Holography in Flat Space:

Note Title 6/25/2009

Algebraic Geometry + the S-Matrix

with Freddy Cachazo
Cliff Cheung
Jared Kaplan

to appear soon

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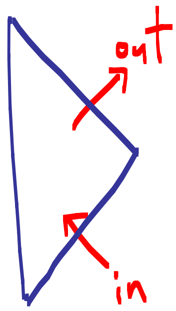
Goal: Discover Dual Theory for S-Matrix



Get S without evolution through spacetime

Evidence it exists: incredible properties of amplitudes, totally obscured by usual insistence on manifest locality.

"WEAK-WEAK" duality $\Longrightarrow$ Explicitly see the emergence of spacetime; "decode the hologram" perturbatively!

Kinematics

- $p^2 = 0 \Longrightarrow p_{\alpha\dot\alpha} = \lambda_\alpha \tilde\lambda_{\dot\alpha}$
- $M(t_i \lambda_i, t_i^{-1} \tilde\lambda_i, h_i) = t_i^{2h_i} M(\lambda_i, \tilde\lambda_i, h_i)$
- ex

$$M_{YM}^{++\cdots \overset{i}{-} \cdots \overset{j}{-} \cdots +} = \frac{\langle ij\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$$

- Maximal SUSY

$|\eta\rangle = e^{\bar{Q}\eta}|-\rangle,\ |\tilde\eta\rangle = e^{Q\tilde\eta}|+\rangle$

- $M\left(\lambda_i, \tilde\lambda_i, \begin{matrix}\eta_i \\ \text{or} \\ \tilde\eta_i\end{matrix}\right)$ no discrete labels!

- $M_n = \sum_{k=0}^{n} M_{n;k}$ $\leftarrow$ $\tilde\eta$ charge

$M_{n;k=0,1,n-1,n} = 0$; $k=2$ "MHV"
$k=3$ "NMHV"
etc.

Twistor Space : Kinematics as simple as possible

$$M(\ldots W \ldots) = \int \ldots d^2\lambda \ldots e^{i\lambda\tilde{\mu}} M(\ldots \lambda, \tilde{\lambda}, \tilde{\eta}, \ldots)$$

$$M(\ldots Z \ldots) = \int \ldots d^2\tilde{\lambda} \ldots e^{i\tilde{\lambda}\mu} M(\ldots \lambda \tilde{\lambda}, \eta, \ldots)$$

$$W_A \equiv \begin{pmatrix} \tilde{\mu} \\ \tilde{\lambda} \\ \hline \tilde{\eta} \end{pmatrix}, \quad Z^A \equiv \begin{pmatrix} \lambda \\ \mu \\ \hline \eta \end{pmatrix}$$

Conf. Grp : $SL(4, \mathbb{R})$

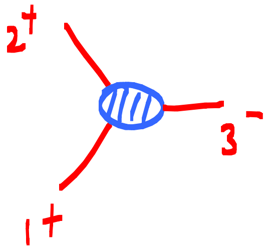
$$\mathcal{M}(tW) = t^{2(s-1)} \mathcal{M}(W)$$

$$\mathcal{M}(tZ) = t^{2(s-1)} \mathcal{M}(Z)$$

SYM : Functions of weight 0 on $\mathbb{RP}^{3|4}$

SUGRA : Functions of weight 2 on $\mathbb{RP}^{3|8}$

Twistor Space Amplitudes Amazingly Simple



YM: $\dfrac{[12]^3}{[13][23]}\,\delta^4(\Sigma\lambda\tilde{\lambda})$ $\qquad$ $\mathrm{sgn}\,W_1Z_3\ \mathrm{sgn}\,W_2Z_3\ \mathrm{sgn}\,[12]$

GR: $\left(\dfrac{[12]^3}{[13][23]}\right)^2\delta^4(\Sigma\lambda\tilde{\lambda})$ $\qquad$ $|W_1Z_3|\,|W_2Z_3|\ |\,[12]\,|$

Symmetries

- Cyclic / (perm. for grav)

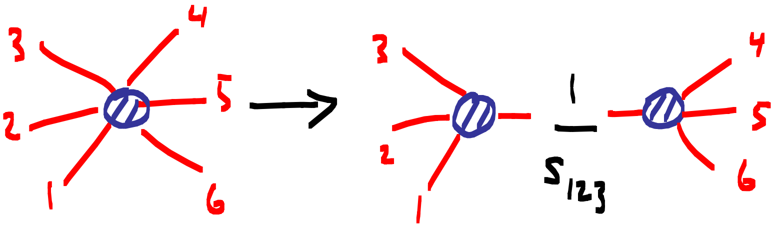
$$\mathcal{M}[\lambda_i, \tilde{\lambda}_i, \tilde{\eta}_i]$$
$$\parallel$$
$$\mathcal{M}[\lambda_{i+1}, \tilde{\lambda}_{i+1}, \tilde{\eta}_{i+1}]$$

