Numerical results for the exact spectrum of planar ABJM theory

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based on: F.L-M, arXiv: 1110.5869

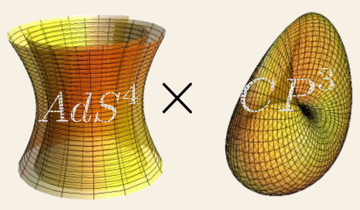
Summary

- We numerically solve the Thermodynamic Bethe Ansatz equations for a short operator in planar ABJM theory
- This provides the operator's exact scaling dimension at intermediate coupling

ABJM duality

Aharony,Bergman,Jafferis,Maldacena 2008

$\mathcal{N}=6$ 3d superconformal $SU(N)\times SU(N)$ Chern-Simons $\Longleftrightarrow$ superstrings in $AdS^4\times CP^3$



An AdS4/CFT3 correspondence

Planar limit: $N\to\infty$

't Hooft coupling λ $\Longleftrightarrow$ string tension $T\propto\sqrt{\lambda}$

operator anomalous dimensions γ_i $\Longleftrightarrow$ spectrum of string state energies E_i

The problem we study: finding this spectrum

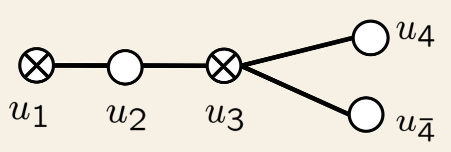
- Integrability $\Longrightarrow$ hope for exact solution of the problem
- Similarities with N=4 SYM / $AdS_5\times S^5$

Asymptotic Bethe ansatz (ABA)

Gromov, Vieira 2008

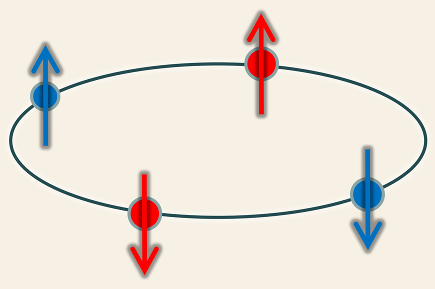
Asymptotic spectrum (at $L\to\infty$) - from ABA equations based on alternating spin chain

L ~ [# of elementary fields in the operator]



The operator we study: $\mathcal{O}_{20}=\mathrm{Tr}\left(Y^{[1}Y^{\dagger}_{[4}Y^{2]}Y^{\dagger}_{3]}\right)$

Y^A, $Y^{\dagger}_A$ are scalar fields; irrep **20** of $SU(4)_R$



ABA description:

su(2): $L=2$, two Bethe roots $u_4=u_{\bar{4}}=0$

sl(2): $L=1$, same Bethe roots

$$\gamma_{ABA}=\sqrt{1+16h(\lambda)^2}-1$$

$h(\lambda)$ is an interpolating function of the coupling

$$h(\lambda)=\lambda+(-8+2\zeta(2))\lambda^3+O(\lambda^5)=\sqrt{\tfrac{\lambda}{2}}+h_0+O(1/\sqrt{\lambda})$$

Y-system and TBA

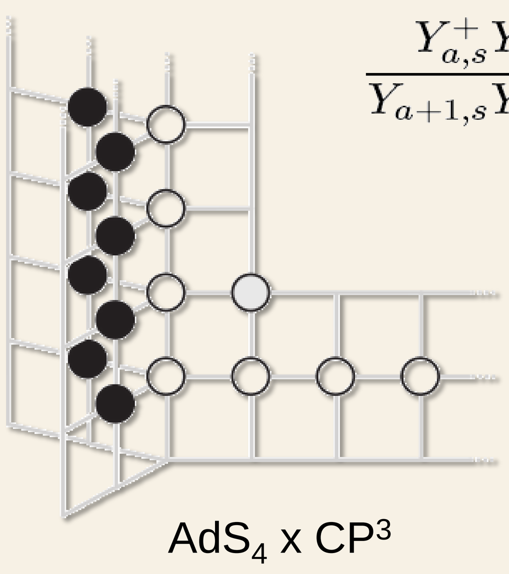
ABA result gets finite-volume wrapping corrections for finite L

