Potsdam
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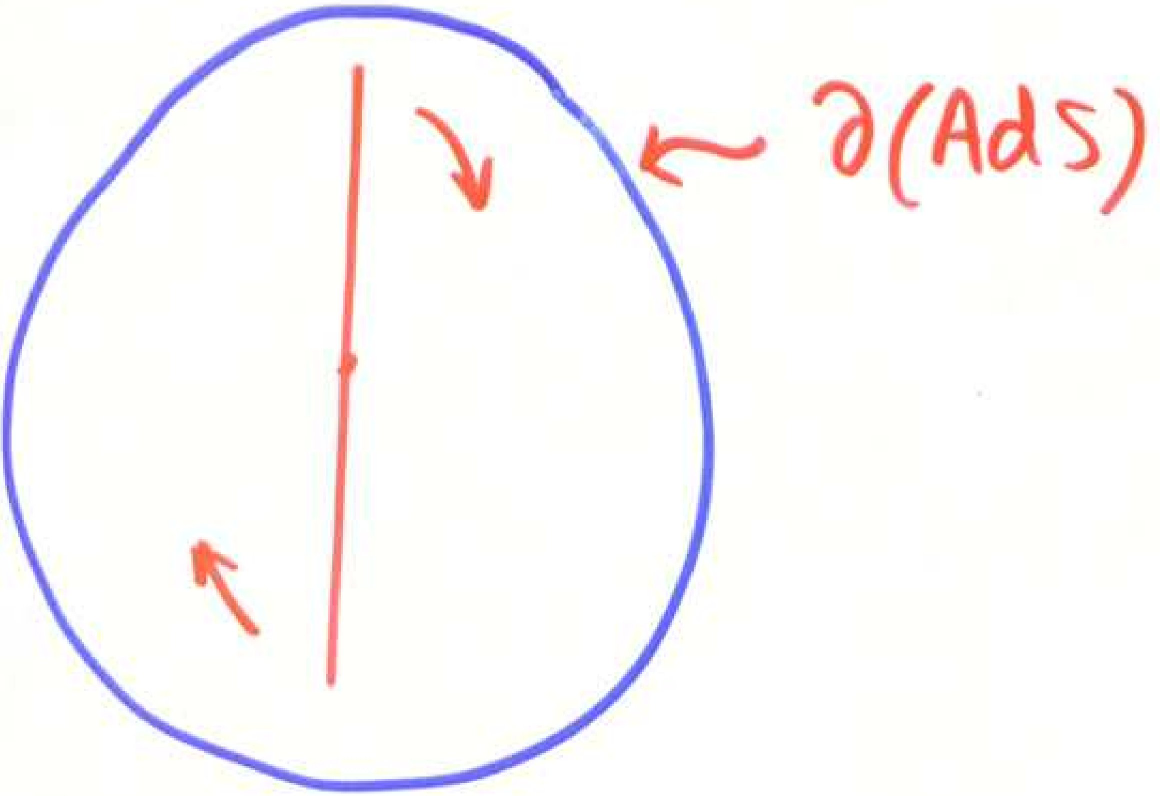
N. Dorey
Cambridge

# Giant Holes

ND arXiv 0805.4387
ND + M. Losi " 0812.1704
+ work in progress

# Spinning string in AdS, GKP



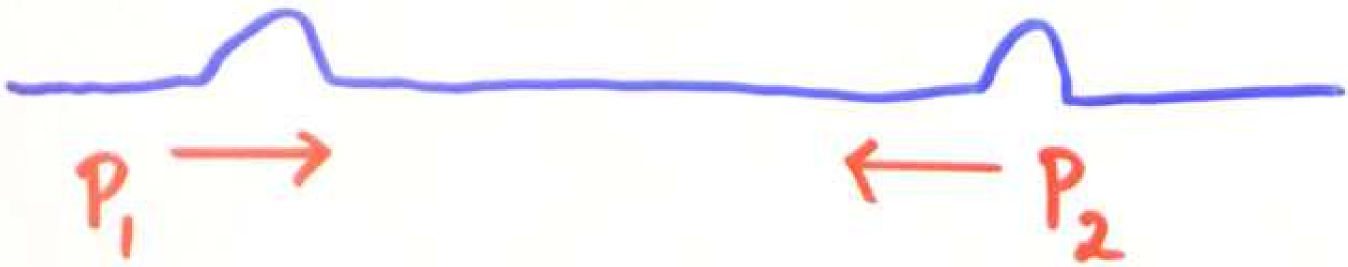

Angular momentum $S \to \infty$

$$\Delta - S \simeq 2\Gamma(\lambda) \log S + O(S^0)$$

cusp anomalous dimension $\Gamma(\lambda) \simeq \frac{\sqrt{\lambda}}{2\pi} + \dots$

- String long but far from BMN vacuum

length: $\ell \sim (\Delta - S) \sim \log S$




- asymptotic states?
- factorized scattering?

