Integrable Sigma Models related to ADS/CFT

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**with N.Gromov, K.Sakai, P.Vieira, hep-th/0603043,
and with N.Gromov, hep-th/0605026**

Motivation

- AdS/CFT correspondence:
- Spin chain in 4D *N*=4 SYM ↔ superstring σ-model on $AdS_5 x S_5$
- Week-strong duality in 'tHooft's coupling $\lambda = N g^2$.
- Signs of integrability on both sides of duality:
  - SYM: **[Lipatov'94],[Faddeev,Korchemsky'95] [Minahan,Zarembo'02], [Beisert,Staudacher'03], [Staudacher'04], [Beisert,Kristjansen,Staudacher'02], [Beisert,Dippel,Staudacher'04]**
  - String: • **[Bena,Roiban,Polchinski'02],[Beisert,V.K.,Sakai,Zarembo'05], [Arutyunov,Frolov,Staudacher], [V.K.,Marshakov,Minahan,Zarembo'04] [Beisert,Staudacher'05], [Beisert'05],[Janik06].**
  - **How to reconcile SYM spin chain with continuous worldsheet of σ-model?**

• Quantization of (super)string

- **Proposal** (inspired by [Mann,Polchinski'05], [Rej,Serban,Staudaher'05]): **integrable superstring sigma model on AdS5xS5 is a**

  Inhomogeneous dynamical spin chain (IDSC).

- **Correct classical limit (algebraic curve for finite gap) reproduced for the full compact SO(6) subsector of full AdS5xS5.** [Gromov, V.K.,Sakai,Vieira'06],

New method relating quantum and classical integrability.

- **Similar limit in spin chains**: [Sutherland'94], [Beisert,Minahan,Staudaher,Zarembo'03]
- **Strings in S_3xR subsector: we reproduce the asymptotic string "Bethe ansatz" (AFS) from our model.** [Gromov, V.K.'06]

“Toy” model: σ-model on $S^3 x R_1$

[Frolov,Tseytlin’02]

$$S = \frac{\sqrt{\lambda}}{4\pi}\int d\sigma d\tau \left[(\partial_\mu X_a)^2 - (\partial_\mu t)^2\right], \qquad X_1^2+\ldots+X_4^2 = 1$$

- Gauge for AdS “time”: $t(\sigma,\tau) = \frac{1}{2}\kappa_+(\tau+\sigma) + \frac{1}{2}\kappa_-(\tau-\sigma)$
  - Equivalent to SU(2)xSU(2) principal chiral field:

$$S = -\frac{\sqrt{\lambda}}{8\pi}\int d\sigma d\tau \ \mathrm{Tr}\ j_a^2$$

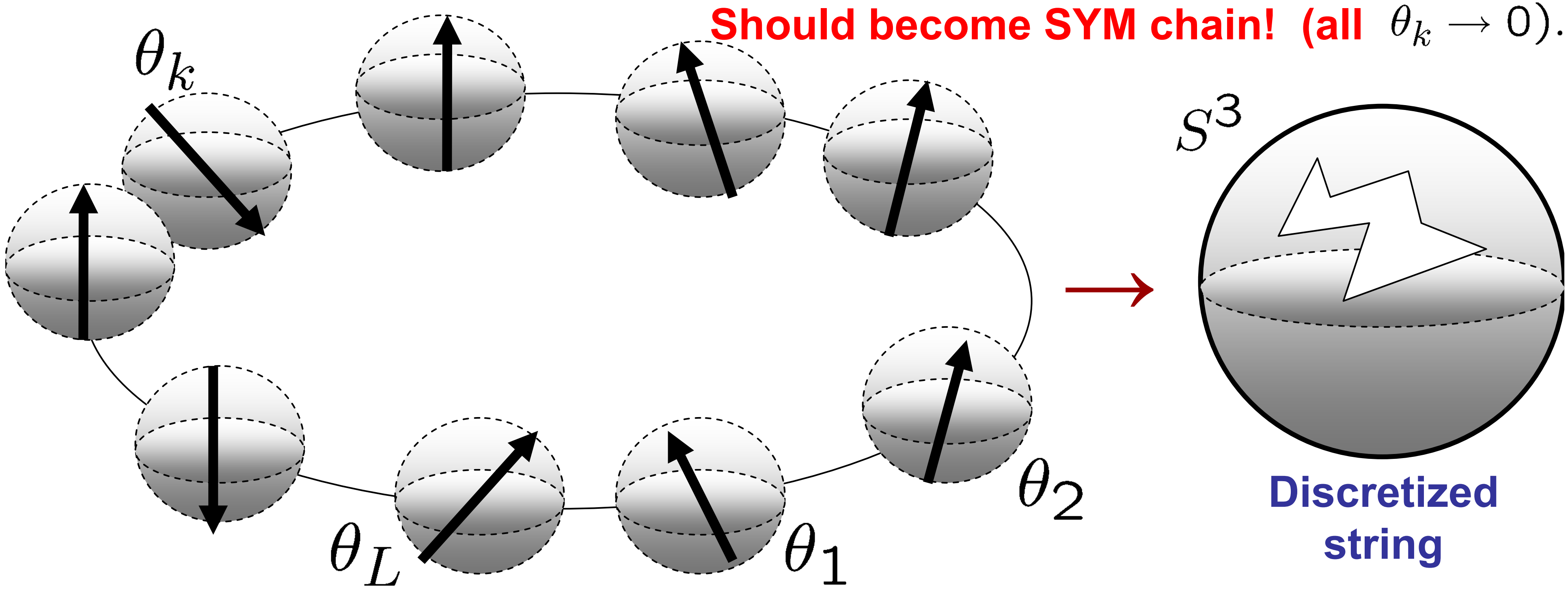
$$j_a = g^{-1}\partial_a g, \qquad g = \begin{pmatrix} X_1 + iX_2 & X_3 + iX_4 \\ -X_3 + iX_4 & X_1 - iX_2 \end{pmatrix} \in SU(2)$$

