Holographic correlator between 1/4 BPS Wilson loop and 1/2 BPS local operator

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1. Introduction:

AdS/CFT correspondence:

Correlator in gauge theory $\leftrightarrow$ Propagator in string theory

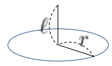
Target: $\langle W(C)O_J(\vec{x})\rangle \quad \sim \quad \exp(-S_{\rm string}) \qquad (J \sim \sqrt{\lambda} \gg 1)$

gauge theory: 1/4 BPS Wilson loop and 1/2 BPS local operator

$$W(C) = {\rm tr P exp} \int d\sigma\left(iA_\mu \dot{x}^\mu + \Phi_i \Theta^i |\dot{x}|\right), \quad O_J = {\rm tr}(\Phi_3 - i\Phi_4)^J$$

$$\vec{\Theta}(\sigma) = (\sin\theta_0 \cos\sigma, \sin\theta_0 \sin\sigma, \cos\theta_0, 0, 0, 0)$$

N.Drukker (06)



string theory: semiclassical string amplitude

1/2BPS WL case $\rightarrow$ K.Zarembo(02)

Gauge theory result:

G.W.Semenoff-K.Zarembo (01), G.W.Semenoff-D.Young (06)

$$\frac{\langle WO_J\rangle}{\langle W\rangle} \propto \left(\frac{r}{r^2+\ell^2}\right)^J \frac{I_J(\sqrt{\lambda'})}{I_1(\sqrt{\lambda'})}, \quad (\lambda' = \lambda\cos^2\theta_0, I_J : \text{modified Bessel function})$$

saddle points for Bessel function:

$$I_J(\sqrt{\lambda'}) = \frac{1}{2\pi i}\int_{\infty - i\pi}^{\infty + i\pi} dz e^{\sqrt{\lambda'}(\cosh z - j' z)} \quad (J = j'\sqrt{\lambda'})$$

$$z = \begin{cases} z_+ \equiv \log(\sqrt{j'^2+1} + j'), & \text{on the path} \\ z_- \equiv -\log(\sqrt{j'^2+1}+j') + \pi i, & \text{not on the path} \end{cases}$$

leading behavior:

$$I_J(\sqrt{\lambda'}) \sim e^{\sqrt{\lambda'}(\cosh z - j'z)}\Big|_{z_+} = e^{\sqrt{\lambda'}(\sqrt{j'^2+1} + j'\log(\sqrt{j'^2+1} - j'))}$$

2. String solution in global coordinate:

global metric and AdS3 × S3 ansatz:

$$ds^2 = L^2\{-\cosh^2\rho dt^2 + d\rho^2 \sinh^2\rho(d\varphi_1^2 + \sin^2\varphi_1 d\varphi_2^2 + \cos^2\varphi_1 d\varphi_3^2)$$
$$+ d\theta^2 + \sin^2\theta d\phi^2 + \cos^2\theta(d\chi_1^2 + \sin^2\chi_1 d\chi_2^2 + \cos^2\chi_1 d\chi_3^2)\}$$

ansatz: $\begin{cases} t = t(\tau), \quad \rho = \rho(\tau), \quad \varphi_1 = \frac{\pi}{2}, \quad \varphi_2 = \sigma, \\ \theta = \theta(\tau), \quad \phi = \sigma, \quad \chi_1 = \frac{\pi}{2}, \quad \chi_2 = \chi_2(\tau). \end{cases}$

(integrable structure for this system is studied in N.Drukker-B.Fiol (06))

constants of motion:

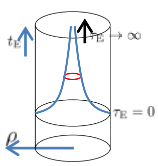
$$\Pi_{\chi_2} = \Pi_t = J = j\sqrt{\lambda}, \qquad (\leftrightarrow \quad \mathcal{O}_J)$$

Euclidean solution ($t_{\rm E} = it, \ \tau_{\rm E} = i\tau$):

AdS5 part:

$$\sinh\rho = \frac{\sqrt{j^2+1}}{\sinh\sqrt{j^2+1}\tau_{\rm E}}$$

$$t_{\rm E} = j\tau_{\rm E} - \frac{1}{2}\log\left(\frac{\cosh(\sqrt{j^2+1}\tau_{\rm E} + \xi)}{\cosh(\sqrt{j^2+1}\tau_{\rm E} - \xi)}\right)$$
$$(\xi = \log(\sqrt{j^2+1} + j))$$



