QCD properties of twist operators in the $\mathcal{N} = 6$ Chern-Simons theory

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QCD & ABJM

In QCD, twist-2 anomalous dimensions $\gamma(N)$ enter the evolution equations of deep inelastic scattering (DIS) and are physically very relevant. The analysis of $\gamma(N)$ suggests, among others, the following three basic predictions about the large N (quasi-elastic) regime:

(a) *Logarithmic scaling.* The large N dominant term is logarithmic, $\gamma(N) \sim f(g) \log N$, where $f(g)$ is the universal cusp anomalous dimension

(b) *Gribov-Lipatov reciprocity.* This is a crossing relation which implies an infinite set of constraints on the subleading terms appearing in the large N expansion of $\gamma(N)$

(c) *Low-Burnett-Kroll wisdom.* The anomalous dimension $\gamma(N)$ develops high powers of $\log N$ increasing with the perturbative order, the leading terms being of the form $(\log(N)/N)^k$. Nevertheless, many of these terms are *inherited* from lower order calculations. This is independent on (b) and can be traced back to quite general old results simply related to gauge invariance

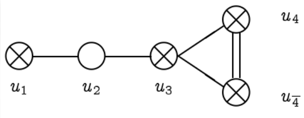
We believe that these are valid motivations for investigating the QCD-inspired properties (a), (b) and (c) in the ABJM theory. This is a three dimensional $U(N) \times U(N)$ gauge theory with four complex scalars in the $(N, \overline{N})$ representation, their fermionic partners, and a Chern-Simons action with levels $+k$, $-k$. This theory has $\mathcal{N} = 6$ superconformal symmetry $\mathfrak{osp}(2, 2|6)$. ABJM can be considered as the low energy theory of N parallel M2-branes at a $\mathbb{C}^4/\mathbb{Z}_k$ singularity. In the large N limit this is M theory on $AdS_4 \times S^7/\mathbb{Z}_k$. For fixed $\lambda = N/k$ we can describe it by type IIA string on $AdS_4 \times \mathbb{CP}^3$ which is classically integrable.The manifest (non abelian) part of the R symmetry is $SU(2) \times SU(2)$. The complex scalars can be written as two doublets transforming as $(2, 1)$ and $(1, 2)$. Under the gauge group they transform as $(N, \overline{N})$ and $(\overline{N}, N)$. At leading order (two loops, λ^2 in 't Hooft coupling λ), the dilatation operator for single trace operators built with these scalars is a $SU(4)$ integrable spin chain

Twist operators

Twist operators can be found in a $\mathfrak{sl}(2)$-like sector of ABJM. At strong coupling and large spin they behave quite similarly to the corresponding ones in $AdS_5 \times S^5$: their dual string state is a folded string rotating in AdS_3 with large spin N and with angular momentum $J \sim \log N$ in $\mathbb{CP}^3$ in close analogy to the folded string solution in $AdS_5 \times S^5$. At weak coupling, they are composite operators in totally different theories. Nevertheless, both $\mathcal{N} = 6$ SCS and $\mathcal{N} = 4$ SYM are integrable and the all-loop Bethe equations in the $\mathfrak{sl}(2)$ sectors are very close. Also, from the leading order analysis of twist-1 operators it seems that maximal transcendentality Ansätze are feasible.

Twist operators in the $\mathfrak{sl}(2)$ sector of ABJM

The all-loop Bethe equations for ABJM can be summarized by the $\mathfrak{osp}(2,2|6)$ diagram; we consider twist operators in the $\mathfrak{sl}(2)$ sector where we excite symmetrically the same number N of u_4 and $u_{\bar{4}}$ roots.