- Virasoro conditions: $\mathrm{tr} j_\pm^2(\sigma,\tau) = 2\kappa_\pm^2$
- Stress-Energy tensor: $E^{\mathrm{cl}} \pm P^{\mathrm{cl}} = -\frac{\sqrt{\lambda}}{8\pi}\int \mathrm{tr}\ [j_0 \pm j_1]^2 d\sigma = \frac{\sqrt{\lambda}}{2}\kappa_\pm^2$
- No time windings: $\kappa_+ = \kappa_- = \kappa = \Delta/\sqrt{\lambda}$

Δ : SYM dimension.

Particles on a ring as dynamical spin chain

Should become SYM chain! (all $\theta_k \to 0$).



Chain of length $\mathcal{L} = 2\pi$

$$E = m \cosh\theta$$
$$p = m \sinh\theta$$

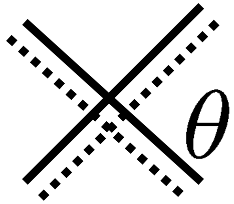
- Large density and energies, classical, conformal limit in asymptotically free theory:

$$\mu = \mathcal{L}m = e^{-\frac{\sqrt{\lambda}}{2}} \to 0, \quad \sqrt{\lambda} \sim L \to \infty, \quad z = \frac{4\pi\theta}{\sqrt{\lambda}} \sim 1$$

S-matrix for SU(2)xSU(2) chiral field

- Equivalent to σ-model on S^3, a subsector of superstring

- S-matrix: $$\hat{S}(\theta) = \hat{S}_L(\theta) \times \hat{S}_R(\theta) \quad \Rightarrow$$



Satisfies the Yang-Baxter eqs., unitarity, crossing and analyticity.

where $$\hat{S}_{L,R}(\theta) = S_0(\theta)\left(P^+_{L,R} + \tfrac{\theta+i}{\theta-i} P^-_{L,R}\right)$$

$$S_0(\theta) = \frac{\Gamma\left(-\frac{\theta}{2i}\right)\Gamma\left(\frac{1}{2}+\frac{\theta}{2i}\right)}{\Gamma\left(\frac{\theta}{2i}\right)\Gamma\left(\frac{1}{2}-\frac{\theta}{2i}\right)} \quad \rightarrow \exp\left(-\frac{i}{\theta}\right), \qquad \theta \rightarrow \pm\infty$$

«Coulomb» asymptotics

[Zamolodchikovs'79], [Wiegmann'84]

- Periodicity condition defining the states:

$$e^{-i\mu \sinh \pi\theta_\alpha} \; |\psi> = \prod_{\beta=\alpha+1}^{L} \hat{S}\left(\theta_\alpha - \theta_\beta\right) \prod_{\gamma=1}^{\alpha-1} \hat{S}\left(\theta_\alpha - \theta_\gamma\right) \; |\psi>$$

- Bethe equations (diagonalization of periodicity condition):

$$\begin{aligned} e^{-i\mu \sinh \pi\theta_\alpha} &= \prod_{\beta\neq\alpha}^{L} S_0^2\left(\theta_\alpha - \theta_\beta\right) \prod_{j}^{K_u} \frac{\theta_\alpha - u_j + i/2}{\theta_\alpha - u_j - i/2} \prod_{k}^{K_v} \frac{\theta_\alpha - v_k + i/2}{\theta_\alpha - v_k - i/2}, \\ 1 &= \prod_{\beta}^{K_u} \frac{u_j - \theta_\beta - i/2}{u_j - \theta_\beta + i/2} \prod_{i\neq j}^{K_u} \frac{u_j - u_i + i}{u_j - u_i - i}, \\ 1 &= \prod_{\beta}^{K_v} \frac{v_k - \theta_\beta - i/2}{v_k - \theta_\beta + i/2} \prod_{l\neq k}^{K_v} \frac{v_k - v_l + i}{v_k - v_l - i}, \end{aligned}$$

- Energy $$E = \frac{\mu}{2\pi} \sum_\alpha \cosh(\pi\theta_\alpha)$$

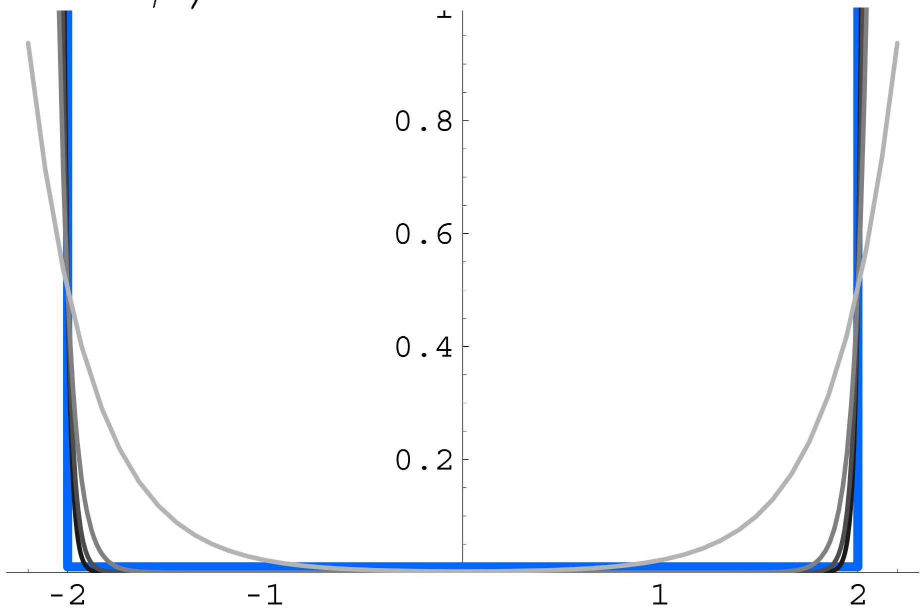
- Momentum $$P = \frac{\mu}{2\pi} \sum_\alpha \sinh(\pi\theta_\alpha) = mL - \sum_p n_p S_p^u - \sum_p n_p S_p^v$$

