z_1, z_2)$!!

- NONTRIVIAL CHECK AT STRONG COUPLING

ARUTYUNOV, FROLOV
0604043

- DRESSING FACTOR AT STRONG COUPLING:

$$S_0(p_k, p_j) = \frac{x_j^- - x_k^+}{x_j^+ - x_k^-} \, \frac{1 - \frac{1}{x_j^+ x_k^-}}{1 - \frac{1}{x_j^- x_k^+}} \cdot \sigma(p_j, p_k)$$

$$e^{i\theta} = \exp\left\{\frac{i\sqrt{\lambda}}{2\pi} \sum_{r=2}^{\infty} \sum_{n=0}^{\infty} c_{r,r+1+2n}(\lambda) \left(q_r(p_j)\, q_{r+1+2n}(p_k) - j \leftrightarrow k\right)\right\}$$

WHERE

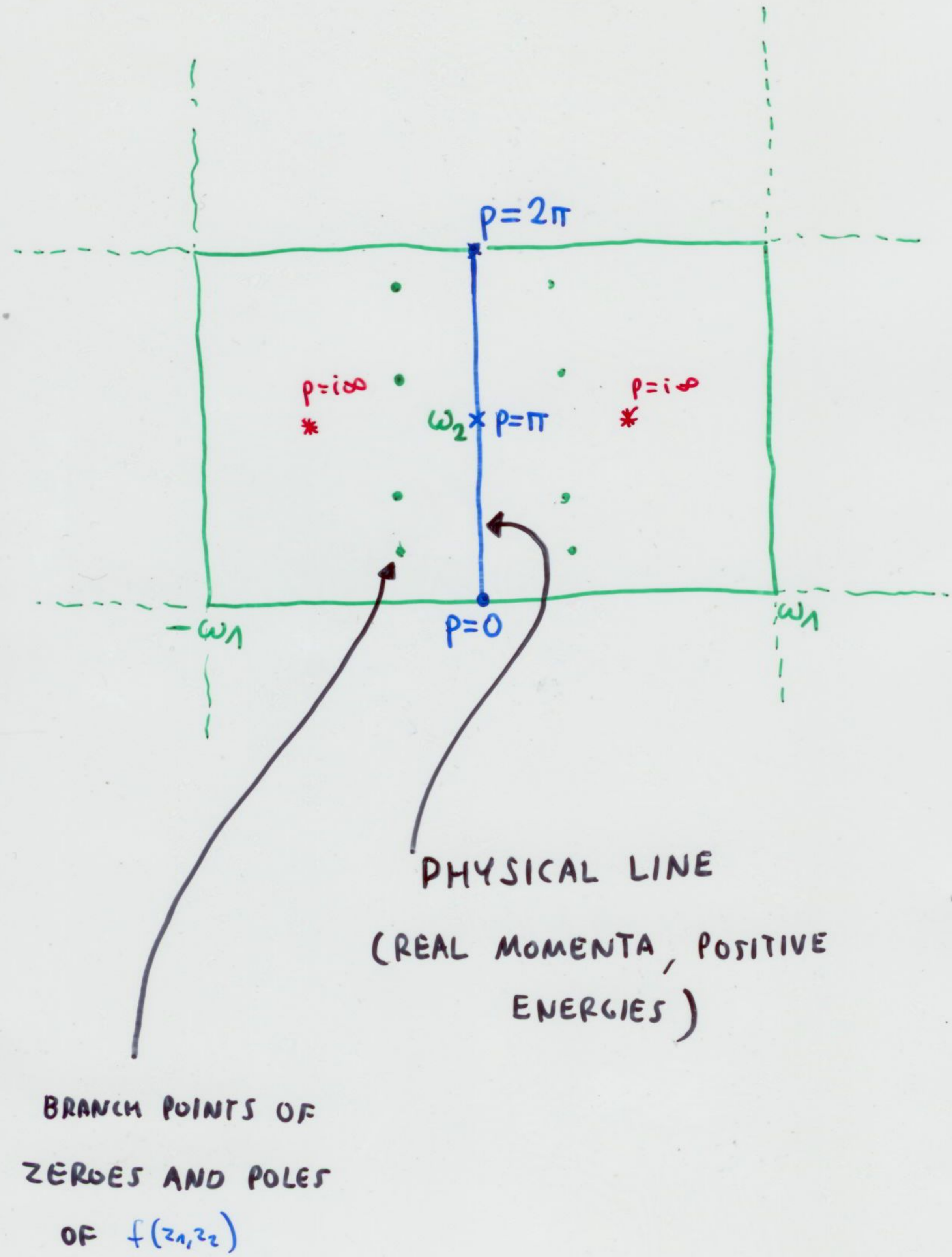
$$c_{r,s} = \delta_{r+1,s} - \frac{8}{\sqrt{\lambda}} \frac{(r-1)(s-1)}{(r+s-2)(s-r)} + \dots$$