- Parity

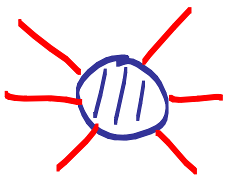
$$\int d^{\mathcal{N}}\tilde{\eta}_i \, e^{\eta_i \tilde{\eta}_i} \mathcal{M}(\lambda_i, \tilde{\lambda}_i, \tilde{\eta}_i)$$
$$\parallel$$
$$\mathcal{M}[\lambda \leftrightarrow \tilde{\lambda}, \tilde{\eta} \to \eta]$$

Singularity Structure: Imprint of Locality

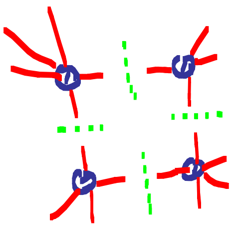
- Trees



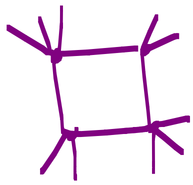
- One Loop



$= \sum_{\text{boxes}}$

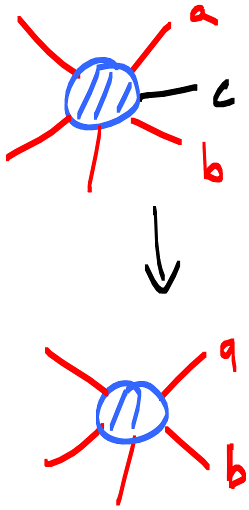


$\times$



(+ subleading, vanish w/ maximal SUSY)

IR Limits



$$\times \frac{\langle ab\rangle}{\langle ac\rangle\langle bc\rangle}$$

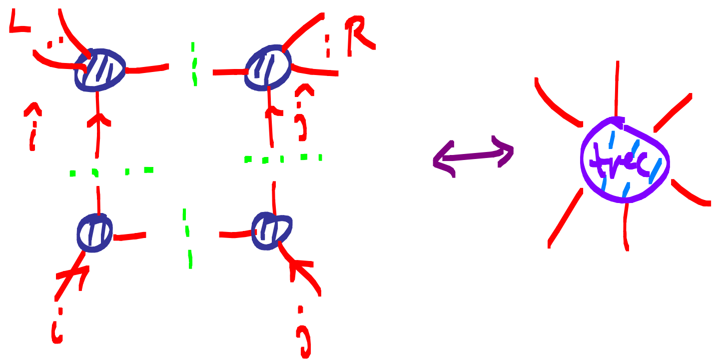
$$M_n^{1\text{-loop}}\Big|_{IR} =$$

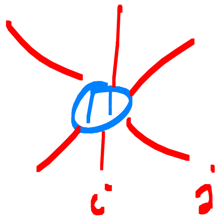
$$\left(\sum_i -\frac{1}{\epsilon^2}\left(-s_{i,i+1}\right)^{\epsilon}\right) M_n^{\text{tree}}$$

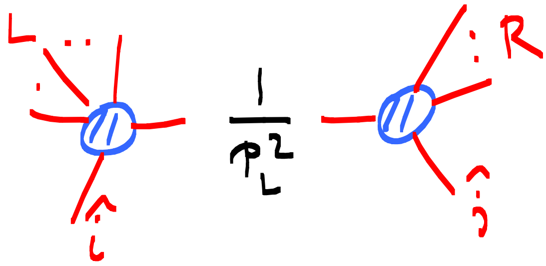
[many eqns...]

BCF

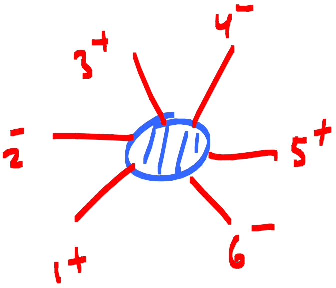
IR eqns @ 1-loop

$\sum_{L,R}$ 

$\Longrightarrow$ 

$= \sum_{L,R}$ 

BCFW 6 pt NMHV



$$= \frac{\langle 46\rangle^4 [13]^4}{[12][23]\langle 45\rangle\langle 56\rangle} \frac{1}{(p_1+p_2+p_3)^2}$$

$$\times \frac{1}{\langle 6|5+4|3]} \frac{1}{\langle 4|5+6|1]}$$

"Spurious" Poles: Don't occur in local theories!