Conformal (classical) limit for U(1) sector

- θ-variables describe unphysical longitudinal motions of the string, and u,v magnon variables – the transverse. Let us first drop the magnons.
- 2D Coulomb charges with coordinates θ_k in potential $\mu \cosh \theta$
- For rescaled variable $z = \frac{4\pi}{\sqrt{\lambda}}\,\theta$ Bethe eq. becomes

$$\mu \sinh\left(\frac{\sqrt{\lambda}}{4} z_\alpha\right) - 2\pi m = -\frac{4\pi}{\sqrt{\lambda}} \sum_{\beta\neq\alpha}^{L} \frac{1}{z_\alpha - z_\beta}$$

Potential becomes a square box on the interval $-2 < z < 2$



Classical limit of U(1) highest weight sector

- Scaling limit: $L \sim \sqrt{\lambda} \to \infty$, Coulomb charges in square box

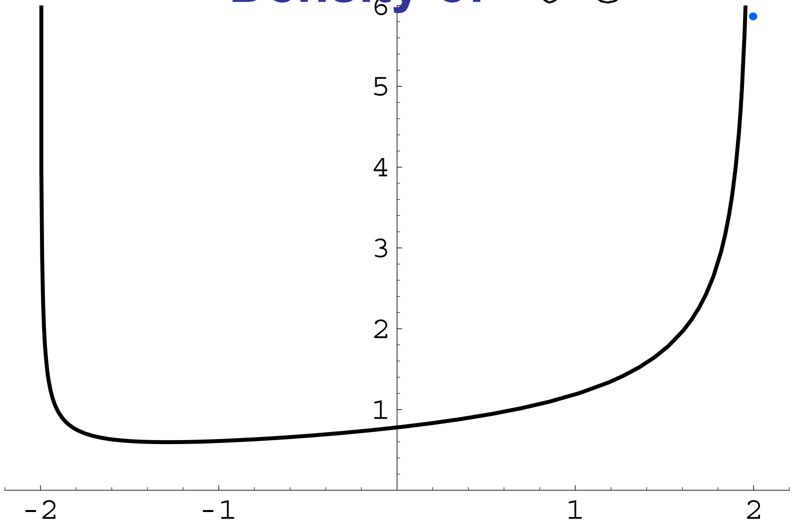
$$\frac{1}{2}\left(G_\theta(z+i0)+G_\theta(z-i0)\right) = -2\pi m, \qquad z \in \mathcal{C}_\theta = (-2,2)$$

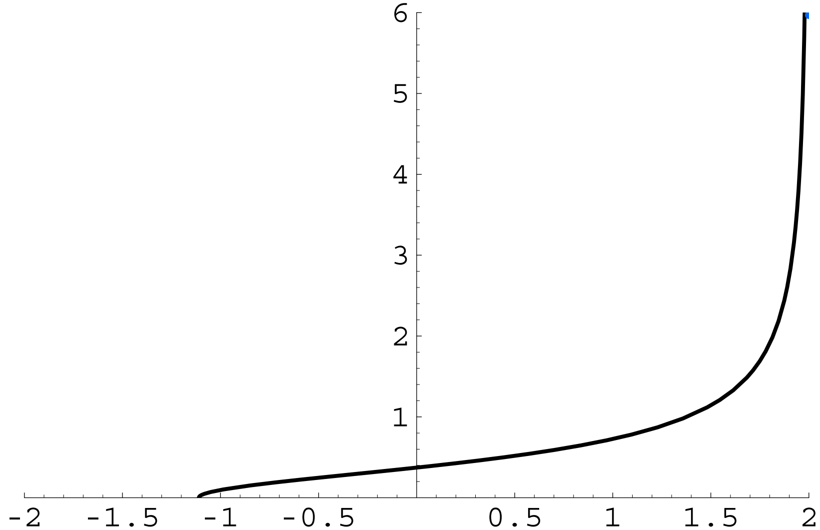
- Solution:

$$G_\theta(z) \equiv \int_{C_\theta} \frac{dy\rho_\theta(y)}{z-y} = \begin{cases} \left(\frac{2\pi m \;\; z+\frac{4\pi L}{\sqrt{\lambda}}}{\sqrt{z^2-4}} - 2\pi m\right), & L > |m|\sqrt{\lambda} \\ 2\pi m \left(\frac{\sqrt{z-2+\frac{4L}{m\sqrt{\lambda}}}}{\sqrt{z-2}} - 1\right), & L \le |m|\sqrt{\lambda} \end{cases}$$

Phase transition!

