Integrability in Gauge and String Theory:

a Preview

why this conference ?

why integrability ?

status and outstanding issues

General aims:

• understand quantum gauge theories at any coupling
[applications to both perturbative and non-perturbative issues]

• understand string theories in non-trivial backgrounds
[e.g. RR ones for flux compactifications]

AdS/CFT duality:

• relates the two questions suggesting solving them together rather than separately is best strategy

• relates simplest most symmetric theories
use of symmetries on both sides to make progress

Integrability:

Existence of powerful hidden symmetries
allowing to solve problem “in principle”

Strategy:

solve simplest most symmetric ("harmonic oscillator") case

then hope to treat other cases "in perturbation theory"

"Harmonic oscillator" (or "Ising", or "WZW"):

planar $\mathcal{N} = 4$ SYM theory = free superstring in $AdS_5 \times S^5$

most symmetric 4-d gauge th. = most symmetric 10-d string th.

$\mathcal{N} = 4$ SYM:

• maximal supersymmetry; conformal invariance;

• integrability? its precise meaning? in which observables?

could be expected in anomalous dimensions

[1-loop gluonic sector – known emergence of XXX spin chain]

• in fact, ∞ of hidden symmetries should play broader role:

"inherited" via AdS/CFT from 2-d integrable QFT –

string σ-model: use 2-d int. QFT to solve 4-d CFT

Superstring in $AdS_5 \times S^5$:

• integrable in "canonical" sense: sigma-model on symmetric space

classical equations admit infinite number of conserved charges

closely related (via Pohlmeyer reduction) to

(super) sine-Gordon and non-abelian Toda eqs

e.g. special motions of strings are described by

the integrable 1-d mechanical systems (Neumann, etc.)

• integrability extends to quantum level:

evidence directly on string-theory side to 2 loops

and also indirectly via AdS/CFT "bootstrap" reasoning

Quantum integrability: should control

• spectrum of string energies on $R \times S^1$

[anom. dim's of 2-d primary operators = vertex operators on $R^{1,1}$]

• correlation functions of vertex operators (to which extent?)*

[closed-string scattering amplitudes]

* cf. flat space; string field theory is not "integrable"

What about open-string sector?

Wilson loops (= disc partition functions)?

definition of "gluon scattering amplitudes"

beyond leading strong-coupling order ?

Integrability = hidden infinite dimensional symmetry

– if valid in quantum string theory –

i.e. at any value of string tension $\frac{\sqrt{\lambda}}{2\pi}$ – any $\lambda = g^2_{\rm YM} N_c$

should be "visible" also – via AdS/CFT – in

perturbative SYM theory

Integrability should then control:

- spectrum of dimensions of gauge-inv. single $\rm tr$ primary operators

[or spectrum of gauge-theory energies on $R \times S^3$]

- correlation functions of these operators (to which extent?)

What about scattering amplitudes and Wilson loops?

Amplitudes – IR divergent; Cusped WL's – UV divergent

Hidden (Yangian) symmetries broken at loop level in a "useful" way?

Are there "better" observables? (from integrability point of view)

Cross-sections? Effective actions?

Relation to correlation functions of gauge-inv. ops.?

Hints from string theory ?

Recent remarkable progress:

Spectrum of states

I. Spectrum of "long" operators = "semiclassical" string states

determined by Asymptotic Bethe Ansatz (2002-2007)

• its final (BES) form found after intricate superposition of information from perturbative gauge theory (spin chain, BA,...) and perturbative string theory (classical and 1-loop phase,...), use of symmetries (S-matrix), and assumption of exact integrability

• consequences checked against all available gauge and string data

Key example: cusp anomalous dimension $\mathrm{Tr}(\Phi D^S \Phi)$

$$f(\lambda \ll 1) = \frac{\lambda}{2\pi^2}\Big[1 - \frac{\lambda}{48} + \frac{11\lambda^2}{2^8 \cdot 45} - (\frac{73}{630} + \frac{4(\zeta(3))^2}{\pi^6})\frac{\lambda^3}{2^7} + ...\Big]$$

$$f(\lambda \gg 1) = \frac{\sqrt{\lambda}}{\pi}[1 - \frac{3\ln 2}{\sqrt{\lambda}} - \frac{K}{(\sqrt{\lambda})^2} - ...]$$

Extensions to subleading terms in large S expansion

[Related talks: Dorey, Freyhult]

II. Spectrum of “short” operators = all quantum string states

Thermodynamic Bethe Ansatz (2005-2009)

• reconstructed from ABA using solely
methods/intuition of 2-d integrable QFT, i.e. string-theory side
(how to incorporate wrapping terms directly on gauge-theory side?)

• highly non-trivial construction – lack of 2-d Lorentz invariance
in the standard “BMN-vacuum-adapted” l.c. gauge

• in few cases ABA “improved” by Luscher corrections is enough:
4- and 5-loop Konishi dimension, 4-loop minimal twist op. dimension

• crucial to check predictions against perturbative gauge and string data

Key example: anomalous dimension of Konishi operator

$$\gamma(\lambda \ll 1) = \frac{12\lambda}{(4\pi)^2}\Big[1 - \frac{4\lambda}{(4\pi)^2} + \frac{28\lambda^2}{(4\pi)^4} - [208 - 48\zeta(3) + 120\zeta(5)]\frac{\lambda^3}{(4\pi)^6} + \ldots\Big]$$

$$\gamma(\lambda \gg 1) = 2\sqrt[4]{\lambda} + \mathrm{b}_0 + \frac{b_1}{\sqrt[4]{\lambda}} + \frac{b_2}{(\sqrt[4]{\lambda})^2} + \frac{b_3}{(\sqrt[4]{\lambda})^3} + \ldots$$

Related talks: Banjok, Frolov, Gromov, Janik, Kazakov, Roiban; Torrielli

Many open questions:

Analytic form of strong-coupling expansion from TBA/Y-system?

Matching onto string spectrum in near-flat-space expansion?

No level crossing?

Strong-coupling expansion is Borel (non)summable?

Exponential corrections $e^{-a\sqrt{\lambda}}$ like in cusp anomaly case?

...

We are just at the beginning of understanding of structure of spectrum

Deeper issues:

Solve string theory from first principles –

- fundamental variables? preserve 2-d Lorentz invariance?
- prove quantum integrability?

lattice version of "supercoset" sigma model?

Another remarkable recent progress:

Amplitudes, Wilson loops and their symmetries

Weak coupling:

various connections to hidden symmetries and integrability

[talks by Arkani-Hamed, Korchemsky, Lipatov, McLoughlin, Plefka]

Strong coupling:

use of integrability of string theory to determine (via relation to WL's)
leading contributions to certain gluon scattering amplitudes

[talk by Maldacena]

Extensions and generalizations:

• $\mathcal{N} = 6$ supersymmetric 3-d Chern-Simons-matter theory
dual to superstring in $AdS_4 \times CP^3$
[talks by Nepomechie, Rey, Sieg; Zarembo]

• integrability methods applied to other
$d = 4$ supersymmetric gauge theories and other quantities
[talks by Gaiotto; Shatashvili]

• Integrability in interesting related systems:
[talks by Bazhanov; Bobenko; Saleur]

• Mathematical propertes of related perturbation-theory coefficients:
[talk by Vermaseren]

We are going to hear about many new exciting developments,
see closely related interesting posters,
and have stimulating discussions!