$$+ \{i \to i+2\} + \{i \to i+4\}$$

Remarkable 6-term Id

$$\frac{\langle 46\rangle^4 [13]^4}{[12][23]\langle 45\rangle\langle 56\rangle}\frac{1}{(p_1+p_2+p_3)^2} \times \frac{1}{\langle 6|5+4|3]}\frac{1}{\langle 4|5+6|1]} + \{i\to i+2\} + \{i \to i+4\}$$

$$= \frac{\langle 3|(2+4)|6]^4}{[23][34]\langle 56\rangle\langle 61\rangle}\frac{1}{(p_5+p_6+p_1)^2} \times \frac{1}{\langle 1|6+5|4]}\frac{1}{\langle 5|6+1|2]} + \{i\to i+2\} + \{i\to i+4\}$$

Guarantees: Parity, Cyclicity, No Spurious Poles

7-pt 12 terms

8-pt 20 terms

40 terms

⋮ ⋮

SOME POWERFUL MATHEMATICAL STRUCTURE IS AT WORK!

The Conjectured Duality

$$Q_{n,k} = \int \frac{d^{n\times k} C_{a\alpha}}{(12\cdots k)\cdots(n1\cdots k-1)} \prod_{\alpha=1}^{k} \delta^{4|4}\left[C_{a\alpha} \mathcal{W}_a\right]$$

Integral over Grassmannian $G(n,k)$

$(m_1 \cdots m_k) = \epsilon^{\alpha_1 \cdots \alpha_k} C_{m_1 \alpha_1} \cdots C_{m_k \alpha_k}$

Manifestly Cyclically Invariant

k linear relations on n twistors

- Claim: after we make this sharply defined – trivial to back to momentum space – multi-dimensional contour integral.

- Residues compute 1-loop leading singularities (+ hence all 1-loop amps) in $\mathcal{N}=4$ YM!

- (Includes all tree amps too by BCF logic)

- $$C = \begin{array}{c} \overset{\longleftarrow\; n \;\longrightarrow}{\begin{pmatrix} c_{11} & \cdots & c_{1n} \\ \vdots & & \\ c_{k1} & \cdots & c_{kn} \end{pmatrix}} \end{array} \updownarrow k \rightarrow \text{k-plane in n-space} \in G(k,n)$$

- "Gauge Symmetry" $C_{a\alpha} \rightarrow C_{a\beta} L^{\beta}_{\alpha}$, any $k \times k$ L

- "Gauge Fix": columns I to some basis e.g.

$$C = \begin{pmatrix} 1 & 0 & 0 & c_{14} & \cdots & c_{17} \\ 0 & 1 & 0 & c_{24} & \cdots & c_{27} \\ 0 & 0 & 1 & c_{34} & \cdots & c_{37} \end{pmatrix} \quad \text{or} \quad C = \begin{pmatrix} 1 & c_{21} & 0 & c_{41} & 0 & c_{61} & c_{71} \\ 0 & c_{23} & 1 & c_{43} & 0 & c_{63} & c_{73} \\ 0 & c_{25} & 0 & c_{45} & 1 & c_{65} & c_{75} \end{pmatrix}$$

Can choose any GFing we like. [Different Charts on Grassmannian]

- Leaves us with $k(n-k)$ variables.

- Obvious mapping between k-plane and $\perp$ $(n-k)$ plane.

Given e.g.

$$C = \left(\mathbb{1}_{k\times k} \middle| \, c \right), \text{ define } *C = \left(\mathbb{1}_{n-k \times n-k} \middle| -c^T \right)$$

- Symmetry $k \leftrightarrow (n-k)$ is just parity

- After GFing, trivial to go back to momentum space:

$$Q_{n,k} = \int \frac{d^{(n-k)k}\, C}{(12..k)\,..\,(n..k-1)}\, \delta^2[*C\lambda]\,\delta^2[C\tilde{\lambda}]\,\delta^4[C\tilde{\eta}]$$

- Parity Manifest!

Note: for gluon amplitudes, convenient to gauge fix columns I corresponding to particles of negative helicity — then intgd is always the same, its just form of C that changes.

e.g. $1^- 2^- 3^+ 4^- 5^+ 6^+$

$$C = \begin{pmatrix} 1 & 0 & c_{31} & 0 & c_{51} & c_{61} \\ 0 & 1 & c_{32} & 0 & c_{52} & c_{62} \\ 0 & 0 & c_{34} & 1 & c_{54} & c_{64} \end{pmatrix}$$

$1^- 2^+ 3^- 4^+ 5^- 6^+$

$$C = \begin{pmatrix} 1 & c_{21} & 0 & c_{41} & 0 & c_{61} \\ 0 & c_{23} & 1 & c_{43} & 0 & c_{63} \\ 0 & c_{25} & 0 & c_{45} & 1 & c_{65} \end{pmatrix}$$

Diff. helicities: integrate same fn. on different charts of Grassmannian!