## Motivation

- Dual to spin chains of _fixed length_
- Relation to gluon scattering amplitudes?

$O(6)$ $\sigma$-model?

Alday + Maldacena

# Plan

Large-S limit,

- Gauge Theory $\lambda \ll 1$
- String Theory $\lambda \gg 1$

# One-loop Gauge Theory

$sl(2)$ sector, Korchemsky, ....

$$\hat{O} \sim \mathrm{Tr}_N \left[ \mathcal{D}_+^{s_1} Z \, \mathcal{D}_+^{s_2} Z \ldots \mathcal{D}_+^{s_J} Z \right]$$

spin $S = \sum_i s_i$

- Bethe ansatz

$S$ magnons, rapidities $u_a$ $a = 1, \ldots, S$

$$\left( \frac{u_a + i/2}{u_a - i/2} \right)^J = \prod_{b \neq a} \left( \frac{u_a - u_b + i}{u_a - u_b - i} \right)$$

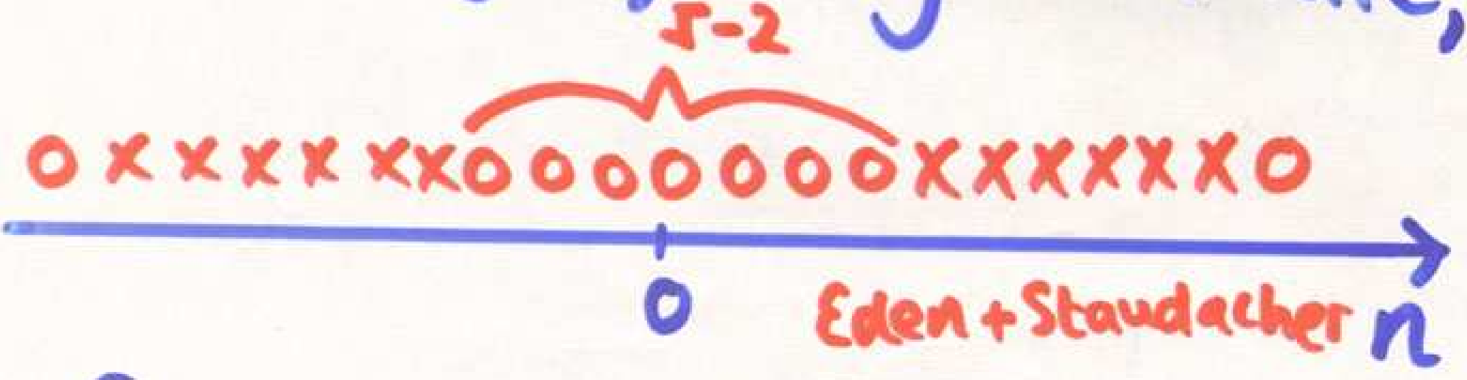
Anomalous dimension,

$$\gamma = \Delta - S - J = \frac{\lambda}{8\pi^2} \sum_{a=1}^{S} \frac{1}{u_a^2 + 1/4}$$

- Large spin limit,

$$\boxed{S \to \infty \, , \quad J \text{ fixed}}$$

Distribution of magnon mode numbers $\{n_a\}$ in "groundstate",




S magnons x

J holes o

Reformulate Bethe ansatz in terms of holes of rapidity

$u = \delta_i \quad i = 1, \ldots, J$ Korchemsky et al

$$\gamma = \frac{\lambda}{8\pi^2}\left[2\log 2 + \sum_{i=1}^{J} \varepsilon(\delta_i)\right]$$

hole energy,

$$\varepsilon(\delta) = \psi\left(\tfrac{1}{2} + i\delta\right) + \psi\left(\tfrac{1}{2} - i\delta\right) + 2\gamma_E$$

$\{\delta_i\}$ quantized by "dual" BAE

• Large S scaling,

- $K \geq 2$ "big" holes

$$|\delta_i| \sim S \qquad i = 1, \dots, K$$

- $J-K$ "small" holes

$$|\delta_i| \sim 1/\log S \qquad i = K+1, \dots, J$$

• Anomalous dimension dominated by big holes

$\psi(x) \sim \log x + \dots$

$$\gamma \approx \frac{\lambda}{4\pi^2}\left[ K \log S + f(\ell_1, \dots, \ell_{K-1}) + O\left(\frac{1}{\log^2 S}\right)\right]$$