• **Density of** $\theta' s$





Phase transition at $\kappa_- = 0$

• Energy and Momentum

E, P expressed through residues $\kappa_\pm$ of G_θ at $z = \pm 2.$

$$E = \frac{\sqrt{\lambda}}{4}(\kappa_+^2 + \kappa_-^2) = \frac{L^2}{4\pi\sqrt{\lambda}} + 4\pi\sqrt{\lambda}m^2$$

$$P = \frac{\sqrt{\lambda}}{4}(\kappa_+^2 - \kappa_-^2) = mL$$

• Note that $G_\theta(z) \simeq \frac{2\pi\sqrt{\lambda}}{\sqrt{z \pm 2}} \kappa_\pm, \qquad z \to \pm 2.$

Classical (scaling) limit of full Bethe equations

- Define resolvents:

$$G_\theta(z) \equiv \sum_{\beta=1}^{L} \frac{1}{\frac{\sqrt{\lambda}}{4\pi}z - \theta_\beta} \quad , \quad G_u(z) \equiv \sum_{i=1}^{J_u} \frac{1}{\frac{\sqrt{\lambda}}{4\pi}z - u_i} \quad , \quad G_v(z) \equiv \sum_{l=1}^{J_v} \frac{1}{\frac{\sqrt{\lambda}}{4\pi}z - v_l}$$

and quasi-momenta:

$$p_1 = -p_2 = G_u - \frac{1}{2}G_\theta \qquad p_3 = -p_4 = G_v - \frac{1}{2}G_\theta$$

- Use $\log S_0(Mz) \simeq \frac{-i}{zM}, \qquad \log \frac{zM + i/2}{zM - i/2} \simeq \frac{i}{xM}, \qquad M = \frac{\sqrt{\lambda}}{4\pi} \to \infty$

and get

Classical Bethe eqs.

$$z \in C_u : \qquad p_1{}^{+} - p_2{}^{-} = 2\not{G}_u - G_\theta = 2\pi n_u$$

$$z \in C_\theta : \qquad p_2{}^{+} - p_3{}^{-} = -G_u - G_v + \not{G}_\theta = 2\pi m$$

$$z \in C_v : \qquad p_3{}^{+} - p_4{}^{-} = 2\not{G}_v - G_\theta = 2\pi n_v$$

$$z \in C_\theta : \qquad p_4{}^{+} - p_1{}^{-} = -G_u - G_v + \not{G}_\theta = 2\pi m$$

- They define an algebraic curve with 4 sheets!

- **Left and right global charges:** $\frac{4\pi}{\sqrt{\lambda}}Q_R = \frac{L}{2} - J_u, \quad \frac{4\pi}{\sqrt{\lambda}}Q_L = \frac{L}{2} - J_v$

$$p_1(z) \simeq -\frac{2\pi Q_L}{\sqrt{\lambda}}\frac{1}{z}, \qquad p_2(z) \simeq \frac{2\pi Q_L}{\sqrt{\lambda}}\frac{1}{z}$$

$$p_3(z) \simeq -\frac{2\pi Q_R}{\sqrt{\lambda}}\frac{1}{z}, \qquad p_4(z) \simeq \frac{2\pi Q_R}{\sqrt{\lambda}}\frac{1}{z}$$

- Energy and momentum through the poles at $z = \pm 2$ (obtained similarly to U(1) subsector):

$$p_{1,3}(z) = \mp\frac{\pi\kappa_\pm}{\sqrt{\pm z - 2}}, \qquad z \to \pm 2.$$

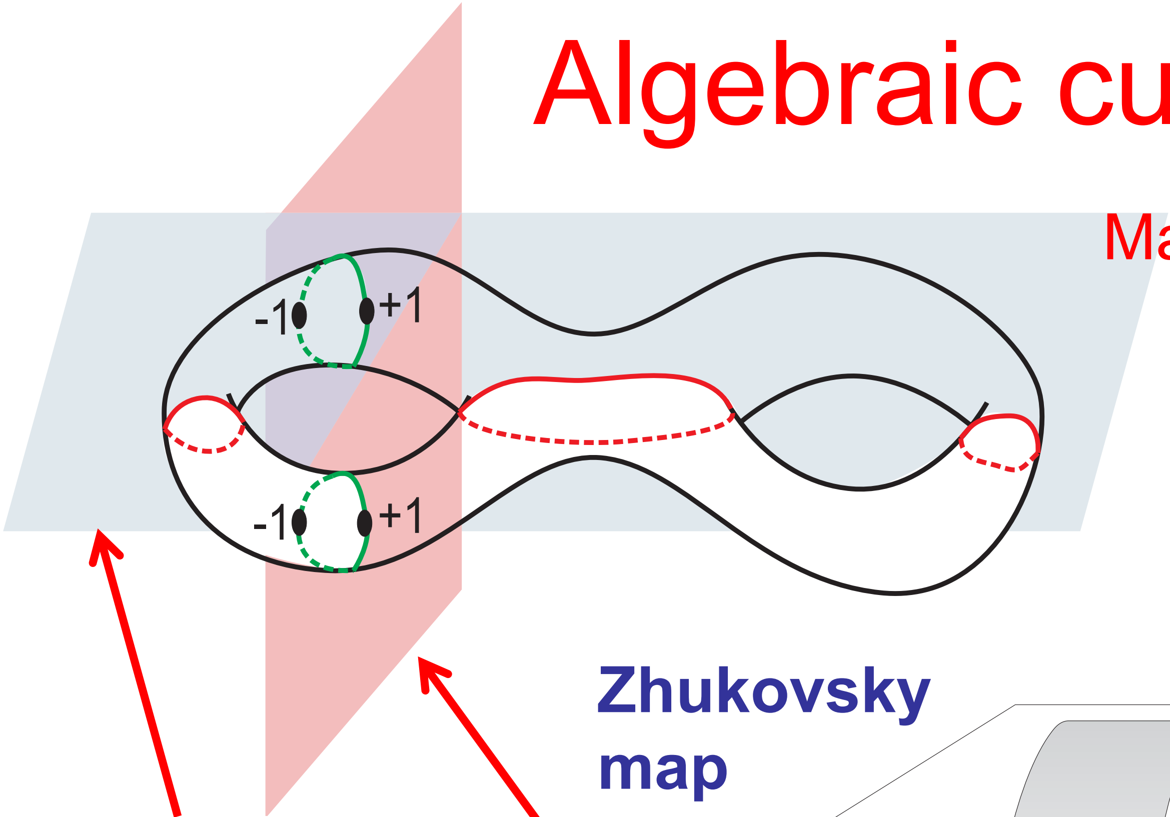
$$E \pm P = \frac{\sqrt{\lambda}}{2}\kappa_\pm^2$$