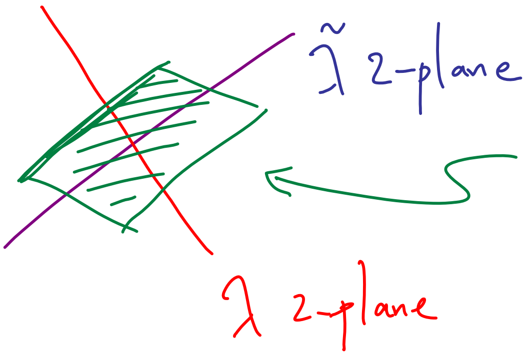
The condition $C\lambda = 0\,, \quad C^{\perp}\tilde{\lambda} = 0$

$\Longleftrightarrow \quad \lambda_i - c_{iI}\lambda_I = 0\,, \quad \tilde{\lambda}_I + c_{iI}\tilde{\lambda}_i = 0$

Can only be satisfied if mom. conserved

$$\lambda_i\tilde{\lambda}_i + \lambda_I\tilde{\lambda}_I = 0\ !$$

Geometrically: $\vec{\lambda}_\alpha$, $\vec{\tilde{\lambda}}_{\dot\alpha}$ n vectors.



$\subset$ k-plane, orthog. to $\tilde\lambda$, must contain λ

$\Longrightarrow$ $\vec{\lambda}_\alpha \cdot \vec{\tilde{\lambda}}_{\dot\alpha} = 0$ = mom conservation!

So, we are left with $k(n-k)-(2n-4)$

$= (k-2)(n-k-2)$ parameters; solns of

$$\lambda_i - c_{iI}(\tau_A)\lambda_I = 0\,, \quad \tilde{\lambda}_I + c_{iI}\tilde{\lambda}_i(\tau_A) = 0$$

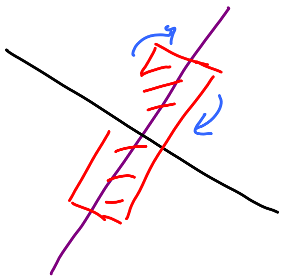
$$c_{iI}(\tau_A) = c^*_{iI} + d_{iIA}\tau^A\,, \quad A=1,\dots,(k-2)(n-k-2)$$

[one to one correspondence with $G(k-2, n-4)$]

Note: no such plane exists for $k=0,1,n-1,n$!
Thats why these amps vanish.

$k=2$, $n-2$ [MHV + $\overline{\text{MHV}}$], plane fixed + we get right answer.

Otherwise



$(k-2)\,[n-k-2]$ directions k-plane can rotate in!

We can factor out momentum conservation:

$$\delta^2[*C\,\lambda]\,\delta^2[C\tilde{\lambda}] = \delta^4(\vec{\lambda}\cdot\vec{\tilde{\lambda}}) \times \mathcal{J}(\lambda,\tilde{\lambda}) \times$$

$$\int d^{(k-2)\times(n-k-2)}\tau \;\delta^{k(n-k)}\left[c_{iI} - c_{iI}(\tau)\right]$$

Then $Q_{n,k} = \delta^4(\Sigma p)\, \mathcal{Q}_{n,k}$ where

$$\mathcal{Q}_{n,k} = \int \frac{d^{(k-2)(n-k-2)}\tau}{(1\,2 \ldots k)[\tau] \ldots (n\,1 \ldots k-1)[\tau]}$$

At this point everything can be complexified.

Now $c_{iI}(\tau) = c^*_{iI} + \tau_{iI}$

where $\tau_{iI} \lambda_{I\alpha} = 0$. Thinking of these as a collection of k-vectors, they are all orthogonal to the 2 k-vectors $(\lambda_I)_{\alpha=1, \alpha=2}$.

$\Longrightarrow$ $(1\,2 \ldots k)$ deg. @ most $\min[(k-2), (n-k-2)]$ in the τ_A. NMHV: deg. 1.

First interesting case: 6 pt NMHV $(n=6, k=3)$.