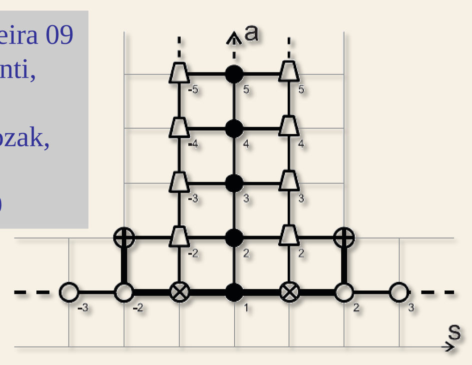
Exact spectrum at any volume and coupling – from an infinite set of functional (Y-system/Hirota) or integral (Thermodynamic Bethe Ansatz) equations

$$\frac{Y^+_{a,s}Y^-_{a,s}}{Y_{a+1,s}Y_{a-1,s}}=\frac{(1+Y_{a,s+1})(1+Y_{a,s-1})}{(1+Y_{a+1,s})(1+Y_{a-1,s})}\qquad Y^{\pm}_{a,s}\equiv Y_{a,s}(u\pm\tfrac{i}{2})$$

Gromov,Kazakov,Vieira 09; Bombardelli,Fioravanti, Tateo 09; Gromov,F.L-M 09

Gromov,Kazakov,Vieira 09; Bombardelli,Fioravanti, Tateo 09; Gromov,Kazakov,Kozak, Vieira 09; Arutyunov,Frolov 09





$AdS_4\times CP^3$ Black nodes $\Longrightarrow$ energy $AdS_5\times S^5$

$$E=2\sum_a\int_{-\infty}^{\infty}\frac{du}{2\pi i}\frac{\partial\epsilon^{mir}_a}{\partial u}\log\left(1+Y^{mir}_{a,0}(u)\right)+2\sum_j\epsilon^{ph}(u_{4,j})$$

+ "exact Bethe equations": $Y^{ph}_{1,0}(u_{4,j})=-1$ sl(2) sector: $Y_{a,0}=Y_{\bar{a},0}$

Leading wrapping for $\mathcal{O}_{20}$ from Y-system: $E_{wrap}=(32-16\zeta_2)h(\lambda)^4$

Confirmed by 4-loop calculation! Minahan,Sax,Sieg 09 Gromov,Kazakov,Vieira 09

TBA equations

$$\log\frac{Y_\otimes}{\mathbf{Y}_\otimes}=+K_{m-1}*\log\frac{1+1/Y_{\bigcirc_m}}{1+Y_{\triangle_m}}\frac{1+\mathbf{Y}_{\triangle_m}}{1+1/\mathbf{Y}_{\bigcirc_m}}+2\mathcal{R}^{(0m)}*\log(1+Y_{\bullet_m})\quad(3.13)$$

$$\log\frac{Y_\oplus}{\mathbf{Y}_\oplus}=-K_{m-1}*\log\frac{1+1/Y_{\bigcirc_m}}{1+Y_{\triangle_m}}\frac{1+\mathbf{Y}_{\triangle_m}}{1+1/\mathbf{Y}_{\bigcirc_m}}-2\mathcal{B}^{(0m)}*\log(1+Y_{\bullet_m})\quad(3.14)$$

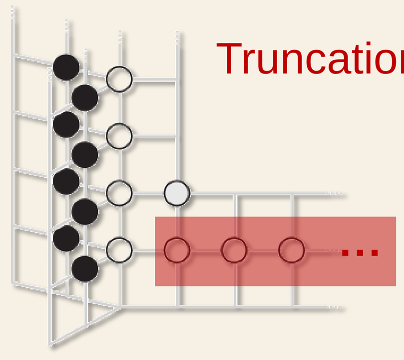
$$\log\frac{Y_{\triangle_n}}{\mathbf{Y}_{\triangle_n}}=-K_{n-1,m-1}*\log\frac{(1+Y_{\triangle_m})}{(1+\mathbf{Y}_{\triangle_m})}-K_{n-1}*\log\frac{1+Y_\otimes}{1+1/Y_\oplus}\frac{1+1/\mathbf{Y}_\oplus}{1+\mathbf{Y}_\otimes}+2\mathcal{M}_{nm}*\log(1+Y_{\bullet_m})\quad(3.15)$$

$$\log\frac{Y_{\bigcirc_n}}{\mathbf{Y}_{\bigcirc_n}}=K_{n-1,m-1}*\log\frac{(1+1/Y_{\bigcirc_m})}{(1+1/\mathbf{Y}_{\bigcirc_m})}+K_{n-1}*\log\frac{1+Y_\otimes}{1+1/Y_\oplus}\frac{1+1/\mathbf{Y}_\oplus}{1+\mathbf{Y}_\otimes}\quad(3.16)$$

