

Regge Limit of Gauge Theories & Gravity

(with Links to Integrability)

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Regge Limit of Gauge Theories & Gravity

Let us complete the picture ...

INTEGRABILITY IN GAUGE AND STRING THEORY

ETH Zurich, 20 - 24 August 2012

Speakers include
Nima Arkani-Hamed
Gleb Arutyunov
Benjamin Basso
Nikolay Gromov
Ben Hoare
Tomasz Lukowski
Vasily Pestun
Agustin Sabio Vera
Emery Sokatchev
Konstantin Zarembo

Organisers
Niklas Beisert
Marius de Leeuw
Giovanni Felder
Matthias Gaberdiel

Advisory Committee
Vladimir Kazakov
Gregory Korchemsky
Juan Maldacena
Matthias Staudacher
Arkady Tseytlin
Pedro Vieira

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Regge Limit of Gauge Theories & Gravity

... with a Pomeron exchange



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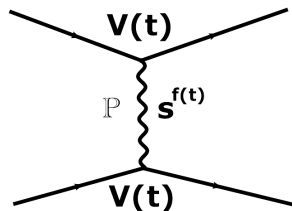
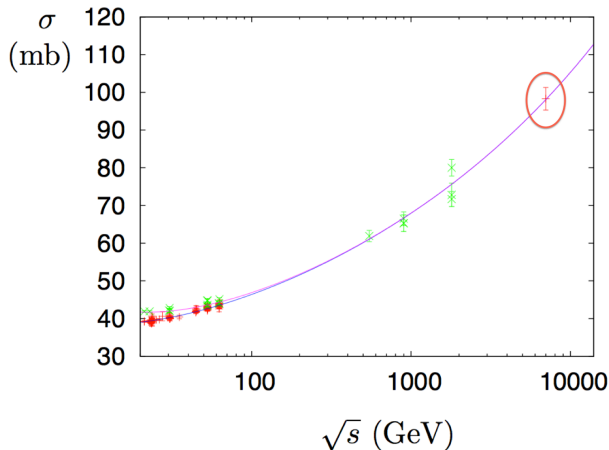
Regge Limit of Gauge Theories & Gravity

- 1 Brief Introduction
- 2 BFKL as a Matrix Random Walk
- 3 Five-point Amplitude in Einstein-Hilbert Gravity
- 4 N -SUGRA - Double Logs in Energy

1. Brief Introduction

Hadron-hadron total cross-sections rise @ $\sqrt{s} = 7$ TeV @ LHC:

$$\sigma^{\text{TOT}} = 98.3 \pm 0.2 (\text{stat}) \pm 2.8 (\text{syst}) \text{ mb}$$

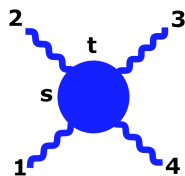


Consistent with Regge theory (Soft Pomeron Exchange): $\sigma^{\text{TOT}} \sim s^{0.1}$

1. Brief Introduction

Regge theory is pre-QCD

Microscopic picture of the Pomeron in terms of quarks & gluons?



Elastic amplitude with Mandelstam invariants s , t

Regge limit in perturbative QCD: $s \gg t$, Q^2

We need a hard scale $Q^2 \gg \Lambda_{\text{QCD}}^2$ to allow for a perturbative expansion $\alpha_s(Q^2) \ll 1$

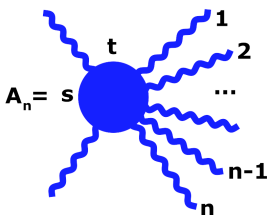
1. Brief Introduction

All-orders resummation of $\alpha_s(Q^2) \log\left(\frac{s}{Q^2}\right)$ terms: How?

Multi-Regge limit: $s \gg s_i \gg t_i \sim Q^2$ (Regge limit in all sub-channels)

Equivalent to strong ordering in rapidity of emitted particles:

$$Y \sim \log s \quad y_i \gg y_{i-1} \quad \mathbf{k}_i^2 \sim \mathbf{k}_{i-1}^2 \sim Q^2$$



Allows for resummation of longitudinal phase space:

$$\alpha_s^n(Q^2) \int_0^Y dy_1 \int_0^{y_1} dy_2 \dots \int_0^{y_{n-1}} dy_n \sim \frac{(\alpha_s(Q^2) Y)^n}{n!}$$

$$\sigma = \sum_n |A_n|^2$$

1. Brief Introduction

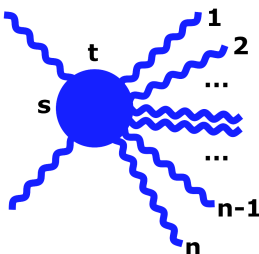
Regge limit in perturbative QCD: $s \gg t, Q^2$