As in the $\mathcal{N} = 4$ case, the integer L can be identified with the twist of the operator. The Bethe equations involve the deformed spectral parameters $x^{\pm}$ defined by

$$x^{\pm} + \frac{1}{x^{\pm}} = \frac{1}{h}\left(u \pm \frac{i}{2}\right),$$

where $h(\lambda)$ is the interpolating coupling appearing in the one-magnon dispersion relation. For twist L operators they read

$$\left(\frac{x_k^+}{x_k^-}\right)^L = -\prod_{j\neq k}^{N} \frac{u_k - u_j + i}{u_k - u_j - i}\left(\frac{x_k^- - x_j^+}{x_k^+ - x_j^-}\right)^2 \sigma_{\mathrm{BES}}^2 .$$

The only difference compared with $\mathcal{N} = 4$ SYM is the extra minus sign. This will turn out to be definitely relevant to our analysis. The factor σ_{BES} is the dressing phase which will play no role at the perturbative order explored in this paper.

Twist-1

The two-loop problem & The four-loop ABA result

The two- and four-loops anomalous dimension can be computed exactly (using maximal and uniform trascendentality) and reads

$$\gamma_2^{\mathrm{ABA}}(N) = \sum_k \frac{2}{u_k^2 + \frac{1}{4}} = 4\left[S_1(N) - S_{-1}(N)\right] \quad \text{with the usual def} \quad S_{a,b,\dots}(N) = \sum_{n=1}^{N} \frac{(\mathrm{sign}\,a)^n}{n^{|a|}} S_{b,\dots}(n)$$

$$\gamma_4^{\mathrm{ABA}}(N) = -16(S_{-3} - S_3 + S_{-2,-1} - S_{-2,1} + S_{-1,-2} - S_{-1,2} - S_{1,-2} + S_{1,2} - S_{2,-1} + S_{2,1} + \\ + S_{-1,-1,-1} - S_{-1,-1,1} - S_{1,-1,-1} + S_{1,-1,1}).$$

Also, we notice the following remarkable *shift symmetry*

$$\gamma_{2,4}^{\mathrm{ABA}}(2\,n+1) = \gamma_{2,4}^{\mathrm{ABA}}(2\,n+2), \qquad n \in \mathbb{N}.$$

The four-loop Wrapping Contribution

The full anomalous dimension of twist-1 operators receives a wrapping contribution at four loops

$$\gamma_4(N) = \gamma_4^{\mathrm{ABA}}(N) + \gamma_4^{\mathrm{wrap}}(N) \qquad \gamma_4^{wrap} = \gamma_2(N) \cdot \mathcal{W}(N),$$

with

$$\mathcal{W}(N) = \sum_{Q=1}^{\infty} \int_{-\infty}^{\infty} dq\, \mathcal{W}(q,Q,N), \quad \mathcal{W}(q,Q,N) = -\frac{1}{2\,\pi}\, \frac{4}{q^2 + Q^2}\, \mathcal{S}(q,Q,N)\, \mathcal{M}(q,Q,N),$$

and (Q_N is the leading order Baxter function)

$$\mathcal{S}(q,Q,N) = (-1)^Q \frac{Q_N\left(\frac{q-i(Q-1)}{2}\right)}{Q_N\left(\frac{q-i(Q+1)}{2}\right) Q_N\left(\frac{q+i(Q-1)}{2}\right) Q_N\left(\frac{q+i(Q+1)}{2}\right)},$$

$$\mathcal{M}(q,Q,N) = 2\sum_{j=0}^{Q-1}\left[\frac{Q_N\left(\frac{q-i(Q-1)+2\,i\,j}{2}\right)}{Q_N\left(\frac{q-i(Q-1)}{2}\right)}\right]^2 \left[\frac{1}{2\,j-i\,q-Q} - \frac{1}{2\,(j+1)-i\,q-Q}\right].$$

This formula takes into account the different $SU(2|2)$ structure of the S-matrix as compared with $\mathcal{N}=4$. Under summation over Q, the integral can be evaluated in terms of the kinematical residue

$$\mathcal{W}(N) = 2\,\pi\,i\sum_{Q=1}^{\infty}\operatorname*{Res}_{q=i\,Q}\mathcal{W}(q,Q,N) \rightarrow \mathcal{W}(N) = -2\,\zeta_2 + r_N,$$

where r_N is a rational number. Unfortunately, we have been unable to find a closed formula for r_N. However, we can show that at large N the leading term in $\mathcal{W}$ is

$$\mathcal{W}(N) = -\frac{2\,\log\,2}{N} + \text{subleading}.$$

In particular, this proves that the cusp anomaly is not modified by the wrapping which goes like $(\log\,N)/N$ at large N.