Look @ $1^+ 2^- 3^+ 4^- 5^+ 6^-$

$$C = \begin{pmatrix} \times & 1 & \times & 0 & \times & 0 \\ \times & 0 & \times & 1 & \times & 0 \\ \times & 0 & \times & 0 & \times & 1 \end{pmatrix}$$

$$\lambda_i - c_{iI} \lambda_I = 0 \,, \quad \tilde{\lambda}_I + c_{iI} \tilde{\lambda}_i = 0$$

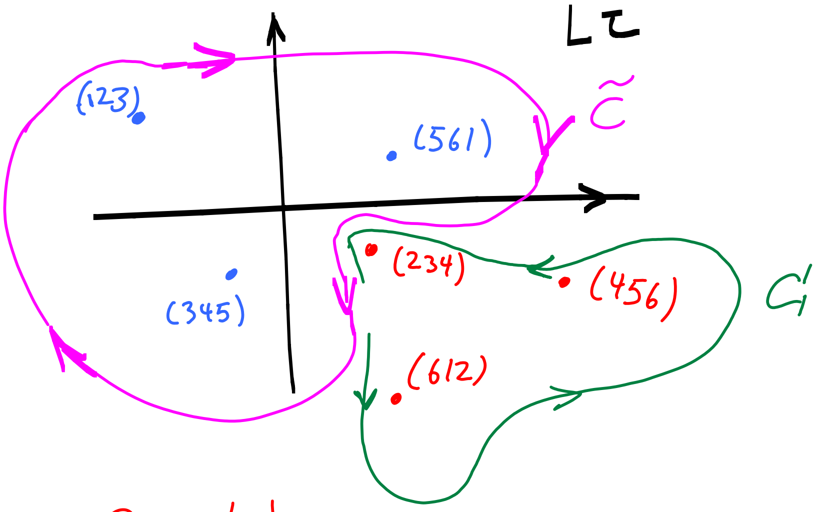
$$\Longrightarrow \quad c_{iI}(\tau) = c^{*}_{iI} + \epsilon_{ijk} \epsilon_{IJK} \langle JK \rangle [jk] \tau \,;$$

$$(\text{Jacobian } \mathcal{J} = 1).$$

So

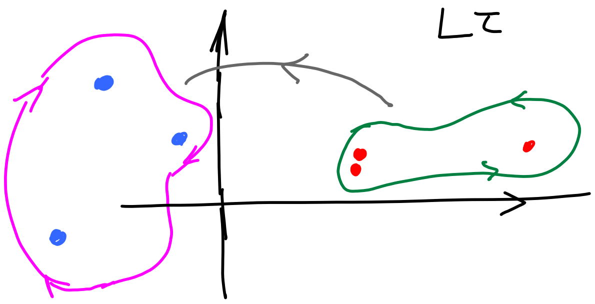
$$Q_{6,3} = \int d\tau \frac{1}{(123)(234)\cdots(612)(\tau)}$$

each factor linear in τ



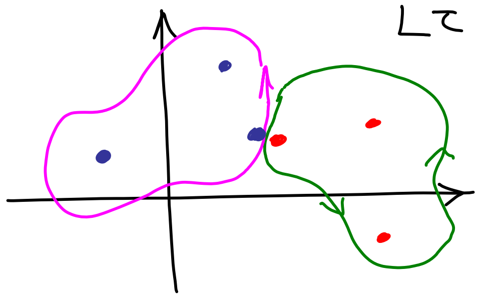
- residues : BCFW terms
- residues : P[BCFW] terms
- Cauchy : BCFW = P[BCFW] = Remarkable 6-term identity!

Spurious Poles



Contour can be deformed away from singularity

Physical Poles



Can't deform contour to avoid singularity

LOCALITY $\leftrightarrow$ CONTOUR DEFORMATION

- For all other cases we have more than 1 complex variable. What is a residue?

$$f(z) = \frac{g(z)}{A_1(z) \cdots A_n(z)} \qquad z = (z_1, \ldots, z_n)$$

$$\underset{A_1(z_*) = \cdots = A_n(z_*) = 0}{\operatorname{res}} f(z_*) = \frac{g(z_*)}{\det \left. \frac{\partial A_i}{\partial z_j} \right|_{z_*}}$$

(note: alternating in $(1, 2, \ldots, n)$).

- Also higher-dim gen of Cauchy's thm:

$$f(z) = \frac{g(z)}{A_1(z) \cdots A_n(z)}$$

[note a given $f(z)$ can be written in this form in many ways]

then

$$\sum_{\substack{z_* \\ A_1(z_*) = \cdots = A_n(z_*) = 0}}' \operatorname{res} f(z_*) = 0 \quad \{\text{if } \deg g \text{ is small enough}\}$$

- First new case: $n=7$, $k=3$ (7 pt NMHV).

Look e.g. @ $1^+2^-3^+4^-5^+6^-7^-$.

$$Q_{7,3} = \int \frac{d\tau_1\, d\tau_2}{(123) \cdots (712)}$$

each linear in τ_1, τ_2

There are $\binom{7}{2} = 21$ residues.