Multi-Regge limit: Regge limit in all sub-channels [Only gluons]

Quasi-multi-Regge limit: one sub-channel with no limit [Quarks appear]

No ordering in rapidity in two nearby emitted particles:

$$Y \sim \log \frac{s}{s_0} \quad y_i \gg y_{i-1} \quad y_i \sim y_{i+1} \quad \mathbf{k}_i^2 \sim \mathbf{k}_{i-1}^2 \sim Q^2$$



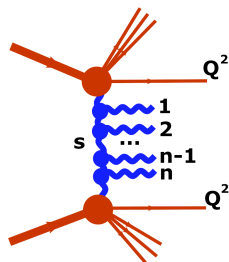
Fixes "factorization scale" s_0 ; $b \sim \log \frac{s_0'}{s_0}$:

$$\alpha_s^n(Q^2) \int_0^{Y'+b} dy_1 \int_0^{y_1} dy_2 \dots \int_0^{y_{n-1}} dy_n \\ \sim \underbrace{\frac{(\alpha_s(Q^2) Y')^n}{n!}}_{\text{LO}} + b \underbrace{\frac{\alpha_s^n(Q^2)}{(n-1)!} Y'^{n-1}}_{\text{NLO}} + \dots$$

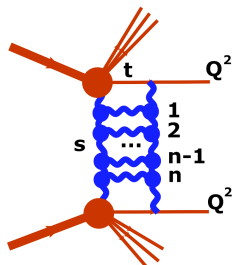
1. Brief Introduction

Rich Phenomenology:

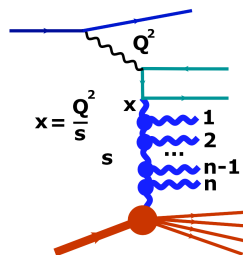
Multijets



Rapidity Gaps



Deep Inelastic Scattering



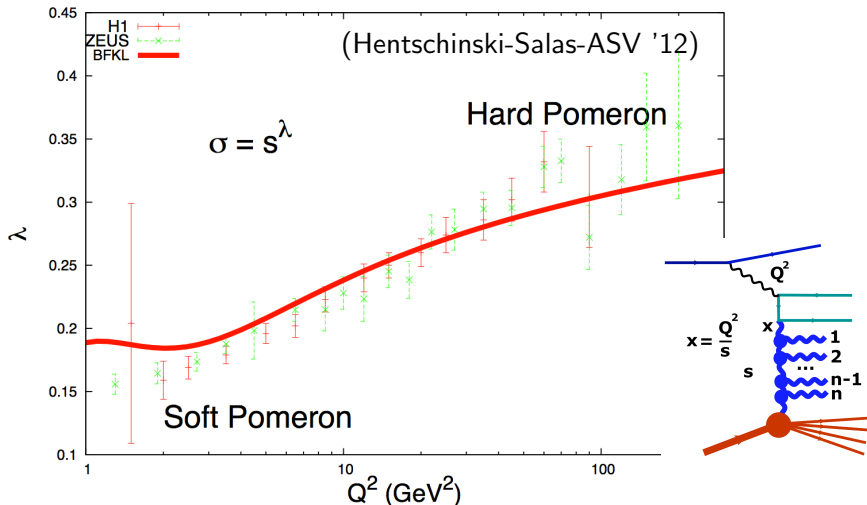
For these we obtain $\sigma \simeq s^{0.3}$

Hadron-Hadron (Soft Pomeron is Non-Perturbative): $\sigma^{\text{TOT}} \simeq s^{0.1}$

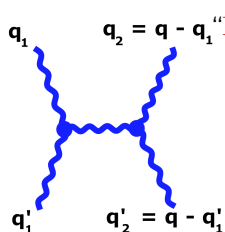
We have calculated a Perturbative or Hard (BFKL) Pomeron.

1. Brief Introduction

To fit data: BFKL+Collinear Resummation+Optimal Renormalization



1. Brief Introduction



“Reggeized Propagators” $\simeq g^2 N_c \delta^{(2)}(\vec{q}_1 - \vec{q}_1') \delta^{(2)}(\vec{q}_2 - \vec{q}_2')$

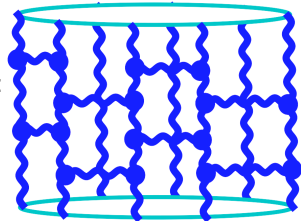
$$\times \left(\int d^2 \vec{r} \frac{\vec{q}_1^2}{\vec{r}^2 (\vec{q}_1 - \vec{r})^2} + \int d^2 \vec{r} \frac{\vec{q}_2^2}{\vec{r}^2 (\vec{q}_2 - \vec{r})^2} \right)$$