$$\log\frac{Y_{\bullet_n}}{\mathbf{Y}_{\bullet_n}}=-\mathcal{B}^{(n0)}*\log\frac{(1+1/Y_\oplus)}{(1+1/\mathbf{Y}_\oplus)}+\mathcal{R}^{(n0)}*\log\frac{(1+Y_\otimes)}{(1+\mathbf{Y}_\otimes)}+\mathcal{N}_{nm}*\log\frac{(1+Y_{\triangle_m})}{(1+\mathbf{Y}_{\triangle_m})}+\left(2\bar{\mathcal{S}}_{nm}-\mathcal{R}^{(11)}_{nm}+\mathcal{B}^{(11)}_{nm}\right)*\log(1+Y_{\bullet_m})\quad(3.17)$$

$$\log Y_n(u)=\Phi_n(u)+\int dv\,K_{n,m}(u,v)\log(1+Y_m(v))$$

- We solve these equations iteratively
- Obtain energy as a function of $h(\lambda)$ for $0\le h(\lambda)\le 1$
- Numerics confirm that Bethe roots remain at zero



Truncation: use $Y_{1,s}=\left(\frac{T^+_{1,s}T^-_{1,s}}{T_{1,s+1}T_{1,s-1}}-1\right)^{-1}$, then Y-system is solved by $T_{1,s}=s+K_s*f_R$

Gromov,Kazakov,IGST2010; Gromov,Kazakov,Leurent,Volin,2011

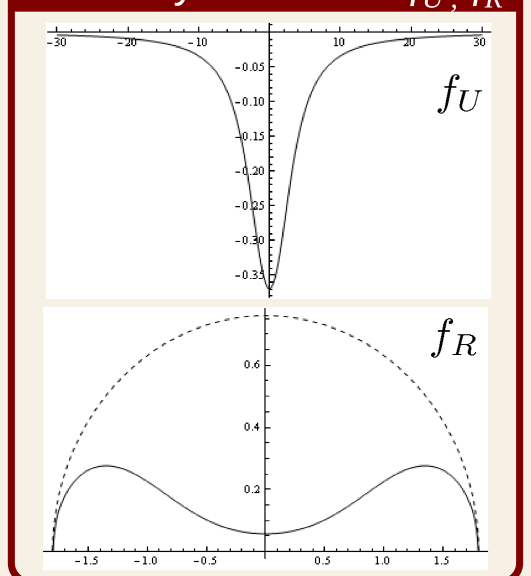
All $\{Y_{1,s}\}$ (s=2,3,...) replaced by 1 function f_R !

$$\frac{1+Y_{1,1}}{1+1/Y_{2,2}}=\frac{(1+K_1^+*_{p.v.}f_R+f_R/2)(1+K_1^-*_{p.v.}f_R+f_R/2)}{(1+K_1^+*_{p.v.}f_R-f_R/2)(1+K_1^-*_{p.v.}f_R-f_R/2)}$$