$O(S^0)$ small holes

quantum numbers $\ell_i \in \mathbb{Z}^+$

# One-loop spin chain

$$\hat{O} \sim \mathrm{Tr}_N\left[\mathcal{D}_+^{S_1} Z\, \mathcal{D}_+^{S_2} Z \ldots \mathcal{D}_+^{S_J} Z\right]$$

$$x \cdots \uparrow \qquad \uparrow \qquad \cdots\cdots \uparrow \cdots x$$

- $S \to \infty$ is classical limit

$$\underset{\text{quantum spin}}{\hat{L}_i^A} \longrightarrow \underset{\text{classical spin}}{\mathcal{L}_i^A}$$

$$\{\mathcal{L}_i^A, \mathcal{L}_j^B\}_{P.B.} = 2\delta_{ij}\epsilon^{ABC}\mathcal{L}_j^C$$

- Anomalous dimensions

$$\gamma = \frac{\lambda}{8\pi^2}\sum_{j=1}^{J} \log\left|\vec{\mathcal{L}}_j + \vec{\mathcal{L}}_{j+1}\right|$$

WKB quantisation, "$\hbar$" $= 1/S$

$$\gamma \approx \frac{\lambda}{4\pi^2}\left[J \log S + f(\ell_1, \ldots, \ell_{J-1}) + O\left(1/\log^2 S\right)\right]$$

# Semiclassical Strings

... on $AdS_3 \times S^1$ $\subset AdS_5 \times S^5$

$(\Delta, S)$ $\quad J$

- Static conformal gauge

$\Rightarrow$ $SL(2,\mathbb{R})$ P.C.M.

$g(\sigma,\tau) \in SL(2,\mathbb{R}) \simeq AdS_3$

$$S_\sigma = \frac{\sqrt{\lambda}}{4\pi} \int_\Sigma d^2\sigma \, \mathrm{tr}[j_+ j_-]$$

$j_\pm = g^{-1} \partial_\pm g$

- eigenvalues $\lambda_\pm = e^{\pm i p(x)}$

of monodromy,

$$\Omega[x;\tau] = P\exp\left[\frac{1}{2}\int_0^{2\pi} d\sigma \left(\frac{j_+}{x-1} + \frac{j_-}{x+1}\right)\right]$$