“Emission” $\simeq \delta^{(2)}(\vec{q}_1 + \vec{q}_2 - \vec{q}_1' - \vec{q}_2') \frac{g^2 N_c}{\vec{q}_1^2 \vec{q}_2^2}$

$$\times \left(\frac{\vec{q}_1^2 \vec{q}_2'^2 + \vec{q}_2^2 \vec{q}_1'^2}{\vec{k}^2} - (\vec{q}_1 + \vec{q}_2)^2 \right)$$

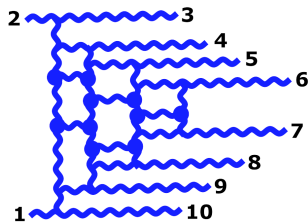
SL(2, \mathbb{C}) invariance (Lipatov)

Unitarization procedure: iterate the kernel (BKP):
 Mapping to Periodic XXX Heisenberg
 Spin Chain (Lipatov, Faddeev-Korchemsky)



1. Brief Introduction

Calculation of Infrared Finite Remainder Function in MHV, planar, $\mathcal{N} = 4$ SYM Scattering Amplitudes for arbitrary number of loops and legs in the Multi-Regge limit (Bartels-Lipatov-ASV '08, Fadin, Prygarin, Kormilitzin):



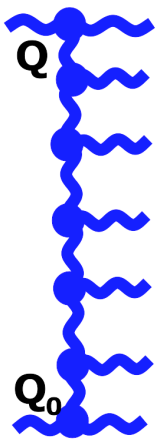
- A similar XXX Open Heisenberg Spin Chain appears (Lipatov)
- Progress MRK+Symbols (Dixon, Drummond, Duhr, Henn, Pennington)
- Strong coupling MRK amplitudes in (Bartels-Schomerus-Sprenger)

Regge Limit of Gauge Theories & Gravity

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- 2 BFKL as a matrix Random walk
with Romagnoni (Madrid)
arXiv:1111.4553
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2. BFKL as a matrix Random walk

Closer look at the ladder: **Forward case:** $\vec{q} = 0$



Y Integrate over azimuthal angle, BFKL equation:

$$\frac{\partial \varphi(Q^2, Y)}{\alpha \partial Y} = \int_0^\infty \frac{dq^2}{|q^2 - Q^2|} \left\{ \varphi(q^2, Y) - \frac{2 \min(q^2, Q^2)}{q^2 + Q^2} \varphi(Q^2, Y) \right\}$$

with initial condition $\varphi(Q^2, Y = 0) = \delta(Q^2 - Q_0^2)$

Discretize virtuality space:

$$Q^2 = N \Delta, \quad q^2 = n \Delta, \quad \phi_n \equiv \varphi(l^2, Y)$$

Unbounded Virtualities

$$\frac{\partial \phi_N}{\alpha \partial Y} = \underbrace{\sum_{n=1}^{N-1} \frac{\phi_n}{N-n}}_{\text{Gluon Emission}} + \underbrace{\sum_{n=N+1}^{\infty} \frac{\phi_n}{n-N}}_{\text{Unbounded Virtualities}} - \underbrace{2h(N-1)\phi_N}_{\text{No Emission}}$$

$$h(N) = \sum_{l=1}^N \frac{1}{l} = \psi(N+1) - \psi(1)$$

2. BFKL as a matrix Random walk

(Minahan/Zarembo)

Dilatation operator of planar $\mathcal{N} = 4$ SYM \leftrightarrow XXX spin chain Hamiltonian

(Beisert)

Hamiltonian for 1-loop AD of spin $S - 1$ operators of $\mathfrak{sl}(2)$ closed sector:

$$\mathcal{H}_{1,2}^{\mathfrak{sl}(2)}(a_1^\dagger)^{N-1}(a_2^\dagger)^{S-N}|00\rangle = -\lambda \sum_{l=1}^S \left(\frac{(1 - \delta_l^N)}{|l - N|} - (h(N - 1) + h(S - N)) \delta_l^N \right) (a_1^\dagger)^{l-1} (a_2^\dagger)^{S-l} |00\rangle$$

$$\lambda = \frac{g^2 N_c}{8\pi^2}$$

$(a^\dagger)^n |0\rangle = \frac{1}{n!} (\mathcal{D})^n \Phi \rightarrow$ 1 site in a 1-dim lattice \rightarrow n -th excited state
Excitations classified in $s = -\frac{1}{2}$ representation of $\mathfrak{sl}(2)$