$E = \frac{\mu}{2\pi}\sum_\alpha \cosh(\pi\theta_\alpha)$ - energy (AdS time generator)

$$P = \frac{G(0)}{2\pi} = \frac{\mu}{2\pi}\sum_\alpha \sinh(\pi\theta_\alpha) = \sum_a n_a S_a^u + \sum_b n_b S_b^v - mL = 0$$

level matching for filling fractions

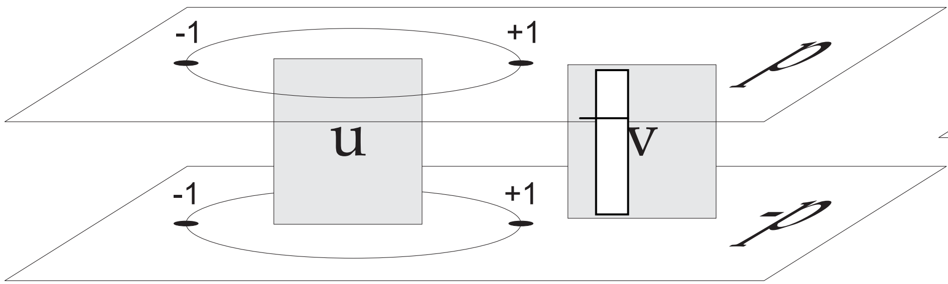
Algebraic curve



Map from Bethe ansatz to KMMZ finite gap solution

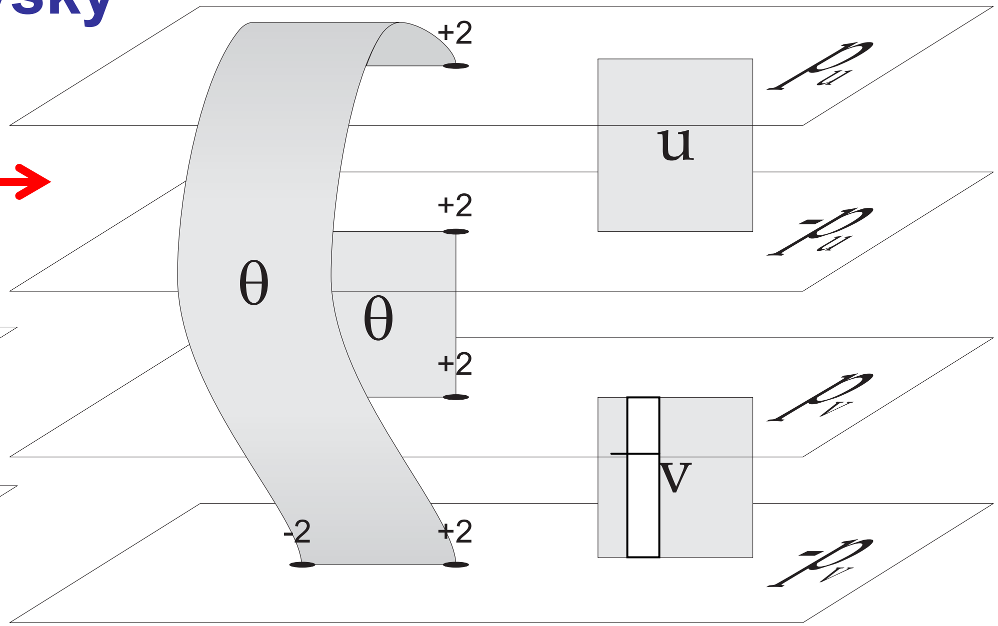
Zhukovsky map

$$x + \frac{1}{x} = z$$



From classical finite gap

[KMMZ, Minahan'05]



From Bethe ansatz

Recovery of classical KMMZ solutions for all string motions:

- By Zhukovsky map: $z = x + \frac{1}{x}, \quad x_\pm = \frac{1}{2}\left(z \pm \sqrt{z^2 - 4}\right)$

we reproduced from the full quantum theory the finite gap KMMZ equation for classical sigma model:

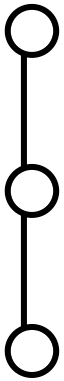
$$\tilde{p}\!\!\!/(x) = \pi n_{u,v}, \qquad x \in C_{u,v}.$$

$$\tilde{p}(x) \sim -\frac{2\pi Q_R}{\sqrt{\lambda}\, x}, \quad x \to \infty, \qquad \tilde{p}(x) \sim 2\pi m + \frac{2\pi Q_L}{\sqrt{\lambda}} x, \quad x \to 0.$$

$$\tilde{p}(x) \sim -\frac{\pi \kappa_\pm}{x \mp 1}, \quad x \to \pm 1$$

- The IDSC reproduces correctly the classical string limit!