The six-loop ABA result

$\gamma_6^{\rm ABA}(N) =$ a very long formula ... the *shift symmetry* is broken. We do not know whether it must be an all-order property of the anomalous dimension. To any extent, the six loop result is affected by next-to-leading wrapping contributions ...

Twist-2

The two-loop problem & The four-loop ABA result

Again, the anomalous dimension can be computed exactly and reads

$$\gamma_2^{\rm ABA}(N) = \sum_k \frac{2}{u_k^2 + \frac{1}{4}} = 4\left[S_1(N) + S_{-1}(N)\right].$$

As in the twist-1 case, there is a *shift symmetry* since $\gamma_2^{\rm ABA}$ enjoys the exact property

$$\gamma_2^{\rm ABA}(2\,n+1) = \gamma_2^{\rm ABA}(2\,n), \qquad n \in \mathbb{N}.$$

After some calculation, we obtain

$$\begin{aligned} \gamma_4^{ABA}(N) \quad = \quad & 16S_{-3} + 16S_3 - 8S_{-2,-1} - 8S_{-2,1} - 16S_{-1,-2} + \\ & -16S_{-1,2} - 16S_{1,-2} - 16S_{1,2} - 8S_{2,-1} - 8S_{2,1}. \end{aligned}$$

Eq. (1) is proved by means of the NLO Baxter equation and has again the exact property

$$\gamma_4^{\rm ABA}(2\,n+1) = \gamma_4^{\rm ABA}(2\,n), \qquad n \in \mathbb{N}.$$

The six-loop ABA result

$$
\begin{aligned}
\gamma_6^{\mathrm{ABA}}(N) = {} & 128\left(S_{-5}+S_{5}\right) - 192\left(S_{-1,-4}+S_{1,-4}\right) - 192\left(S_{-1,4}+S_{1,4}\right) - 256\left(S_{-2,-3}+S_{2,-3}\right) \\
& -256\left(S_{-2,3}+S_{2,3}\right) - 160\left(S_{-3,-2}+S_{3,-2}\right) - 160\left(S_{-3,2}+S_{3,2}\right) - 128\left(S_{-4,-1}+S_{4,-1}\right) \\
& -128\left(S_{-4,1}+S_{4,1}\right) + 96\left(S_{-1,-3,-1}+S_{1,-3,-1}\right) + 96\left(S_{-1,-3,1}+S_{1,-3,1}\right) \\
& +96\left(S_{-1,-2,-2}+S_{1,-2,-2}\right) + 96\left(S_{-1,-2,2}+S_{1,-2,2}\right) + 128\left(S_{-1,-1,-3}+S_{1,-1,-3}\right) \\
& +128\left(S_{-1,-1,3}+S_{1,-1,3}\right) + 128\left(S_{-1,1,-3}+S_{1,1,-3}\right) + 128\left(S_{-1,1,3}+S_{1,1,3}\right) \\
& +96\left(S_{-1,2,-2}+S_{1,2,-2}\right) + 96\left(S_{-1,2,2}+S_{1,2,2}\right) + 96\left(S_{-1,3,-1}+S_{1,3,-1}\right) + 96\left(S_{-1,3,1}+S_{1,3,1}\right) \\
& +96\left(S_{-2,-2,-1}+S_{2,-2,-1}\right) + 96\left(S_{-2,-2,1}+S_{2,-2,1}\right) + 80\left(S_{-2,-1,-2}+S_{2,-1,-2}\right) \\
& +80\left(S_{-2,-1,2}+S_{2,-1,2}\right) + 80\left(S_{-2,1,-2}+S_{2,1,-2}\right) + 80\left(S_{-2,1,2}+S_{2,1,2}\right) \\
& +96\left(S_{-2,2,-1}+S_{2,2,-1}\right) + 96\left(S_{-2,2,1}+S_{2,2,1}\right) + 32\left(S_{-3,-1,-1}+S_{3,-1,-1}\right) \\
& +32\left(S_{-3,-1,1}+S_{3,-1,1}\right) + 32\left(S_{-3,1,-1}+S_{3,1,-1}\right) + 32\left(S_{-3,1,1}+S_{3,1,1}\right) \\
& -32\left(S_{-1,-1,-2,-1}+S_{1,-1,-2,-1}\right) - 32\left(S_{-1,-1,-2,1}+S_{1,-1,-2,1}\right) - 32\left(S_{-1,-1,2,-1}+S_{1,-1,2,-1}\right) \\
& -32\left(S_{-1,-1,2,1}+S_{1,-1,2,1}\right) - 32\left(S_{-1,1,-2,-1}+S_{1,1,-2,-1}\right) - 32\left(S_{-1,1,-2,1}+S_{1,1,-2,1}\right) \\
& -32\left(S_{-1,1,2,-1}+S_{1,1,2,-1}\right) - 32\left(S_{-1,1,2,1}+S_{1,1,2,1}\right).
\end{aligned}
$$