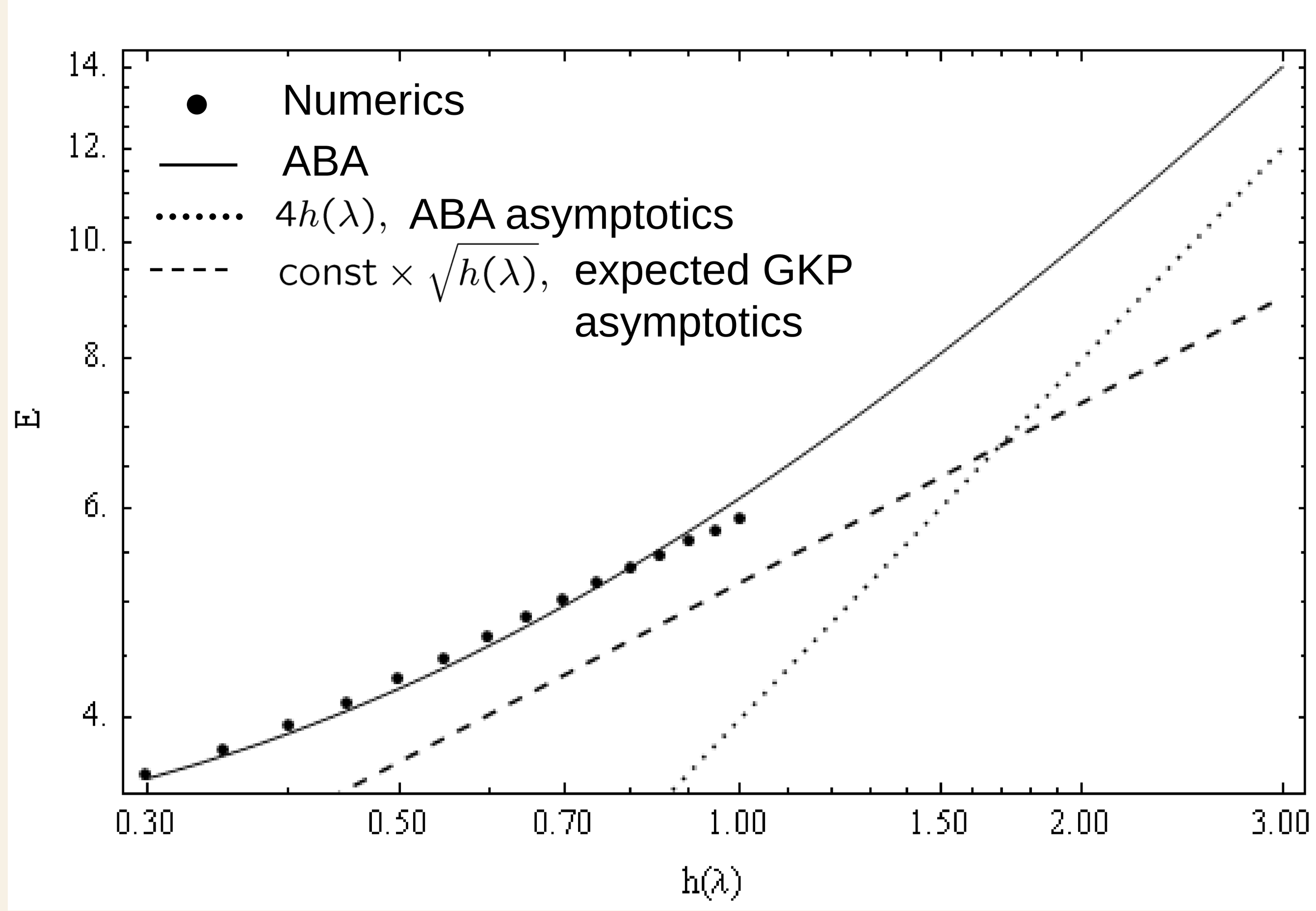
Same in upper part: $\{Y_{a,1}\}\Longrightarrow f_U$ but need a cutoff on black nodes (keep 6-7 of them).

Also subtract large L solution of Y-system; in the end precision for energy is $\pm10^{-3}$