SO(6) σ-model



$$e^{-i\mu \sinh \frac{\pi\theta_\alpha}{2}} = \prod_{\beta\neq\alpha}^{L} S_0(\theta_\alpha-\theta_\beta) \prod_{j=1}^{K_2} \frac{\theta_\alpha - u_j^{(2)} + i/2}{\theta_\alpha - u_j^{(2)} - i/2}$$

$$1 = \prod_{j\neq i}^{K_1} \frac{u_i^{(1)} - u_j^{(1)} + i}{u_i^{(1)} - u_j^{(1)} - i} \prod_{j=1}^{K_2} \frac{u_i^{(1)} - u_j^{(2)} - i/2}{u_i^{(1)} - u_j^{(2)} + i/2}$$

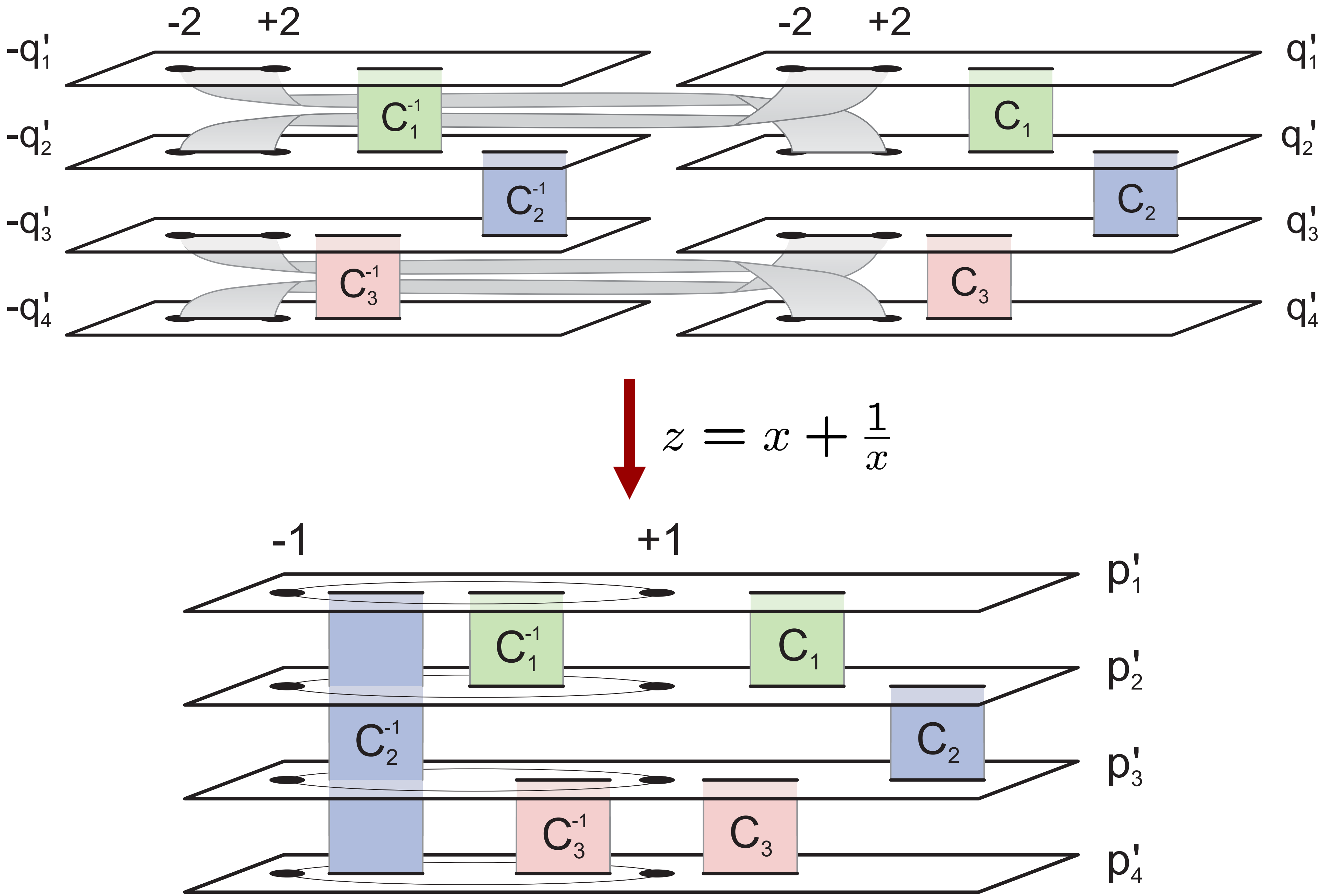
$$\prod_{\alpha=1}^{L} \frac{u_i^{(2)} - \theta_\alpha + i/2}{u_i^{(2)} - \theta_\alpha - i/2} = \prod_{j\neq i}^{K_2} \frac{u_i^{(2)} - u_j^{(2)} + i}{u_i^{(2)} - u_j^{(2)} - i} \prod_{j=1}^{K_3} \frac{u_i^{(2)} - u_j^{(3)} - i/2}{u_i^{(2)} - u_j^{(3)} + i/2} \prod_{j=1}^{K_1} \frac{u_i^{(2)} - u_j^{(1)} - i/2}{u_i^{(2)} - u_j^{(1)} + i/2}$$

$$1 = \prod_{j\neq i}^{K_3} \frac{u_i^{(3)} - u_j^{(3)} + i}{u_i^{(3)} - u_j^{(3)} - i} \prod_{j=1}^{K_2} \frac{u_i^{(3)} - u_j^{(2)} - i/2}{u_i^{(3)} - u_j^{(2)} + i/2}$$

- Bethe eqs. follow the Dynkin diagram pattern.