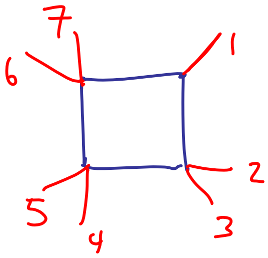
· They are computed just by solving linear equations, but can get interesting!

e.g. $(123)(456) = \dfrac{\left([7|(2+4)|3\rangle\langle 54\rangle + [76]\langle 65\rangle\langle 34\rangle\right)^4}{\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 56\rangle[71]}$

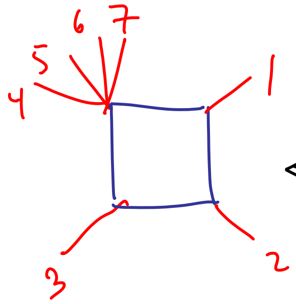
$$\times \frac{1}{[1|(2+3)|4\rangle[7|5+6|4\rangle\langle 4|(5+6)(7+1)|2\rangle\langle 4|(2+3)(7+1)|6\rangle}$$

+ 20 other horrible guys!

. Miraculously, these 21 residues exactly match the 21 objects first found in '04 by Zvi, Lance, David et. al. in the 1-loop 7 pt $\mathcal{N}=4$ amp!



$\longleftrightarrow \quad (123)(671)$



$\longleftrightarrow \quad (123)\left[(345)+(567)+(712)\right] + (456)(567)$

Now, the Global Residue Thm implies many identities. [Actually overkill in this case since all factors linear — just repeated use of Cauchy gives same thing].

$(i) \longleftrightarrow [i\ i{+}1\ i{+}2]$

$(i)\wedge(j) \longleftrightarrow \mathrm{res}\big|_{(i)=(j)=0}$

$$f(z_1,z_2) = \frac{1}{\underbrace{(1)}_{A_1}\ \underbrace{[(2)(3)\cdots(7)]}_{A_2}} \Longrightarrow (1)\cdot(2) + \cdots + (1)\cdot(7) = 0$$

+ cyclic exhaust all residue id.

These "basic" identities have a direct physical interpretation: IR equations!

e.g. look at $\log((p_1+p_2+p_3)^2)$ term in 1-loop IR eqn $=$ $(1)\cdot(2) + (1)\cdot(3) + \cdots + (1)\cdot(7) = 0$.

LOCALITY $\longleftrightarrow$ CONTOUR DEFORM.

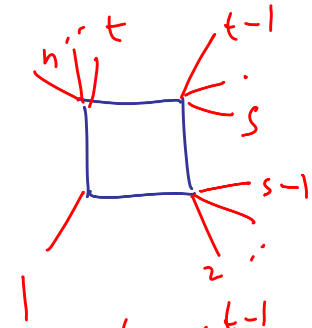
Take

$$f(\tau_1, \tau_2) = \frac{1}{\underbrace{(1)(2)(3)(4)}_{A_1}\ \underbrace{(5)(6)(7)}_{A_2}}$$

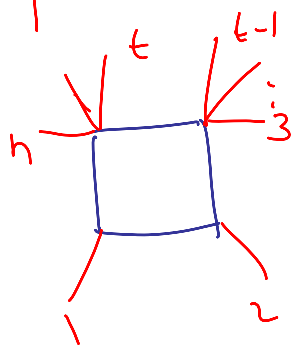
$4 \times 3 = 12$ term identity which guarantees cyclic.
+ absence of spurious poles for 7 pt tree amp!
(Can actually show there is a unique such object $\longrightarrow$ build local 7pt amp).

• We know the explicit map between residues + boxes for all NMHV amplitudes:

e.g.



$$(n-1)\wedge(1)\wedge\left[(2)\wedge\cdots\wedge(s-3)\right]\wedge\left[(s)\wedge\cdots\wedge(t-3)\right]\wedge\left[(t)\wedge\cdots\wedge(n-2)\right]$$



$$\left[(n-1)+(2)\right]\wedge\left[(3)\wedge\cdots\wedge(t-3)\right]\wedge\left[(t)\wedge\cdots\wedge(n-2)\right]$$

Residue Thm Relations:

$$(i_1) \wedge \cdots \wedge (i_{n-6}) \wedge \sum_k (k) = 0.$$

A Collection of 6-term identities.