are conserved $\forall x \in \mathbb{C}$

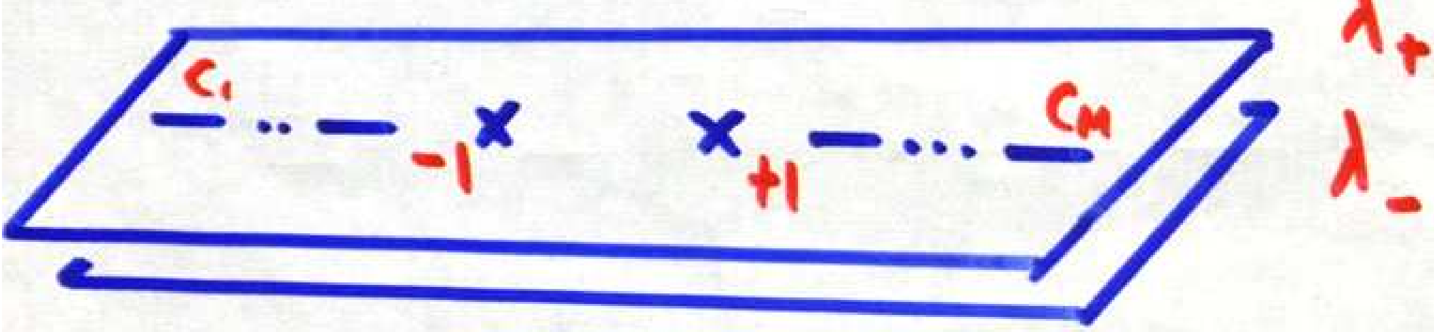
# M-gap solutions

KMMZ

find $(\Sigma, dp)$

genus $M-1$

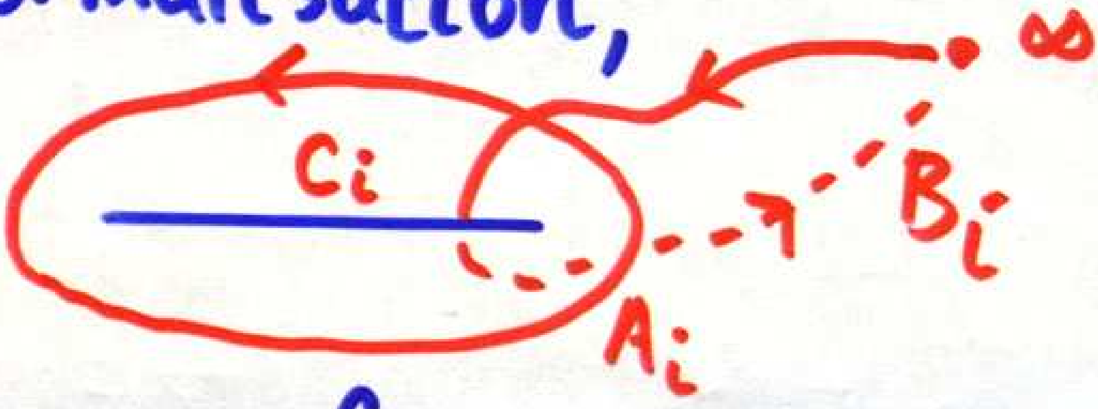
meromorphic differential




- singularities,

$$dp \to -\frac{\pi J}{\sqrt{\lambda}} \frac{dx}{(x \mp 1)^2} \qquad x \to \pm 1^{\pm}$$

- normalisation,




$$\int_{A_i} dp = 0 \,, \quad \int_{B_i} dp = 2\pi n_i \qquad i = 1, \ldots, M$$

mode number

Find M dimensional space of solutions $(\Sigma, dp)$ ...

- Asymptotics,

$$dp \to -\frac{2\pi}{\sqrt{\lambda}}(\Delta + S)\frac{dx}{x^2} \qquad x \to \infty$$

$$\to -\frac{2\pi}{\sqrt{\lambda}}(\Delta - S)\, dx \qquad x \to 0$$

- Semiclassical quantization

ND + Vicedo

$$\frac{1}{2\pi i} \cdot \frac{\sqrt{\lambda}}{4\pi} \oint_{A_i} \left(x + \frac{1}{x}\right) dp = l_i \in \mathbb{Z}^+$$