Remarkably, *shift symmetry* is not broken. Wrapping effects are expected to show up at this order.

Reciprocity and LBK wisdom: Large N analysis

Our main aim is the analysis of possible QCD-inspired properties showing up in $\gamma_{2n}^{\text{ABA}}(N)$: mimicking the $\mathcal{N}=4$ case, we shall work out the expansion of the anomalous dimensions at large N and look for peculiar properties

$$\gamma(N) = \alpha(N) + (-1)^N\,\beta(N),$$

where α and β have a smooth expansion in $1/N$ with possible logarithmic enhancements. We shall consider the even N case. The general form of the large N expansion is expected to be

$$\gamma(N) = f_{CS}(h)\,\log\,N + \sum_{a=1}^{\infty}\frac{1}{N^a}\sum_{b=0}^{a} g_{a,b}(h)\,\log^b\,N.$$

We have already checked that the leading cusp logarithm is in agreement with property (a) Concerning properties (b)-(c), they are conveniently expressed in terms of the function P defined order by order in h by the functional relation

$$\gamma(N) = P\left(N+\frac{1}{2}\,\gamma(N)\right).$$

The large N expansion of P is similar to that of γ and reads

$$P(N) = f_{CS}(h)\,\log\,N + \sum_{a=1}^{\infty}\frac{1}{N^a}\sum_{b=0}^{a} p_{a,b}(h)\,\log^b\,N.$$

The Gribov-Lipatov reciprocity and LBK cancellations can be concisely expressed as follows.

- *Gribov-Lipatov reciprocity.* There is a constant κ such that the large N expansion of $P(N)$ runs in integer inverse powers of $J^2 = N\,(N+\kappa)$.
- *Low-Burnett-Kroll cancellations.* Some (maximal) logarithms are missing in Eq. (8). This implies that there are inheritance relations among the logarithms of Eq. (8).

We recall once again that these seemingly technical conditions have a clear physical origin in the QCD context and are widely checked in $\mathcal{N}=4$. It remains to look for their manifestation in ABJM, at least at the level of the multi-loop asymptotic anomalous dimensions.

Twist-1

We define $\overline{n} = N\, e^{\gamma_E}$ and consider even N. The expansion of the two loop anomalous dimensions is

$$\begin{aligned}
\gamma_2^{\rm ABA} &= 4\log(2\,\overline{n}) + \frac{2}{3\,n^2} - \frac{7}{15\,n^4} + \frac{62}{63\,n^6} - \frac{127}{30\,n^8} + \cdots \\
\gamma_4^{\rm ABA} &= \left(-\frac{4}{3}\pi^2\log(2\,\overline{n}) - 12\zeta_3\right) + \frac{8\log(2\,\overline{n})}{n} + \left(2\log(2\,\overline{n}) - \frac{2\pi^2}{9} + 2\right)\frac{1}{n^2} + \\
&+ \left(\frac{4}{3} - \frac{8}{3}\log(2\,\overline{n})\right)\frac{1}{n^3} + \left(-\frac{5}{2}\log(2\,\overline{n}) + \frac{7\pi^2}{45} + \frac{1}{12}\right)\frac{1}{n^4} + \\
&+ \left(\frac{56}{15}\log(2\,\overline{n}) - \frac{62}{45}\right)\frac{1}{n^5} + \left(7\log(2\,\overline{n}) - \frac{62\pi^2}{189} - \frac{269}{60}\right)\frac{1}{n^6} + \\
&+ \left(\frac{914}{315} - \frac{248}{21}\log(2\,\overline{n})\right)\frac{1}{n^7} + \left(-\frac{285}{8}\log(2\,\overline{n}) + \frac{127\pi^2}{90} + \frac{76613}{2016}\right)\frac{1}{n^8} + \cdots
\end{aligned}$$

