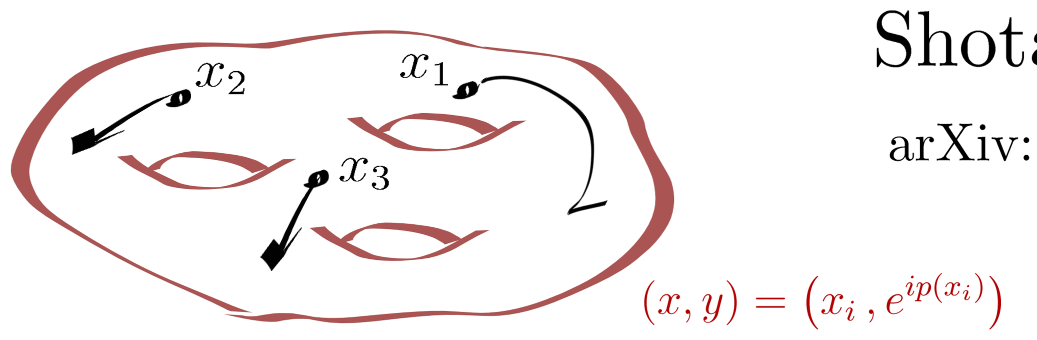
Evaluation of wave functions for three point functions



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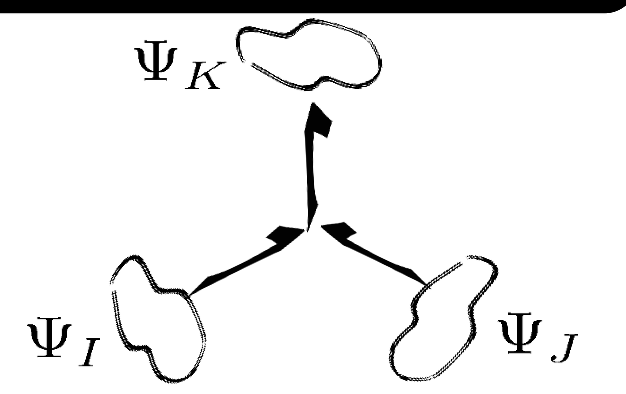
arXiv:1205.6060 [hep-th]



Brief summary

What?

- Developed a method to evaluate the constribution of vertex operators in the holographic correlation functions.

How?

- Use the state-operator correspondence of the worldsheet CFT.
- Construct action-angle variables using the Sklyanin's separation of variables.
- Construct wave functions from action-angle variables.

1. Introduction

Correlation functions in N=4 SYM

Since $\mathcal{N} = 4$ SYM is conformal,

$$\langle \mathcal{O}_I(x_1)\mathcal{O}_J(x_2)\rangle = \frac{\delta_{IJ}}{|x_{12}|^{2\Delta_I}} \qquad \langle \mathcal{O}_I(x_1)\mathcal{O}_J(x_2)\mathcal{O}_K(x_3)\rangle = \frac{C_{IJK}}{|x_{12}|^{\Delta_I+\Delta_J-\Delta_K}|x_{23}|^{\Delta_J+\Delta_K-\Delta_I}|x_{31}|^{\Delta_K+\Delta_I-\Delta_J}}$$

Δ_I : Encodes infomation on the spectrum. Well-studied by integrability-based methods.

C_{IJK} : Encodes the information on the dinamics. Not yet clear whether it can be solved by the integrability.

Correlation functions from classical strings

Correlation functions at $\lambda \to \infty$ can be evaluated by classical strings. To see this, first consider GKP-Witten relation for non-BPS operators.

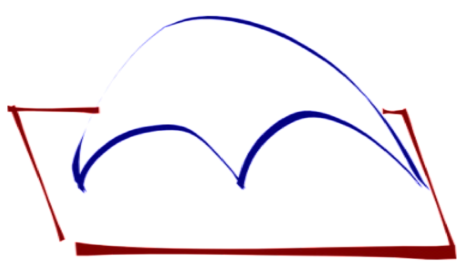
$$\langle \mathcal{O}_1(x_1)\,\mathcal{O}_2(x_2)\,\mathcal{O}_3(x_3)\rangle_{\text{gauge theory}} = \frac{1}{\text{Möbius}}\int \prod_i d^2 z_i \,\langle V_1[X^\mu(z_1)]\,V_2[X^\mu(z_2)]\,V_3[X^\mu(z_3)]\rangle_{\text{worldsheet}}$$

In $\lambda \to \infty$ limit, the worldsheet correlation function is dominated by a saddle point trajectory.

$$\langle V_1(z_1)V_2(z_2)\cdots\rangle = \int \mathcal{D}X\, V_1(z_1)V_2(z_2)\cdots e^{-S_{\text{string}}} \qquad S_{\text{string}} = \sqrt{\lambda}\int d^2z\,\partial X^\mu \bar{\partial} X_\mu$$