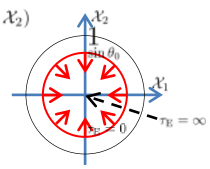
S5 part:

$$\sin\theta = \frac{\sqrt{j^2+1}}{\cosh\sqrt{j^2+1}(\tau_{\rm E} + \tau_0)} \quad \left(\sin\theta_0 = \frac{\sqrt{j^2+1}}{\cosh\sqrt{j^2+1}\tau_0}\right)$$

$$\chi_2 = -ij\tau_{\rm E} + \frac{i}{2}\log\frac{\sinh(\sqrt{j^2+1}(\tau_{\rm E}+\tau_0) + \xi)\sinh(\sqrt{j^2+1}\tau_0 - \xi)}{\sinh(\sqrt{j^2+1}(\tau_{\rm E}+\tau_0) - \xi)\sinh(\sqrt{j^2+1}\tau_0 + \xi)}$$

embedding coordinate: $\mathcal{X}_1 \pm i\mathcal{X}_2 = \sin\theta e^{\pm i\phi}$ $\mathcal{X}_3 \pm i\mathcal{X}_4 = \cos\theta e^{\pm i\chi_2}$

$(\mathcal{X}_1, \mathcal{X}_2)$ $(\mathcal{X}_3, \mathcal{X}_4)$



"rotation"

$\Pi_{\chi_2} = J$

3. BPS condition:

BPS condition in gauge theory $\rightarrow$ G.W.Semenoff-D.Young (06)

projection for string world sheet (Euclidean):

$$\frac{i}{\sqrt{\det g}}\partial_{\tau_{\rm E}} X^M \partial_\sigma X^N \hat{\Gamma}_M \hat{\Gamma}_N \sigma_3 \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} = \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}$$

Killing spinor in AdS5 × S5:

$$\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} = e^{\frac{\rho}{2}\epsilon\Gamma_*\Gamma_1} e^{\frac{t_{\rm E}}{2}\epsilon\Gamma_*\Gamma_E} e^{\frac{\varphi_2}{2}\Gamma_{13}} e^{\frac{\theta}{2}\epsilon\Gamma_*\Gamma_5} e^{\frac{\phi}{2}\Gamma_{56}} e^{\frac{\chi_2}{2}\epsilon\Gamma_*\Gamma_8} \begin{pmatrix} \epsilon_1^0 \\ \epsilon_2^0 \end{pmatrix}$$

preserved supersymmetry:

$(1 - i\Gamma_{\rm E}\Gamma_8)\begin{pmatrix} \epsilon_1^0 \\ \epsilon_2^0 \end{pmatrix} = 0$ $\leftrightarrow$ S5 rotation (local operator)

$$\left.\begin{array}{l} (1 - \Gamma_{1356})\begin{pmatrix} \epsilon_1^0 \\ \epsilon_2^0 \end{pmatrix} = 0 \\ (1 + i(\sin\theta_0\Gamma_{16} + \cos\theta_0\Gamma_{13})\sigma_3)\begin{pmatrix} \epsilon_1^0 \\ \epsilon_2^0 \end{pmatrix} = 0 \end{array}\right\} \leftrightarrow \text{1/4 BPS string (Wilson loop)}$$

N.Drukker (06)

1/8 BPS (agree with the expectation from gauge theory)

4. Solutions in Poincare AdS:

Poincare AdS: $ds^2 = L^2\frac{dY^2 + d\vec{X}^2}{Y^2} = L^2\frac{dY^2 + dR^2 + R^2 d\Omega_3^2}{Y^2}$

global $\leftrightarrow$ Poincare: $Y = \frac{e^{t_{\rm E}}}{\cosh\rho}, \quad R = e^{t_{\rm E}}\tanh\rho$

solution with O_J at infinity (obtained by above transformation):

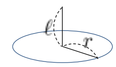
$$Y(\tau_{\rm E}) = e^{j\tau_{\rm E}}\left[\sqrt{j^2+1}\tanh(\sqrt{j^2+1}\tau_{\rm E} + \xi) - j\right] \quad Y(0) = 0, \quad Y(\infty) = \infty$$

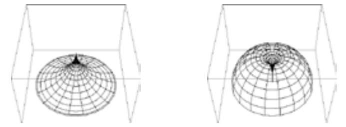
$$R(\tau_{\rm E}) = \frac{e^{j\tau_{\rm E}}\sqrt{j^2+1}}{\cosh(\sqrt{j^2+1}\tau_{\rm E} + \xi)} \quad R(0) = 1, \quad R(\infty) = 0$$