For twist-1, we do not discuss the six-loop result which is heavily affected by the wrapping corrections. The two loop result is parity invariant under the transformation ($\kappa = 0$ in the Gribov-Lipatov reciprocity)

$$n \longrightarrow -n, \quad \log n = \frac{1}{2}\log(n^2) \longrightarrow \log n\,.$$

This is not a symmetry of the four loop result. Nevertheless, we can look at the four loop P function ($P = \sum_{n=1}^{\infty} P_{2n}\, h^{2n}$)

$$P_4 = \gamma_4 - \frac{1}{2}\gamma_2\,\gamma_2'\,.$$

and its expansion is

$$P_4 = \left(-\frac{4}{3}\pi^2 \log(2\,\overline{n}) - 12\zeta_3\right) + \left(2\log(2\,\overline{n}) - \frac{2\pi^2}{9} + 2\right)\frac{1}{n^2} +$$
$$+ \left(-\frac{5}{2}\log(2\,\overline{n}) + \frac{7\pi^2}{45} + \frac{1}{12}\right)\frac{1}{n^4} + \left(7\log(2\,\overline{n}) - \frac{62\pi^2}{189} - \frac{269}{60}\right)\frac{1}{n^6} +$$
$$+ \left(-\frac{285}{8}\log(2\,\overline{n}) + \frac{127\pi^2}{90} + \frac{76613}{2016}\right)\frac{1}{n^8} + \cdots$$

We see that P_4 is indeed parity invariant ! This structure implies that all terms in γ_4 odd under $n \longrightarrow -n$ are precisely *inherited* from the two-loop anomalous dimension

$$\gamma_4^{\rm odd} = \frac{1}{2}\,\gamma_2\,\gamma_2'.$$

We can summarize the result by saying that *the twist-1 ABA anomalous dimension is reciprocity respecting under $n \longrightarrow -n$ up to four loops.* Instead, no LBK cancellation is observed. The logarithmic enhancement which are observed in γ_4 are the same as in P_4. This means that the single logarithms appearing in γ_4 are not related to the lowest order γ_2.

Twist-2

For twist-2, we use the variable $n = N/2$ and we find, with $\overline{n} = n\,e^{\gamma_E}$, the expansions at 2 and 4 loops

$$\gamma_2^{\rm ABA} = 4\log\overline{n} + \frac{2}{n} - \frac{1}{3\,n^2} + \frac{1}{30\,n^4} - \frac{1}{63\,n^6} + \frac{1}{60\,n^8} + \cdots,$$
$$\gamma_4^{\rm ABA} = \left(4\zeta_3 - \frac{4}{3}\pi^2\log\overline{n}\right) + \left(4\log\overline{n} - \frac{2\pi^2}{3} - 4\right)\frac{1}{n} + \left(-2\log\overline{n} + \frac{\pi^2}{9} + 5\right)\frac{1}{n^2} +$$
$$+ \left(\frac{2\log\overline{n}}{3} - \frac{29}{9}\right)\frac{1}{n^3} + \left(\frac{4}{3} - \frac{\pi^2}{90}\right)\frac{1}{n^4} + \left(-\frac{2\log\overline{n}}{15} - \frac{19}{225}\right)\frac{1}{n^5} +$$
$$+ \left(-\frac{19}{60} + \frac{\pi^2}{189}\right)\frac{1}{n^6} + \left(\frac{2\log\overline{n}}{21} + \frac{41}{588}\right)\frac{1}{n^7} + \left(\frac{17}{63} - \frac{\pi^2}{180}\right)\frac{1}{n^8} + \cdots,$$