- General NMHV tree amplitudes:

Define $(i_1)\otimes \cdots \otimes (i_m) = \begin{cases} (i_1)\wedge\cdots\wedge(i_m) & i_1 < i_2 \cdots < i_m \\ 0 & \text{otherwise} \end{cases}$

Also define $E = (2) + (4) + \cdots$

$\Theta = (1) + (3) + \cdots$

Then $M^{NMHV} = \underbrace{E \otimes \Theta \otimes \cdots \otimes}_{(n-5)} \begin{cases} \Theta \\ E \end{cases}$

- Using the residue theorems, it is easy to prove that this is cyclically invariant:

$$E \otimes \Theta \otimes \cdots = (-1)^{n-5}\, \Theta \otimes E \otimes \cdots$$

Furthermore, LHS + RHS have non-overlapping residues $\rightsquigarrow$ different spurious poles $\rightsquigarrow$ no sp. poles in the amplitude.

Ex: $n=7$. In general, when is

$$\sum_{j,k} f_{j,k}\,(j)\cdot(k)$$

cyclically invariant?

f is ambiguous up to identities

$$0 = \sum_{k,j} \alpha_k\,(j)\cdot(k) = \sum_{k,j} (\alpha_k - \alpha_j)\,(j)\cdot(k)$$

So $\quad f_{j,k} \sim f_{j,k} + \alpha_j - \alpha_k$

Invariant $\quad (\Delta_1 \Delta_2 f) \sim (\Delta_1 \Delta_2 f)$

So $\Delta_1 \Delta_2 f$ better be manifestly cyclic.

$$\Delta_1 \Delta_2 \left[(2)+(4)+(6)\right] \otimes \left[(1)+(3)+(5)+(7)\right]$$

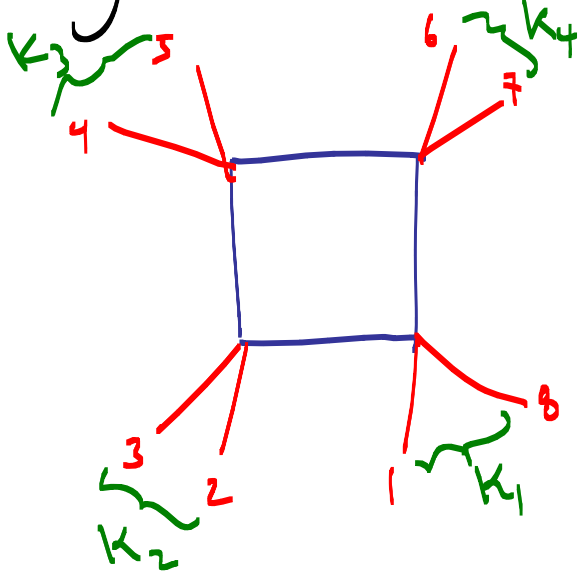
$$= \left[(2)-(1)+(4)-(3)+(6)-(5)+(7)\right] \otimes \left[(3)-(2)+(5)-(4)+(7)-(6)+(1)\right]$$

$$= -\left(\left[(1)-(2)+(3)-(4)+(5)-(6)+(7)\right]\otimes\right)^2$$

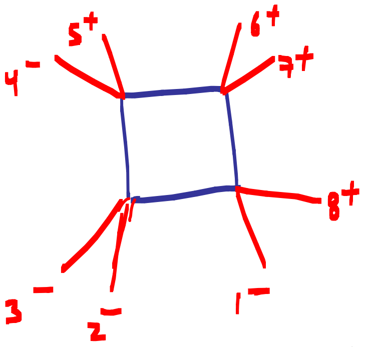
= manifestly cyclic!

LOCALITY $\leftrightarrow$ CONTOUR DEFORMATION

Simplest IR Finite 1-loop:



8 pt N^2MHV

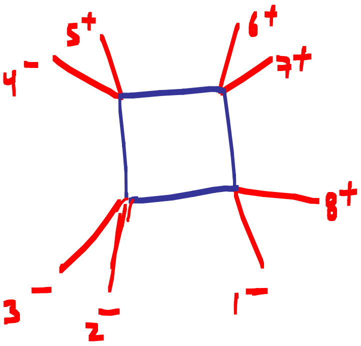


$$= \sum_{\ell^+,\ell^-} \frac{\langle 23\rangle^3 [67]^3 [4|\ell - P_{23}|4\rangle^3 \langle 1\ell\rangle^3}{\langle 45\rangle\langle 81\rangle\langle \ell 2\rangle\langle 3|\ell - P_{23}|4]}$$

$$\times \frac{1}{[4|P_{23}|\ell\rangle\langle \ell|P_{18}|7]\langle 5|\ell - P_{2345}|6] \quad \langle 8|(\ell - P_{18})(\ell - P_{2345})(\ell - P_{23})|4]}$$

where $\ell^2 = (\ell - K_1)^2 = (\ell - K_1 - K_2)^2 = (\ell + K_4)^2$, quad. eqn with

$$\Delta = 1 - 2(\rho_1 + \rho_2) + (\rho_1 - \rho_2)^2 \,; \quad \rho_1 = \frac{K_1^2 K_2^2}{K_{12}^2 K_{23}^2}, \quad \rho_2 = \frac{K_2^2 K_4^2}{K_{12}^2 K_{23}^2}$$



$$= \mathrm{res}\left[\frac{1}{(1234)\cdots(8123)}\right]$$

where $(1234)=(4567)=(6781)=(8123)=0$!