$\lambda \to \infty$ *Dominated by a saddle point*

$$V_1[X_*(z_1)]V_2[X_*(z_2)]V_3[X_*(z_3)]e^{-S[X_*]}$$



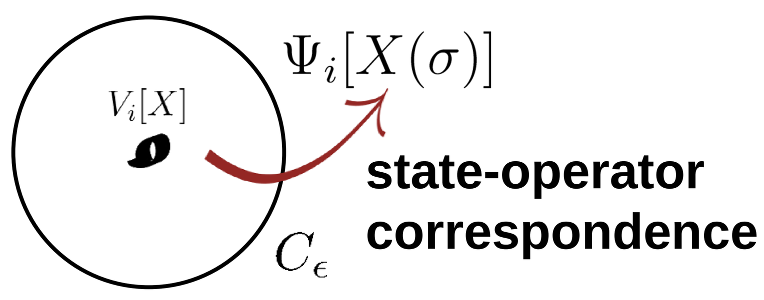
The vertex ops., $V_i[X_*(z_i)]$, are important because...

1. (Correctly renormalized) vertex ops. will cancel the divergence from the action $S[X_*]$.
2. The contribution from the vertex operators is necessary to reproduce the correct spacetime dependence of three point functions.

2. How to construct and evaluate vertex operators for non-BPS states.

Step1. Vertex op. to wave functions

By the state-operator correspondence of the world-sheet CFT, the vertex ops are mapped to the wave functions. It is practically easier to construct wave functions than to construct vertex ops.



Step 2. Classical wave functions from action-angle variables

In the classical limit, the wave functions can be obtained by

$$\Psi[X(\sigma)] \sim e^{iW[X(\sigma)]} \qquad W[X(\sigma)] : \text{solution of the Hamilton-Jacobi eq.}$$

Solving the H-J eq. is difficult in general. However, the solution can be easily constructed if we know the action-angle variables.

$$W = \sum_j J_j \theta_j \qquad J_j : \text{action variable} \qquad \theta_j : \text{angle variable}$$

Step 3. Action-angle variables from Sklyanin's method

Fortunately, a general method to construct action-angle variables for the system with Lax representation is known.

[Sklyanin '95] cf. [Dorey-Vicedo '06]

`Angle-variable`

Angle variables can be constructed from the poles of the normalized eigenvector of the monodromy matrix:

$$\Omega(x)\vec{\psi}(x) = e^{ip(x)}\vec{\psi}(x) \qquad \Omega(x) : \text{monodromy matrix}$$

$$\vec{n}\cdot\vec{\psi}(x) = 1 \qquad \vec{n} : \text{normalization vector}$$

$$\phi_i \sim \sum_j \int^{x_j} \omega_i \qquad \vec{\psi}(x_j) \to \infty$$

ω_i : Abel-Jacobi map associated with the spectral curve

`Action-variable`

$$S_i \sim \oint_{a_i} p(x)d(x + x^{-1})$$

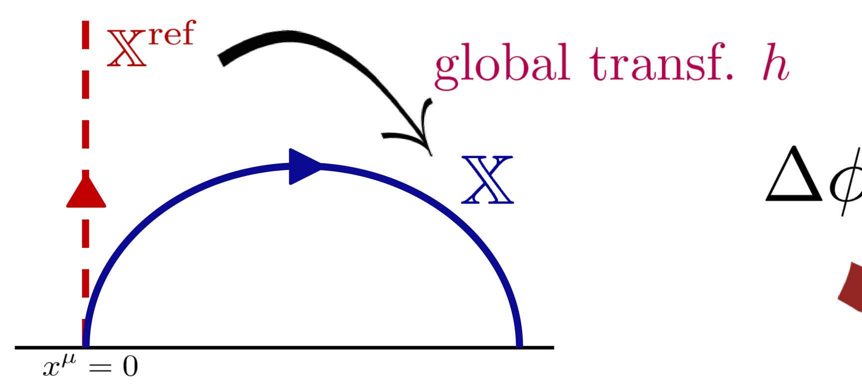
a.k.a. filling fraction

Remark 1:

gauge		string
$\mathcal{O}(x^\mu)$		$\Psi[\phi_j] = e^{i\sum_j S_j\phi_j}$
Charges: $\Delta, S, \cdots$	$\longleftrightarrow$	Spectral curve
x^μ	$\longleftrightarrow$	$\vec{n}$ $\quad (\vec{n}\cdot\vec{\psi} = 1)$

Remark 2:

Overall normalization of the wave function is unimportant. To extract nontrivial information, it is useful to consider the difference from a certain "reference" trajectory.



$$\Delta\phi = \phi - \phi^{\text{ref}}$$

Determined by h

The difference is determined by the global transformation we need to approximate the behavior of the solution around the vertex operator by using the reference solution.

3. Prospect

- Construction of action-angle variables for other sectors (SO(6), full PSU(2,2|4))
- Application to the correlation functions of the Wilson loop and the local operators.
- Application to four point functions.
- Is the Sklyanin's separation of variables useful for the gauge theory calculation?

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