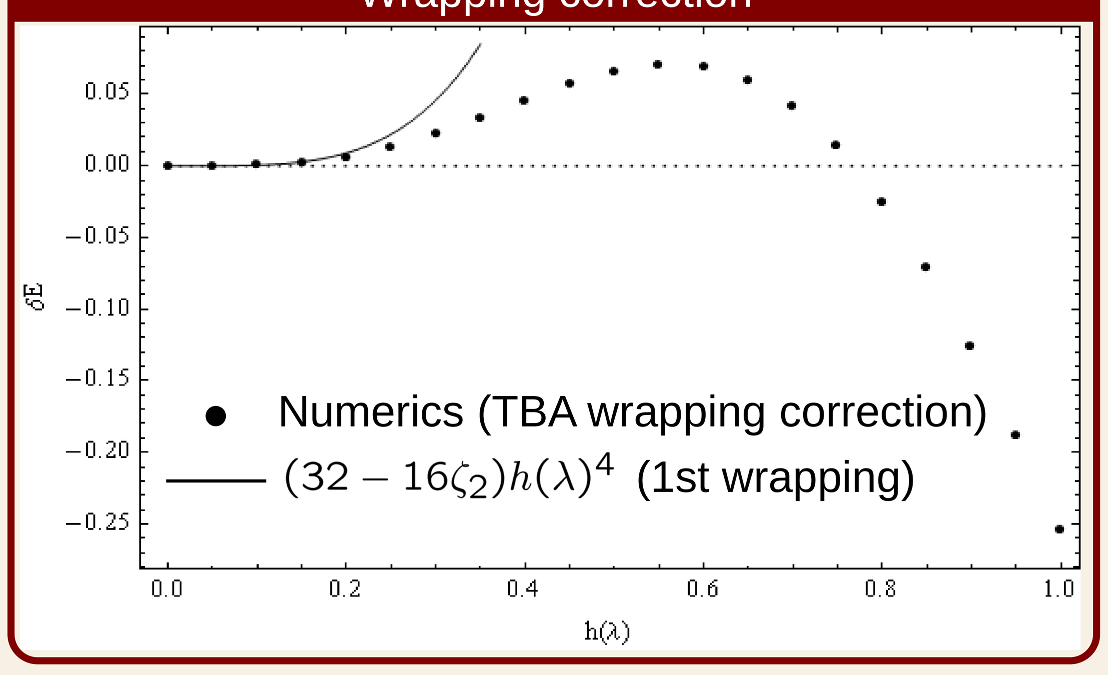
Auxiliary functions f_U, f_R



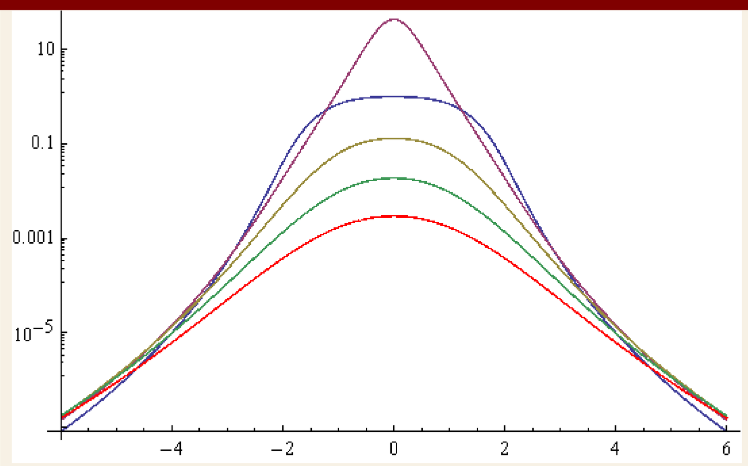
Results: the scaling dimension



Wrapping correction

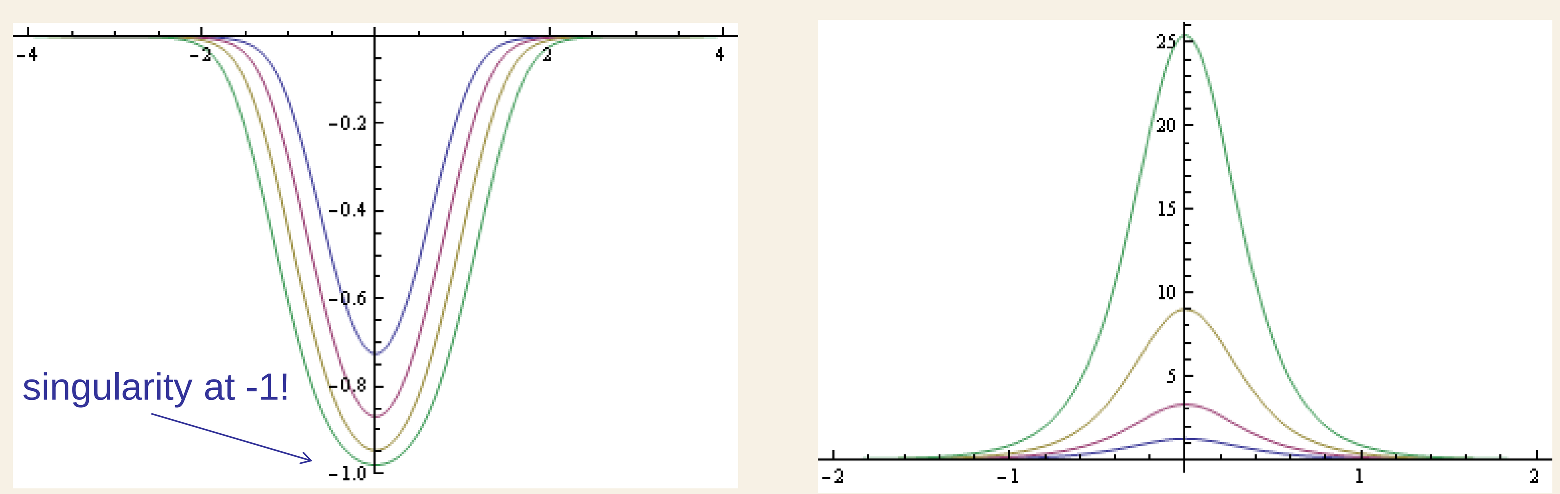


Y-functions



Plot of $(-1)^n\,Y_{n,0}$ for n = 1,2,3,4,5

Y-functions



Y-functions $Y_{1,0}$ (left) and $Y_{2,0}$ (right) for $h(\lambda)$ = 0.6, 0.7, 0.8, 0.9

Conclusions

- First numerical study of TBA for ABJM theory; scaling dimension computed in a region inaccessible by other means
- Agreement to 4 loops with perturbation theory at weak coupling
- Singularity approached by $Y_{1,0}$ – new feature compared to Konishi
- Future directions: increase the coupling using FiNLIE to investigate restoration of $\lambda^{1/4}$ scaling; explore other states

Acknowledgements

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