AFS
0406256

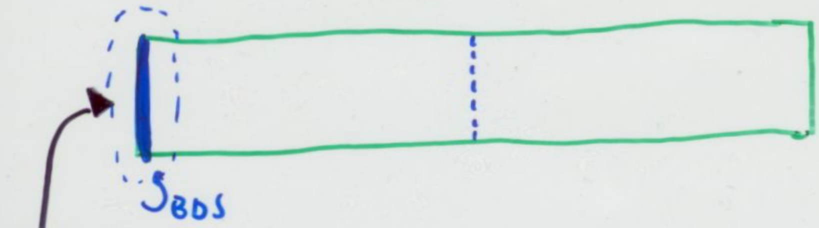
HERNANDEZ LOPEZ
0603204

- THIS DRESSING FACTOR SATISFIES CROSSING EQUATIONS UP TO FIRST TWO ORDERS IN $\frac{1}{\sqrt{\lambda}}$ EXPANSION

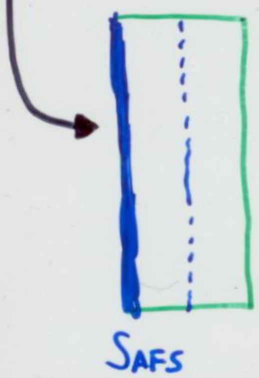
STRUCTURE OF THE GENERALIZED RAPIDITY PLANE



- SMALL λ



- LARGE λ



- DEGENERATIONS:

DISCRIMINANT $\Delta = \alpha^2\beta^2(1+16\alpha\beta)$

$\alpha\beta = \frac{g^2}{2}$

ELLIPTIC CURVE SINGULAR $\Leftrightarrow$ $\Delta = 0$

$\rightarrow$ OCCURS FOR $g^2 = -\frac{1}{8}$ $(\lambda = -\pi^2)$

EXACTLY SAME POSITION AS IN LIPATOV'S TALK!

$$\begin{cases} S_0(z_1,z_2)\,S_0(z_2,z_1)=1 \\ S_0(z_1+\omega_1,z_2)\,S_0(z_1,z_2)=f^2(z_1,z_2) \\ S_0(z_1,z_2-\omega_1)\,S_0(z_1,z_2)=f^2(z_1,z_2) \end{cases}$$

- WHY SOLVING THESE EQUATIONS IS DIFFICULT?

  - USUALLY IN RELATIVISTIC THEORIES SOLVING SUCH EQUATIONS IS SIMPLE

$$s_0(\theta)\,s_0(-\theta)=\frac{\theta^2}{\theta^2+\pi^2}$$

$$s_0(i\pi-\theta)=s_0(\theta)$$

$\rightsquigarrow$ ITERATIVE TECHNIQUE

$\rightarrow$ HERE $f(z_1,z_2)$ IS PERIODIC

  - NONTRIVIAL DEPENDENCE ON BOTH RAPIDITIES

    ONE CANNOT WRITE $f(z_1,z_2)$ AS

$$f(z_1,z_2)\neq F\big(\lambda(z_1)-\lambda(z_2)\big)$$

A SOLUTION WITH LOTS OF BRANCH CUTS:

$$S_0^{\text{cuts}}(z_1, z_2) = \sqrt{\frac{f(z_1, z_2)}{f(z_2, z_1)}} \cdot \Big(f(z_1, z_2)\, f(z_2, z_1) \Big)^{\frac{z_2 - z_1}{\omega_1}}$$

RJ

A. REJ, M. STAUDACHER

- UNPHYSICAL
- BRANCH CUTS APPEAR FOR ALL ZEROES/POLES OF $f(z_1, z_2)$
- USUALLY THERE EXISTS A CORRESPONDING SOLUTION WITHOUT BRANCH CUTS (MEROMORPHIC)