as well as the six-loop result

$$
\begin{aligned}
\gamma_6^{\text{ABA}} &= \left(\frac{44}{45}\pi^4 \log\overline{n} - 88\zeta_5\right) + \frac{-\frac{10}{3}\pi^2\log\overline{n} + 16\log\overline{n} + 4\zeta_3 + \frac{22\pi^4}{45} + \frac{2\pi^2}{3} + 48}{n} + \\
&+ \left(-2\log^2\overline{n} + \frac{5}{3}\pi^2\log\overline{n} + 2\log\overline{n} - 2\zeta_3 - \frac{11\pi^4}{135} - \frac{5\pi^2}{2} - 20\right)\frac{1}{n^2} + \\
&+ \left(2\log^2\overline{n} - \frac{5}{9}\pi^2\log\overline{n} - \frac{292\log\overline{n}}{27} + \frac{2\zeta_3}{3} + \frac{95\pi^2}{54} + \frac{446}{27}\right)\frac{1}{n^3} + \\
&+ \left(-\log^2\overline{n} + \frac{193\log\overline{n}}{18} + \frac{11\pi^4}{1350} - \frac{13\pi^2}{18} - \frac{416}{27}\right)\frac{1}{n^4} + \left(\frac{1}{9}\pi^2\log\overline{n} - \frac{19007\log\overline{n}}{3375} - \frac{2\zeta_3}{15} + \frac{13\pi^2}{450} + \frac{2365123}{202500}\right)\frac{1}{n^5} + \\
&+ \left(\frac{\log^2\overline{n}}{3} + \frac{13\log\overline{n}}{150} - \frac{11\pi^4}{2835} + \frac{59\pi^2}{360} - \frac{19603}{3375}\right)\frac{1}{n^6} + \left(-\frac{5}{63}\pi^2\log\overline{n} + \frac{1006399\log\overline{n}}{463050} + \frac{2\zeta_3}{21} - \frac{643\pi^2}{52920} + \frac{21409879}{64827000}\right)\frac{1}{n^7} + \\
&+ \left(-\frac{1}{3}\log^2\overline{n} - \frac{107\log(\overline{n})}{588} + \frac{11\pi^4}{2700} - \frac{26\pi^2}{189} + \frac{917411}{463050}\right)\frac{1}{n^8} + \cdots .
\end{aligned}
$$

Again, possible structures are best investigated by looking at the P functions. Using $M = N/2$ as argument and inverting the relation

$$
\gamma^{\text{ABA}}(M) = P\left(M + \frac{1}{4}\gamma^{\text{ABA}}(M)\right),
$$

we get the expressions

$$
\begin{aligned}
P_2 &= \gamma_2, \\
P_4 &= \gamma_4 - \frac{1}{4}\gamma_2\,\gamma_2', \\
P_6 &= \gamma_6 - \frac{1}{4}\gamma_4\,\gamma_2' + \frac{1}{16}\gamma_2\,(\gamma_2')^2 - \frac{1}{4}\gamma_2\,\gamma_4' + \frac{1}{32}\gamma_2^2\,\gamma_2''
\end{aligned}
$$

Expanding at large n, we find

$$P_2 = \gamma_2 = 4\log\bar{n} + \frac{2}{n} - \frac{1}{3\,n^2} + \frac{1}{30\,n^4} - \frac{1}{63\,n^6} + \frac{1}{60\,n^8} + \cdots,$$

$$P_4 = \left(4\zeta_3 - \frac{4}{3}\pi^2\log\bar{n}\right) - \left(4 + \frac{2\pi^2}{3}\right)\frac{1}{n} + \left(3 + \frac{\pi^2}{9}\right)\frac{1}{n^2} - \frac{17}{9}\frac{1}{n^3} + \left(\frac{5}{6} - \frac{\pi^2}{90}\right)\frac{1}{n^4} +$$
$$-\frac{14}{225}\frac{1}{n^5} + \left(-\frac{7}{30} + \frac{\pi^2}{189}\right)\frac{1}{n^6} + \frac{152}{2205}\frac{1}{n^7} + \left(\frac{3}{14} - \frac{\pi^2}{180}\right)\frac{1}{n^8} + \cdots,$$