$i = 1, \ldots, M$

$\sum l_i = S$

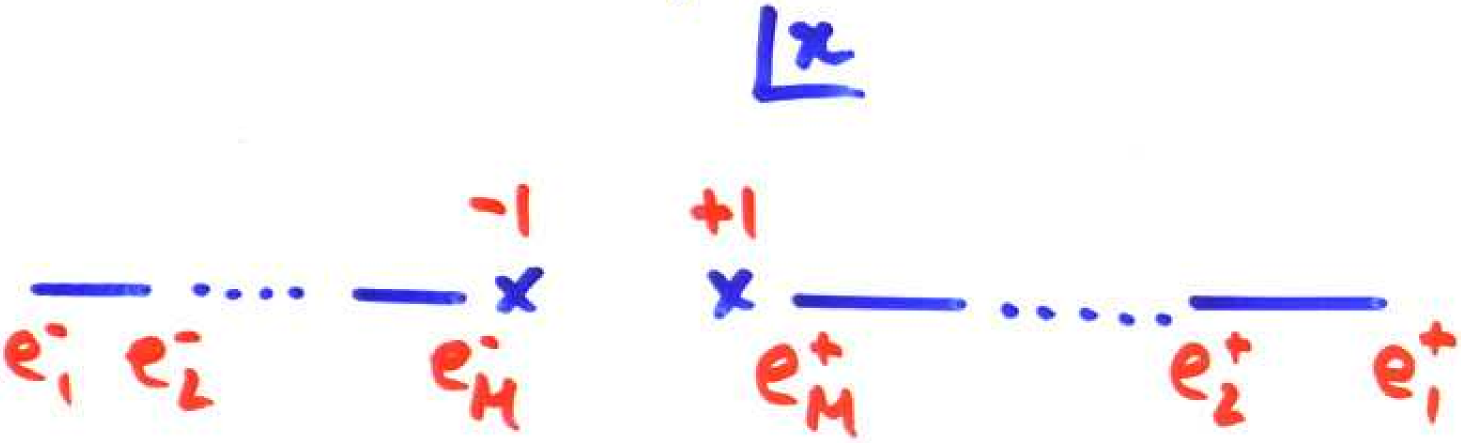
reduces spectrum,

$$\Delta = \Delta(l_1, \ldots, l_M)$$

to quadratures

# Spectral curve    M even

$$\Sigma: \quad y^2 = \prod_{i=1}^{M} (x - e_i^-)(x - e_i^+)$$




- $S \to \infty$, $J$ fixed,
  - preserve normalisation

$$\oint_A dp = 0 \, , \quad \oint_B dp = 2\pi n$$

Sakai + Satoh

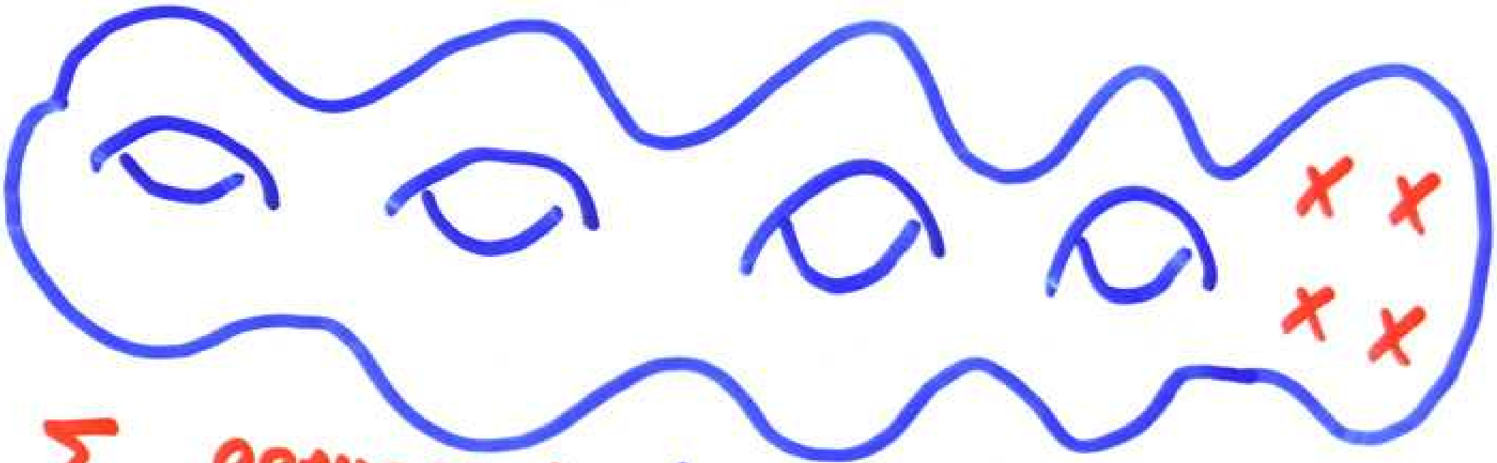
  - branch points scale,

"big" $e_i^{\pm} \sim S$ $\quad i = 1, \ldots, K-1$

"small" $e_i^{\pm} \sim S^0$ $\quad i = K, \ldots, M$

# Degeneration

$$\Sigma \xrightarrow{S\to\infty} \tilde{\Sigma}_{I} \cup \tilde{\Sigma}_{II}$$



$\Sigma$ genus $M-1$

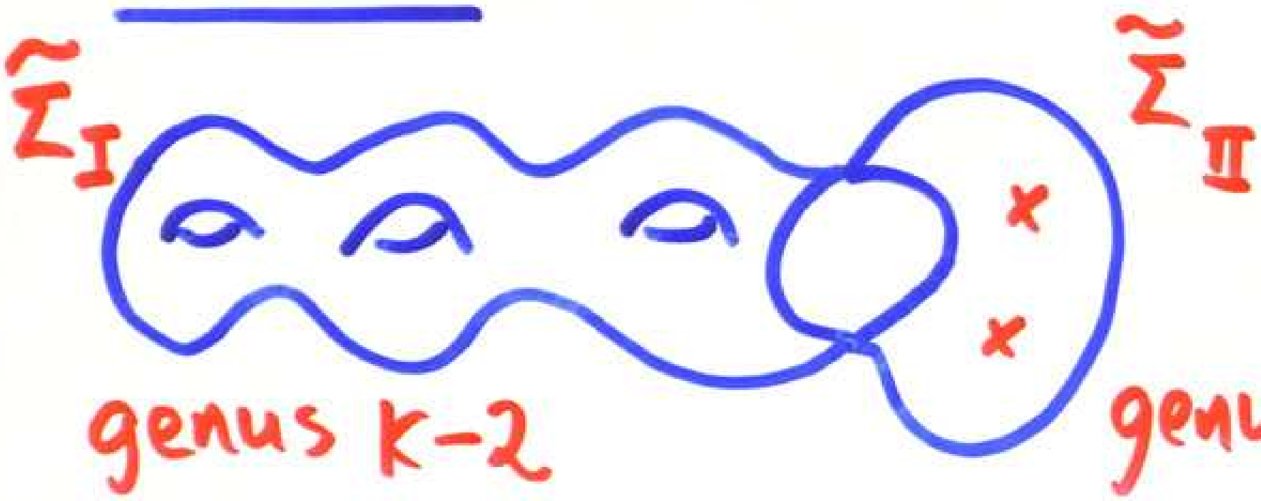
$$\Big\Downarrow \; S\to\infty$$

$\tilde{\Sigma}_{I}$ genus $k-2$ $\qquad$ $\tilde{\Sigma}_{II}$ genus $0$

$$dp = d\tilde{p}_1 + d\tilde{p}_2$$

- Two double poles $\times\ \times$
- $M-k+2$ simple poles $\bullet\bullet\ldots\bullet,\ \infty^{\pm}$