$S_0(\theta)$ is known [(Zamolodchikov)x2 '79].

- Classical algebraic curve coincides, after Zhukovski map, with the finite gap solution of [Beisert,Sakai,V.K'04]


-2
+2
-q'1
-q'2
-q'3
-q'4
-2
+2
q'1
q'2
q'3
q'4
C-1 1
C-1 2
C-1 3
C1
C2
C3
z = x + 1/x
-1
+1
p'1
p'2
p'3
p'4
C-1 1
C-1 2
C-1 3
C1
C2
C3

[Arutynov,Frolov,Staudacher'06]

Asymptotic string Bethe ansatz (AFS) from dynamical chain [Gromov,V.K.'06]

$$e^{-ip(\theta)} = \prod_{\beta(\neq\alpha)}^{L} S_0^2\left(\theta_\alpha - \theta_\beta\right) \prod_j^K \frac{\theta_\alpha - u_j + i/2}{\theta_\alpha - u_j - i/2},$$

$$1 = \prod_\beta^L \frac{u_j - \theta_\beta - i/2}{u_j - \theta_\beta + i/2} \prod_{k(\neq j)}^K \frac{u_j - u_k + i}{u_j - u_k - i}$$

- Note that for $p(\theta) = -L\,\mathrm{arcsin}(\theta/g), \quad S_0^2 = -1$ one gets the Lieb-Wu eqs for Hubbard model describing N=4 SYM at 3 (and may be all) loops. **[Rej,Serban,Staudacher'06]**

- *We restore from dynamical spin chain the asymptotic BA eq.(AFS* **for long spin chain** $L \to \infty$ **and** θ's continuously distributed in a square box (conformal limit).
- AFS appr. contains "giant magnons": non-smooth classical configurations of strings described in [Hofman,Maldacena'06].

- Calculation of density of rapidities θ_α from our BAE's: take log of first BAE and get a Riemann-Hilbert problem

$$\mathcal{G}_\theta(z|\{u_j\}) + 2\pi m = i \sum_{j=1}^{K} \log \frac{z\frac{\sqrt{\lambda}}{4\pi} - u_j + i/2}{z\frac{\sqrt{\lambda}}{4\pi} - u_j - i/2}, \qquad |z| \leq 2$$

- Impose a one cut distribution: analogue of Virasoro conditions.

- Solution, in terms of Zhukovsky variables

$$z = x + 1/x$$
$$x \equiv \frac{1}{2}\left(z + \sqrt{z^2 - 4}\right)$$
$$y_j^\pm = x(u \pm i/2)$$

$$G\left(z(x)\right) = \frac{Ax + B}{x^2 - 1} + 2i \sum_{j=1}^{K} \log \frac{x - 1/y_j^+}{x - 1/y_j^-}$$

with definitions: $A = \sum_{j=1}^{K} \left(\frac{2i}{y_j^+} - \frac{2i}{y_j^-}\right) + \frac{4\pi L}{\sqrt{\lambda}}$, $\quad B = 4\pi m - 2i \sum_{j=1}^{K} \log \frac{y_j^+}{y_j^-}$

- Zero momentum condition imposed: $\prod_j \frac{y_j^+}{y_j^-} = 1$

- AFS formula comes from the second (magnon) BAE: take its log and exclude rapidities θ_α using their density

$$2\pi n_k + i\sum_k \log\frac{u_k-u_j+i/2}{u_k-u_j-i/2} = \oint_{(-2,2)} G(z)\,\log\frac{z\frac{\sqrt{\lambda}}{4\pi}-u_k+i/2}{z\frac{\sqrt{\lambda}}{4\pi}-u_k-i/2}$$

- The calculation gives precisely the AFS formula

$$\left(\frac{y_k^+}{y_k^-}\right)^L = \prod_{j=1}^{K} \underbrace{\left(\frac{y_k^+-y_j^-}{y_k^- - y_j^+}\right)\frac{1-1/(y_j^- y_k^+)}{1-1/(y_j^+ y_k^-)}}_{\frac{u_k-u_j+i}{u_k-u_j-i}}\,\sigma^2(y_k,y_j)$$

where

$$\sigma(y_k,y_j) = \frac{1-1/(y_j^+ y_k^-)}{1-1/(y_j^- y_k^+)}\left(\frac{(y_j^- y_k^- -1)}{(y_j^- y_k^+ -1)}\frac{(y_j^+ y_k^+ -1)}{(y_j^+ y_k^- -1)}\right)^{i(u_j-u_k)}$$

- $\sigma = 1$ corresponds to Hubbard model (all loop SYM).

- Energy (dimension) $\Delta = L + i\frac{\sqrt{\lambda}}{2\pi}\sum_{j=1}^{K}\left(\frac{1}{y_j^+} - \frac{1}{y_j^-}\right)$
- Momentum: $P = \left(m - \frac{i}{2\pi}\sum_{j=1}^{K}\log\frac{y_j^+}{y_j^-}\right)\Delta = 0$

they follow from relativistic formulas for E, P

- In terms of individual momenta of magnons $p = -i\log\frac{y^+}{y^-}$

AFS eqs. are periodic in p: dispersion relation for magnon

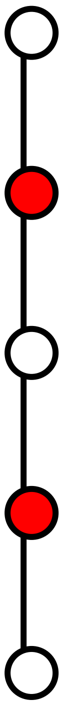
$$i\frac{\sqrt{\lambda}}{2\pi}\left(\frac{1}{y^+} - \frac{1}{y^-}\right) = \sqrt{1 + \frac{\lambda}{\pi^2}\sin^2(p/2)} - 1$$

as well as the effective S-matrix.