K.Zarembo(02)

solution with O_J at finite distance (obtained by isometry):

$$\vec{X}' = \frac{(\ell^2 + r^2)}{\ell^2 + R^2 + Y^2}(R\cos\sigma, R\sin\sigma, 0, -\ell) + (0,0,0,\ell), \quad Y' = \frac{(\ell^2+r^2)Y}{\ell^2 + R^2 + Y^2}$$





5. Semi-classical amplitude:

action: $S_{\rm string} = S_{\rm bulk} + S_{\rm boundary}$

bulk action: $S_{\rm bulk} = \sqrt{\lambda}\left[\frac{1}{0} - \sqrt{j^2 + \cos^2\theta_0}\right]$

boundary terms:

$\tau_{\rm E} = 0$: Legendre transformation for $u = 1/Y$

N.Drukker-D.Gross-H.Ooguri(99)

$$S_{\rm boundary,0} = \frac{\partial L}{\partial \dot{u}}u\Big|_{\tau_{\rm E}=0} = -\frac{\sqrt{\lambda}}{0}$$

$\tau_{\rm E} = \infty$: vertex operator

K.Zarembo(02), A.A.Tseytlin(03)

$$-S_{\rm boundary,\infty} = J\left[\log\frac{Y'}{Y'^2 + (\vec{X}' - \vec{x})^2} + \log\cos\theta e^{-i\chi_2}\right]_{\tau_{\rm E}=\infty}$$

$$\begin{cases} = \log\frac{r}{\ell^2+r^2} + j\infty + \log(\sqrt{j^2+1} - j) \\ = -j\infty - \log(\sqrt{j^2+1} - j) + \log(\sqrt{j'^2+1} - j') \end{cases}$$

result:

$$\exp(-S_{\rm string}) = \left(\frac{r}{\ell^2+r^2}\right)^J \exp\sqrt{\lambda'}\left[\sqrt{j'^2+1} + j'\log(\sqrt{j'^2+1} - j')\right]$$

agree with gauge theory result

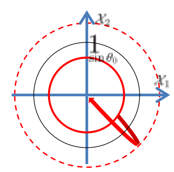
see also the recent paper: S.Giombi-V.Pestun(12)

6. Second solution:

unstable string solution for J=0 $\rightarrow$ N.Drukker (06)

solution: $\sin\theta = \frac{\sqrt{j^2+1}}{\cosh\sqrt{j^2+1}(-\tau_{\rm E} + \tau_0)}$

- the "size" becomes larger than S5
- smooth in embedding coordinate
- preserves the same supersymmetry



changes in string action:

$$S_{\rm bulk} = \sqrt{\lambda}\left[\frac{1}{0} \underline{+} \sqrt{j^2 + \cos^2\theta_0}\right]$$

$$\log\cos\theta e^{-i\chi_2}\Big|_\infty = -j\infty - \log(\sqrt{j^2+1} - j) \underline{-} \log(\sqrt{j'^2+1} - j') \underline{+\pi i}$$

result:

$$\exp(-S_{\rm string}) \propto (-1)^J \exp\sqrt{\lambda'}\left[-\sqrt{j'^2+1} - j'\log(\sqrt{j'^2+1} - j')\right]$$

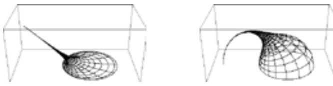
second saddle point for modified Bessel function:

agree

$$e^{\sqrt{\lambda'}\cosh z - Jz}\Big|_{z_-} = (-1)^J e^{\sqrt{\lambda'}(-\sqrt{j'^2+1} - j'\log(\sqrt{j'^2+1} - j'))}$$

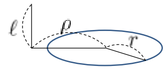
7. Generic configurations:

generic isometry $\rightarrow$ genericstring solutions:



evaluation of string action:

$$\left(\frac{r}{\ell^2+r^2}\right)^J \rightarrow \left(\frac{r}{\sqrt{(\rho^2 + \ell^2 - r^2)^2 + 4\ell^2 r^2}}\right)^J$$



agree with the scaling behavior found previously

D.E.Berenstein-R.Corrado-W.Fischler-J.M.Maldacena (99)
L.F.Alday-A.A.Tseytlin (11)

8. Conclusion:

- rotating string solution extended in S5 as well as AdS5 constructed
- BPS condition istudied. $\rightarrow$ 1/8 of supersymmetry preserved
- leading behavior of modified Bessel func., scaling behavior reproduced
- the second solution which becomes larger than S5 found
- the saddle point of modified Bessel function, which is not on the steepest descent path reproduced (by the second string solution)
- scaling behavior for generic configuration reproduced