• $\underline{K = M}$ ND




- $\tilde{\Sigma}_I$ coincides with curve of one-loop spin chain of length $K = 2, 3, \ldots$

- Semiclassical quantization

$$\Delta - S \approx \frac{\sqrt{\lambda}}{2\pi}\left(K \log S + f(\ell_1, \ldots, \ell_{K-1}) + O\left(1/\log S\right)\right)$$

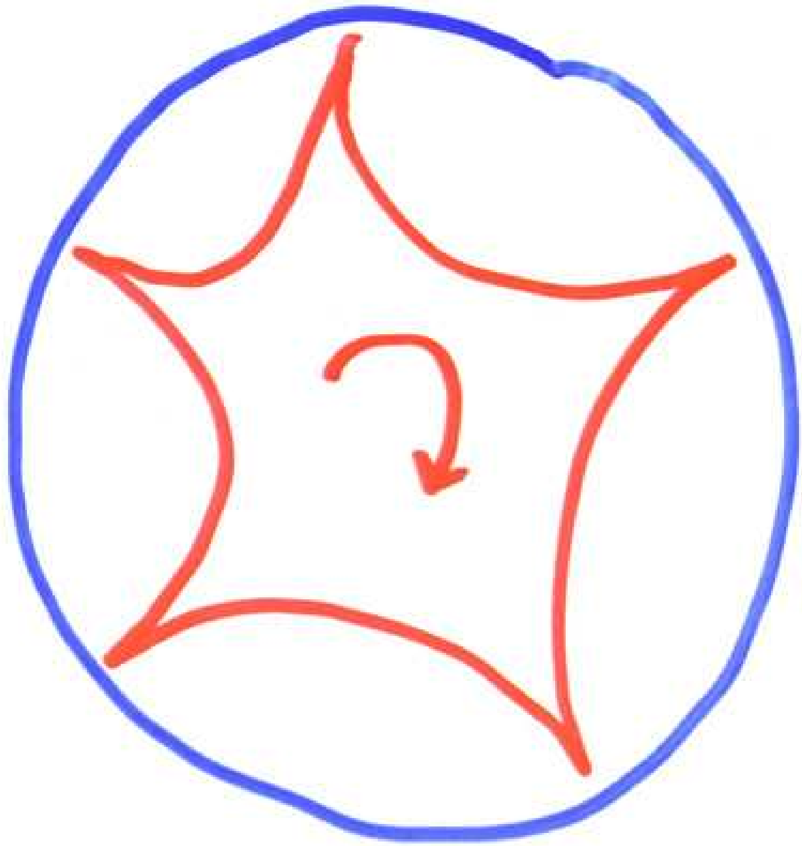
precise agreement with one-loop gauge theory to $O(S^0)$

$$\sqrt{\lambda}/2\pi \longrightarrow \Gamma(\lambda)$$

# Big Spikes

Kruczenski
ND + M. Losi

k spikes approach boundary,



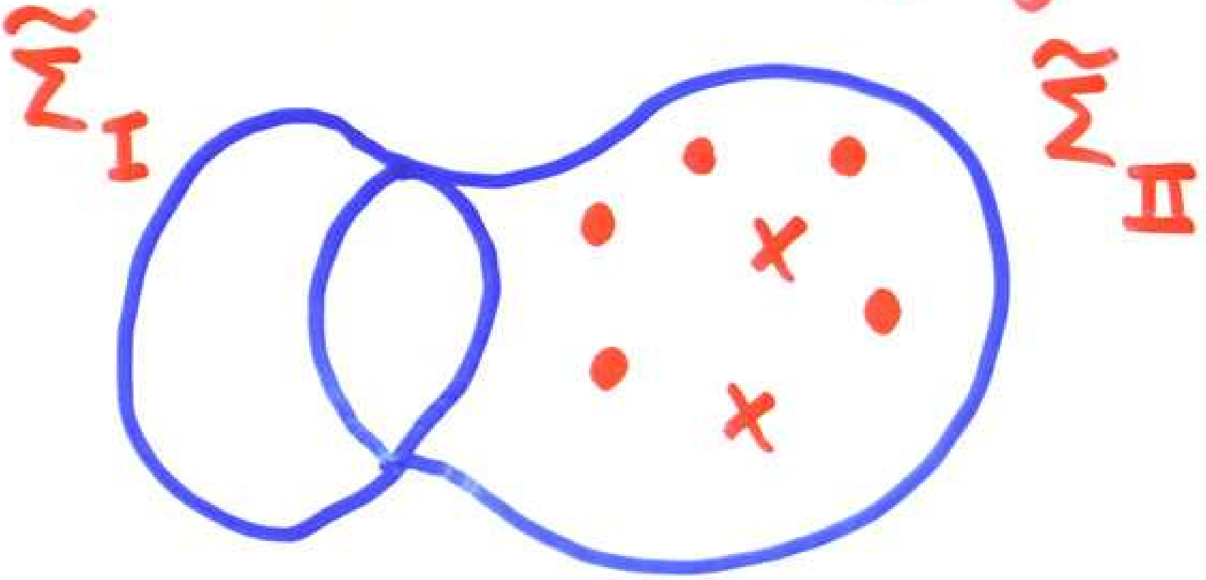
Angular seperation $\Delta\theta_i$:

$i = 1, \ldots, k$

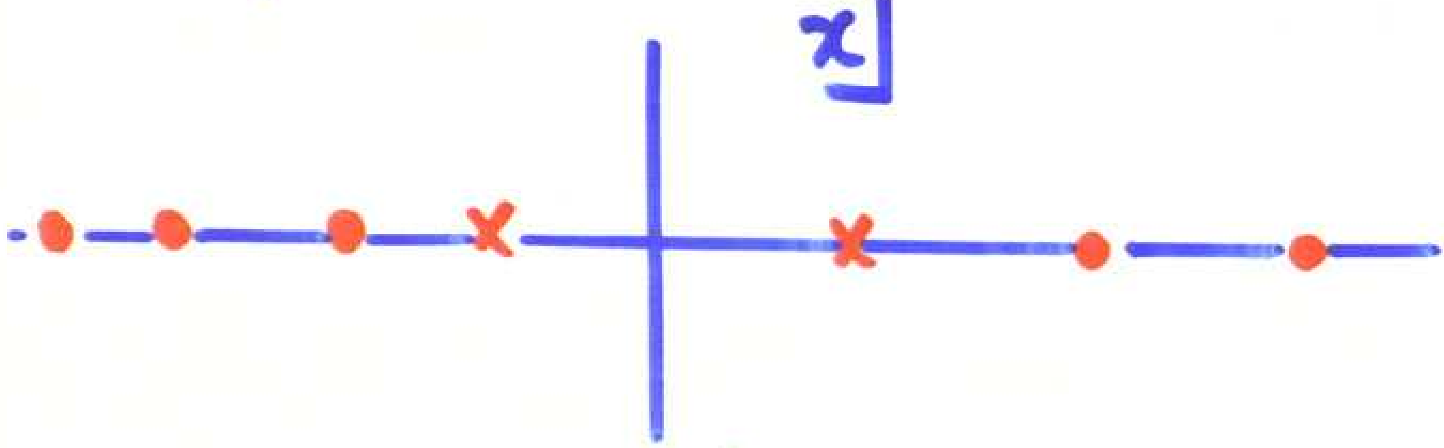
$$\Delta - S \approx \frac{\sqrt{\lambda}}{2\pi}\left[k \log S + \underbrace{\sum_{j=1}^{k} \log\left(\sin\frac{\Delta\theta_j}{2}\right)}_{f(\ell_1, \ldots, \ell_{k-1})} \ldots\right]$$