$$P_6 = \left(\frac{44\pi^4}{45}\log\bar{n} - 88\zeta_5\right) + \left(\left(16 - \frac{2\pi^2}{3}\right)\log\bar{n} + \frac{22\pi^4}{45} + \frac{2\pi^2}{3} + 48\right)\frac{1}{n} +$$
$$\left(\left(-6 + \frac{\pi^2}{3}\right)\log\bar{n} - \frac{11\pi^4}{135} - \frac{7\pi^2}{6} - 16\right)\frac{1}{n^2} +$$
$$\left(\left(\frac{32}{27} - \frac{\pi^2}{9}\right)\log\bar{n} + \frac{47\pi^2}{54} + \frac{203}{27}\right)\frac{1}{n^3} +$$
$$\left(\frac{7}{18}\log\bar{n} + \frac{11\pi^4}{1350} - \frac{7\pi^2}{18} - \frac{140}{27}\right)\frac{1}{n^4} +$$
$$\left(\left(-\frac{1007}{3375} + \frac{\pi^2}{45}\right)\log\bar{n} + \frac{19\pi^2}{1350} + \frac{790123}{202500}\right)\frac{1}{n^5} + \cdots$$

We did not find any simple parity invariance analogous to what is found in $\mathcal{N} = 4$. Nevertheless, LBK cancellations are present. Indeed, the structure of the logarithmic expansion is peculiar. Apart from the cusp anomaly, γ_2 has no logarithms, γ_4 has simple logarithms, and γ_6 has squared logarithms. Instead, P_2 and P_4 are logarithm-free, whereas P_6 has only simple logarithms.

This implies that the leading logarithms in the anomalous dimension are all inherited from the lowest order γ. In more details, one can check the remarkable relations

$$\gamma_4^{\rm ABA}(n) = \log\, n \left[-\frac{4}{3}\,\pi^2 + \frac{d\gamma_2^{\rm ABA}(n)}{dn}\right] + \mathcal{O}(1),$$

$$\gamma_6^{\rm ABA}(n) = \frac{1}{2}\,\log^2\, n\, \frac{d^2\gamma_2^{\rm ABA}(n)}{dn^2} + \mathcal{O}(\log\, n).$$

We can summarize this result by saying that *the twist-2 ABA anomalous dimension has leading order LBK inheritance up to six loops.*

Conclusions

It is natural and puzzling to ask whether QCD-inspired physical properties of $\mathcal{N}=4$ SYM twist operators are robust enough to carry over to the ABJM context. In this paper, we have focused on two non-trivial features of $\mathcal{N}=4$ SYM twist operators, Gribov-Lipatov reciprocity and Low-Burnett-Kroll cancellations. We have shown that these properties show up in a much softer and broken way compared to $\mathcal{N}=4$. Nevertheless, various intriguing remnants of these physical properties are still found in ABJM.

Indeed, the multi-loop analysis of the (asymptotic) anomalous dimensions of twist-1 and 2 operators reveals a curious pattern. Twist-1 operators obey a four loop parity invariance closely related to conventional Gribov-Lipatov reciprocity. Instead, twist-2 operators have no non-trivial parity invariance, but display a variety of LBK cancellations up to six loops.

Were it not for the $\mathcal{N}=4$ case, one could naively conclude that these features are accidental. We believe that this is not the case. Technically, they are due to the partial similarity of the ABJM and $\mathcal{N}=4$ $\mathfrak{sl}(2)$ sectors. Physically, it would be very interesting to look for arguments leading to Gribov-Lipatov reciprocity and Low-Burnett-Kroll wisdom in the case of the $\mathcal{N}=6$ superconformal Chern-Simons theory.