$\rightsquigarrow$

TOY MODEL

TOY MODEL:

$$f(z+1) = \sin \pi z \, f(z)$$

NAIVE SOLUTION:

$$f(z) = (-1)^{\frac{z^2+z}{2}} \underline{(\sin \pi z)^z}$$

• $\exists$ MEROMORPHIC SOLUTION

PRODUCT REPRESENTATION $\quad \sin \pi z = \pi z \prod_{n=1}^{\infty} \left(1 - \frac{z^2}{n^2}\right)$

$$f(z) = \pi^z \, \Gamma(z) \prod_{n=1}^{\infty} \frac{n \, \Gamma(n+z)}{\Gamma(n+1-z)} \, n^{-2z}$$

z		
⋮		
3	ZERO OF ORDER	2
2	ZERO OF ORDER	1
1		
0	POLE OF ORDER	1
−1	⋮	2
−2	⋮	3
−3	POLE ⋮	4

- STRUCTURE OF $f(z_1, z_2)$

$$f(z_1,z_2) = \underbrace{\frac{x^+(z_1) - x^+(z_2)}{\frac{\alpha\beta}{x^-(z_1)} - x^-(z_2)}}_{\text{'EASY' PART}} \cdot \underbrace{\frac{\frac{\alpha\beta}{x^+(z_1)} - x^-(z_2)}{x^-(z_1) - x^+(z_2)}}_{\text{'HARD' PART}}$$

- POLES AND ZEROES ASSOCIATED ONLY WITH z_1 (OR ONLY WITH z_2) CANCEL OUT
- 'EASY' AND 'HARD' DO NOT SATISFY CONSISTENCY CONDITION INDIVIDUALLY
- FOR FIXED z_2, $f(z_1, z_2)$ HAS $1+2$ ZEROES AND $1+2$ POLES
- ZEROES AND POLES OF THE 'EASY' PART KNOWN EXPLICITLY

  ⟶ SOLVE USING ELLIPTIC GAMMA FUNCTION

- PAIRS OF ZEROES (OR POLES) OF THE 'HARD' PART FORM A BRANCHED COVERING OF THE COMPLEX PLANE

BRANCH POINTS $x^{\pm}(z) = \pm 1$ $\left(= \pm\sqrt{\alpha\beta}\right)$

$\rightsquigarrow$ GENERICALLY LEAD TO BRANCH CUTS IN THE SOLUTION

- A 1-PARAMETER FAMILY OF SOLUTIONS WITH SUCH BRANCH CUTS CAN BE CONSTRUCTED

- N. BEISERT'S SOLUTION hep-th/0606214

A-PRIORI NOT BAD ~ 'PURELY KINEMATICAL' BRANCH POINTS

COVERING SPACE OF A TORUS $\rightsquigarrow$ COVERING SPACE OF A 'PUNCTURED' TORUS

BUT INCORRECT STRONG COUPLING LIMIT....

- <u>SOLUTIONS WITHOUT BRANCH CUTS</u>

- ESSENTIAL SINGULARITIES AT ZEROES AND POLES

  OF 'HARD' PART

  \+

  ELLIPTIC GAMMA SOLUTION FOR THE 'EASY' PART

- A-PRIORI NOT BAD:

  - ONLY THE 'EASY' PART HAS ZEROES ON

    THE LINE OF PHYSICAL MOMENTA

  - BUT THE SINGULARITIES OF THE 'HARD' PART

    APPROACH THE $p=0$ POINT

J. MALDACENA NO-GO ARGUMENT : EITHER BRANCH CUTS

OR ESSENTIAL SINGULARITIES

$\downarrow$

- ALLOW FOR SOME BRANCH CUTS WHICH COULD BE INTERPRETED AS ENLARGING THE SPACE OF GENERALIZED RAPIDITIES

- HOPE TO OBTAIN GOOD LIMITS:

  - ARUTYUNOV - FROLOV - STAUDACHER + HERNANDEZ - LOPEZ

    STRONG COUPLING SMALL MOMENTUM AF EXPANSION

  - HOFMAN - MALDACENA

    $\lambda \to \infty$ p FIXED

  - WEAK COUPLING ?

CONCLUSIONS AND OUTLOOK

- FORMULATED CROSSING CONDITIONS FOR THE $AdS_5 \times S^5$ WORLDSHEET S-MATRIX
  - HOPF ALGEBRAIC REFORMULATION OF CROSSING
  - INTRODUCE GENERALIZED RAPIDITY PLANE AS UNIVERSAL COVERING SPACE OF THE S-MATRIX PARAMETERS
  - FORMULATE CROSSING ON THE UNIVERSAL COVER

- SOLUTIONS .. → ANALYTICAL STRUCTURE ??

  → ADMISSIBLE BRANCH CUTS ?

  → BOUND STATES ...

- FIND THE HOPF ALGEBRA STRUCTURE OF THE NONLOCAL CHARGES

- MATHEMATICAL MEANING OF THE QUARTIC CONSTRAINTS ...