quantization: $f(\ell_1, \ldots, \ell_{k-1})$

• $\underline{k=2}$, $\underline{M>k}$ ND + M. Losi in progress

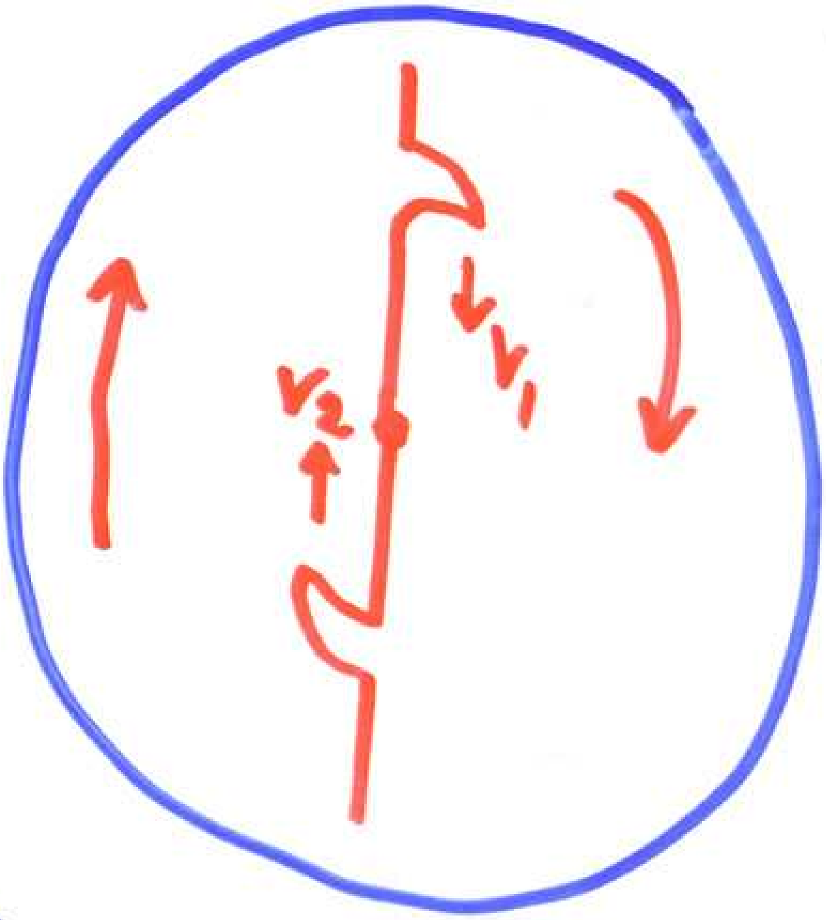



– $d\tilde{p}_2$ has $(M-k)$ extra simple poles at $x = c_i$




Solitons?

# Little Spikes Jevicki et al




- Pohlmeyer reduction

Spikes ≡ sinh - Gordon solitons

velocity: $v_i = 1/c_i < 1$

dispersion relation ?

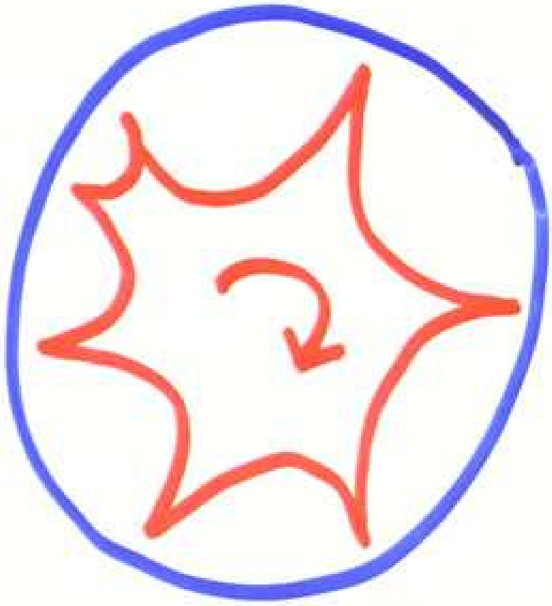
S - matrix ?

# Conclusion

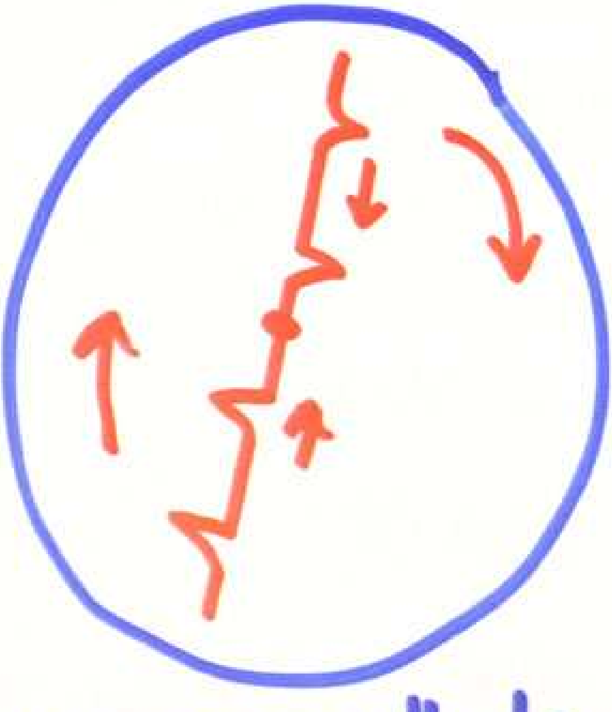
Semiclassical spectrum of string on AdS $S \to \infty$

- Two "branches"

- "big" spikes



- "small" spikes



strong parallels with big and small holes in gauge theory

# Speculation / Conjecture

"Giant Holes"

$$\text{spikes} \xleftrightarrow{AdS/CFT} \text{holes}$$

Test $AdS_5 \times S^5$

quantization of zero modes

$\rightsquigarrow$ particle in $\underline{6}$ of $SO(6)$?