A final comment is deserved by wrapping corrections. We have applied the Kazakov-Gromov-Vieira formalism to the evaluation of the leading wrapping correction to twist-1 operators. It would be very interesting to work out closed formulae for this correction as well as explicit diagrammatic checks.

(a) Log scaling

The anomalous dimensions of composite operators carrying a large Lorentz spin scale (at most) logarithmically with the spin. This result is just one of the facets of a more general Sudakov phenomenon. The logarithmic scaling of anomalous dimensions is a universal feature of all gauge theories ranging from QCD to the maximally supersymmetric $\mathcal{N} = 4$ Yang-Mills (SYM) theory. In particular, in the simplest case of twist-two Wilson operators with large Lorentz spin $N \gg 1$, the anomalous dimension behaves as (in the adjoint representation of the $SU(N_c)$ group)

$$\gamma(\lambda) = 2\Gamma_{\rm cusp}(\lambda) \ln N + \mathcal{O}(N^0) \,,$$

where $\lambda = g^2_{\rm YM} N_c$ is the 't Hooft coupling constant and $\Gamma_{\rm cusp}(\lambda)$ is the so-called cusp anomalous dimension. $\Gamma_{\rm cusp}(\lambda)$ is not universal however and depends on the theory under consideration.

(b) Gribov-Lipatov reciprocity

The scale dependence of QCD parton distribution functions in deep inelastic scattering is governed by the the DGLAP evolution equations.The non perturbative ingredients are the space-like (S) splitting functions $P_S(x)$, related to the anomalous dimensions of twist-2 operatorsthrough a Mellin transformation.

The related crossed process of e^+e^- annihilation into hadrons involves the non perturbative fragmentation functions. In their scale evolution the role of splitting functions is played by the so-called time-like (T) splitting functions $P_T(x)$, which allow to define time-like anomalous dimensions $\gamma_T(N)$ again by a Mellin transformation. A first relation between $P_S(x)$ and $P_T(x)$ is the Drell-Levy-Yan relation

$$\text{Drell-Levy-Yan}: \qquad P_T(x) = -\frac{1}{x}\, P_S\left(\frac{1}{x}\right).$$

This is an analytic continuation from one kernel to the other which passes through the singular point $x = 1$ at the border of the respective disjoint physical regions. It is a relation trivial at one-loop and full of subtleties at higher orders.
A second equation has been proposed by Gribov and Lipatov

$$\text{Gribov-Lipatov}: \qquad P_T(x) = P_S(x) \equiv P(x).$$

Assuming this result and the (true) Drell-Levy-Yan relation, we get the following *reciprocity* for the common function $P(x)$

$$\text{Gribov-Lipatov reciprocity}: \qquad P(x) = -x\, P\left(\frac{1}{x}\right).$$

In Mellin spaceit can be shown that this means (in the sense of asymptotic expansions at large N)

$$P(N) = f(J^2), \qquad J^2 = N\,(N+1), \qquad N \to \infty.$$

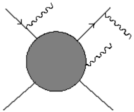
Gribov-Lipatov reciprocity holds at one-loop, but fails at two loops.

(c) LBK Theorems

... old general results which improve the eikonal leading order factorization. They rely on gauge invariance alone and are of a quite general validity; the LBK theorems tell us that the that the soft gluon emission has a *classical* nature.

$$d\sigma \propto d\omega \left(\frac{1}{\omega} + \text{constant} \right),$$

Example: $\varphi^+(p) + \varphi^0(q) \to \varphi^+(p') + \varphi^0(q') + \gamma(k)$



Theorem [eikonal improved]: The soft matrix element is

$$\mathcal{M} = e \left(\frac{p' \cdot \varepsilon}{p' \cdot k} - \frac{p \cdot \varepsilon}{p \cdot k} \right) \mathcal{M}_0 + F\,\partial\,\mathcal{M}_0 + \mathcal{O}(k),$$

LBK taught us that both the singular, and the constant terms, in the photon/gluon emission are universal while the quantum contributions vanish in the $x \longrightarrow 1$ limit as $(1-x)$. beyond the leading order, these structures must be therefore deducible rather than genuine higher order corrections.