*

Full theory: Beisert-Staudacher eqs. :

psu(2,2|4)



$$1 = \prod_k e_{-2}\left(u_j^{(2)} - u_k^{(2)}\right) e_{+1}\left(u_j^{(2)} - u_k^{(3)}\right)$$

$$1 = \prod_k e_{-1}\left(u_j^{(3)} - u_k^{(2)}\right) r_+\left(u_j^{(3)}, u_k^{(4)}\right)$$

$$\left(\frac{x_j^{(4)+}}{x_j^{(4)-}}\right)^L = \prod_k \sigma^2(x_j^{(4)}|x_k^{(4)}) r_-\left(u_j^{(5)}, u_k^{(4)}\right) e_{+2}\left(u_j^{(4)} - u_k^{(4)}\right) r_-\left(u_j^{(3)}, u_k^{(4)}\right)$$

$$1 = \prod_k r_+\left(u_j^{(5)}, u_k^{(4)}\right) e_{-1}\left(u_j^{(5)} - u_k^{(6)}\right)$$

$$1 = \prod_k e_{+1}\left(u_j^{(6)} - u_k^{(5)}\right) e_{-2}\left(u_j^{(6)} - u_k^{(6)}\right)$$

where

$$e_k(u) = \frac{u + ik/2}{u - ik/2}, \quad r_\pm(u, \tilde{u}) = \frac{x - \tilde{x}^\pm}{x - \tilde{x}^\mp}.$$

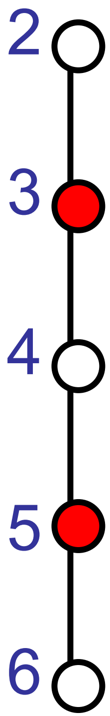
$$u = x + 1/x, \quad x(u) = u + \sqrt{u^2 - 4}, \quad x_\pm = x(u \pm i/2)$$

Can we represent it by inhomogeneous chain?

Full theory: **Naively, we could write the BS eqs. as follows:**

psu(2,2|4)

$$e^{-iP(\theta_\alpha)} = \prod_{\beta=1}^{L} S_0(\theta_\alpha - \theta_\beta) \prod_k e_{+1}(\theta_\alpha - u_k^{(4)})$$



$$1 = \prod_k \; e_{-2}\left(u_j^{(2)} - u_k^{(2)}\right) e_{+1}\left(u_j^{(2)} - u_k^{(3)}\right)$$

$$1 = \prod_k e_{-1}\left(u_j^{(3)} - u_k^{(2)}\right) \; r_+\left(u_j^{(3)}, u_k^{(4)}\right)$$

$$\prod_\beta e_{+1}\left(u_j^{(4)} - \theta_\beta\right) = \prod_k r_-\left(u_j^{(5)}, u_k^{(4)}\right) e_{+2}\left(u_j^{(4)} - u_k^{(4)}\right) r_-\left(u_j^{(3)}, u_k^{(4)}\right)$$

$$1 = \prod_k r_+\left(u_j^{(5)}, u_k^{(4)}\right) \; e_{-1}\left(u_j^{(5)} - u_k^{(6)}\right)$$

$$1 = \prod_k e_{+1}\left(u_j^{(6)} - u_k^{(5)}\right) e_{-2}\left(u_j^{(6)} - u_k^{(6)}\right)$$

The BS eqs are reproduced in the limit $L \longrightarrow \infty.$

Beisert,Staudacher (comments)

But the supersymmetry among multiplets is broken.
May be, a useful building block for the future.

Problems

- Find a dynamical chain reproducing the full perturbation string Bethe ansatz of **[Beisert,Staudacher'05]** (for the moment we reproduced only $S^3 \times R$ sector): Zero-level excitations.

- Define scalar factor $S_0(\theta)$ and dispersion $P(\theta)$ (the cross-symmetry, similar to **[Janik'06]** or perturbative S-matrix calculations of **[Klose,Zarembo'06] might help).**

- Quantum $1/\sqrt{\lambda}$ corrections should reproduce in a regular way the results of **[Schafer-Nameki,Zamaklar'05], [Beisert,Tseytlin'05], [Frolov,Tseytlin'02], [Arutyunov,Frolov'06], [Hernandes,Lopez'06], [Freyhult,Kristjansen,'06]**

- Quantum $1/L$ corrections: see **[Beisert,Tseytlin,Zarembo'05], [Beisert, Freyhult'06] [Gromov,V.K.'05], [Minahan, Tirziu, Tseytlin'05]**

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Observations about the workshop and the subject

Achievements:

- Asymptotic BA and S-matrix for the full AdS/CFT.
- Scalar factor: its origins, crossing eq., quantum corrections.
- Classical superstring is well understood.
- Excellent results up to 3 loops: “theory” versus “experiment” (hopes on 4 loops).
- Interest to the subject from pure “integrists”.

Problems:

- Finite operators from superstring.
- Contradiction between asymptotic BA and superstring predictions, related to finite radius of convergence in λ.
- Tree loop divergency ….
- May be, after all, it is a math. problem for (super)algebraists?