Very Non-trivial Check!

Actually these are quadratic in 4 τ's, get 2 solns, corresponding to $\ell_\pm$!

- We have found an object that unifies all 1-loop $\mathcal{N}=4$ information — the leading singularities — into a single geometric object.
- Most general case involves some deep + beautiful algebraic geometry: residue + intersection theory, the Schubert Calculus.
- Haven't played central role in physics before, but, sitting @ the heart of gluon scattering amplitudes!

For ex: $n=10$, $k=5$

$(12345) \quad \cdots \quad (10\,1234)$ 9 τ's

cubic in τ's

residue: simultaneous soln of 9 cubics in 9 variables...

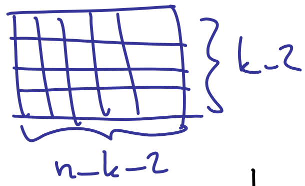
help!

General answer: $D = (k-2)(n-k-2) = \dim G(k-3, n-4)$

\# self-int of σ_1^{D}

elem. Schubert cycle

$\|$

\# of appearances of 

in $(\square)^{D}$

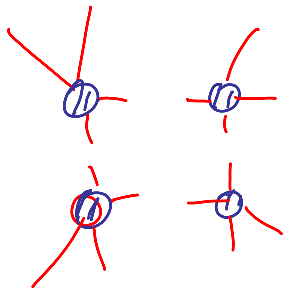
$$= \frac{1!\,2!\,\cdots(k-3)!\;D!}{(n-k-2)!\;\cdots\;(n-5)!} = 42 \text{ for } k=5,\ n=10$$

(WORKS!)

But perhaps our physical interpretation has something to offer mathematicians?

Naively: expect soln's of cubics to involve $\sqrt[3]{\ }$'s.

But in



only $\sqrt{\ }$'s appear!

[4D Locality again!]

So we predict that, no matter how high k, n get, that if the external momenta are in $\mathbb{Q}$, the solns are in $\mathbb{Q} + \sqrt{\mathbb{Q}}$, no higher roots! Check $k=5, n=10$ ✓ (!)

$\Longrightarrow$ A mysterious connection between Galois Theory + Schubert Calculus?

- All of these checks reveal the power of of a weak-weak duality! And we are beginning to see the sorts of structures that allow local spacetime physics to emerge holographically.

Outlook

THE DUAL THRY CLEARLY EXISTS

[Quantum!] S-Matrix

?

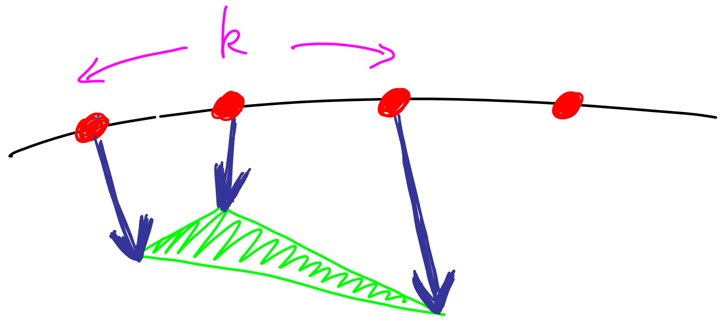
Twistors

Alg. Geometry

WHAT IS THE PHYSICS?

Comments

- Dual Conformal Invariance/Yangian should be natural in this picture! Perhaps our formula is a natural way of generating Yangian invariants + all relations between them.



$$Q_{n,k} = \int dX_\alpha^{(j)}\, dY_\beta^{(j)}\, dz_{\alpha I}\, dC_{a\alpha}\; e^{iS}$$

$$S = X_\alpha^{(j)}\, Y_\beta^{(j)}\, C_{j+\alpha-1,\beta} + W_a\, C_{a\alpha}\, z_\alpha$$

Connection to Hodges' Picture?

- Hodges: NMHV amps are "polytope volume" in T-dual space. Decompose in tetrahedra
  $\Longrightarrow$ BCFW. Spurious Pole cancellation:
  cancellation along faces $\sim$ "$\partial . \partial = 0$".

Us: sp. pole cancellation $\sim$ contour deformation
"$\partial . \partial = 0$". More direct relation?

$\mathcal{N} = 8$ Will Be Much More Interesting!