2. BFKL as a matrix Random walk

XXX $_{-\frac{1}{2}}$ Hamiltonian for $S = 2N - 1$ acts on two $(N - 1)$ -th excited states:

$$\mathcal{H}_{1,2}^{\text{sl}(2)} (a_1^+)^{N-1} (a_2^+)^{N-1} |00\rangle =$$

Bounded Excitations

$$-\lambda \sum_{l=1}^{\overbrace{2N-1}} \left(\frac{(1 - \delta_l^N)}{|l - N|} - 2h(N - 1)\delta_l^N \right) (a_1^+)^{l-1} (a_2^+)^{2N-1-l} |00\rangle$$

BFKL equation can be written as

Unbounded Virtualities

$$\mathcal{H}^{\text{BFKL}} \phi_N = \alpha \sum_{l=1}^{\overbrace{\infty}} \left(\frac{(1 - \delta_l^N)}{|l - N|} - 2h(N - 1)\delta_l^N \right) \phi_l$$

What is the link between these two systems?

2. BFKL as a matrix Random walk

$$\alpha = -\lambda \quad \vec{\phi} \equiv (\phi_1, \dots, \phi_N)$$

$$\hat{\mathcal{H}}_N^{\text{sl}(2)} = \begin{pmatrix} -2h(0) & 1 & \frac{1}{2} & \frac{1}{3} & \cdots & \frac{1}{N-1} \\ 1 & -2h(1) & 1 & \frac{1}{2} & \cdots & \frac{1}{N-2} \\ \frac{1}{2} & 1 & -2h(2) & 1 & \cdots & \frac{1}{N-3} \\ \frac{1}{3} & \frac{1}{2} & 1 & -2h(3) & \cdots & \frac{1}{N-4} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{1}{N-1} & \frac{1}{N-2} & \frac{1}{N-3} & \frac{1}{N-4} & \cdots & -2h(N-1) \end{pmatrix}$$

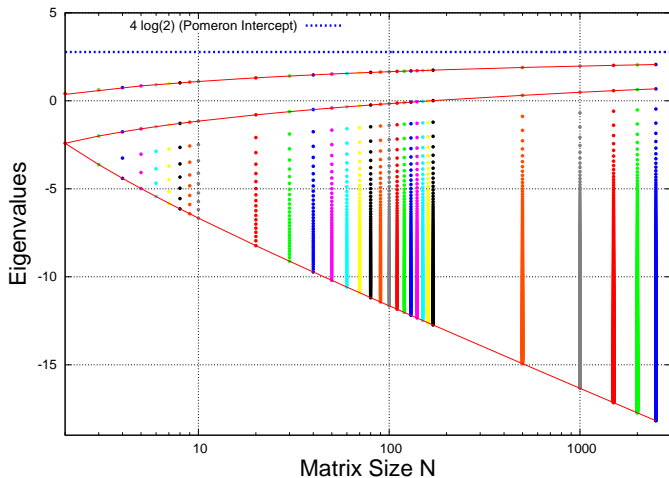
Construct "BFKL-like Equation" for the $\text{sl}(2)$ chain:

$$\frac{\partial \vec{\phi}}{\alpha \partial Y} = \hat{\mathcal{H}}_N^{\text{sl}(2)} \cdot \vec{\phi} \quad \rightarrow \quad \vec{\phi} = e^{\alpha Y \hat{\mathcal{H}}_N^{\text{sl}(2)}} \cdot \vec{\phi}_0$$

with Initial Condition $\vec{\phi}_0$

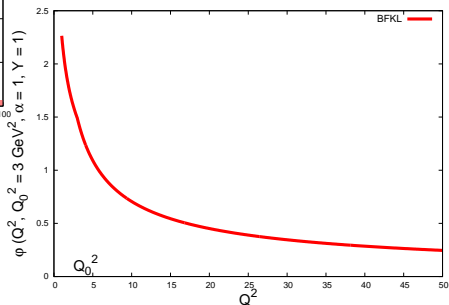
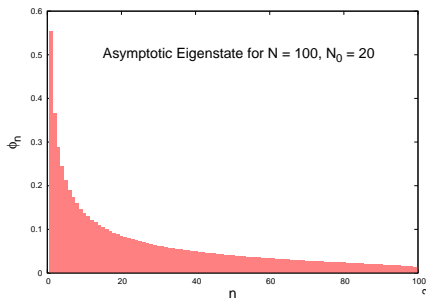
2. BFKL as a matrix Random walk

$$\vec{\phi} = e^{\alpha Y \hat{\mathcal{H}}_N^{\text{sl}(2)}} \cdot \vec{\phi}_0 = \sum_{L=1}^N c_L^{(N)} e^{\alpha Y \lambda_L^{(N)}} \psi_L^{(N)} \xrightarrow{\alpha Y \rightarrow \infty} c_{\text{as}}^{(N)} e^{\alpha Y \lambda_{\text{as}}^{(N)}} \psi_{\text{as}}^{(N)}$$



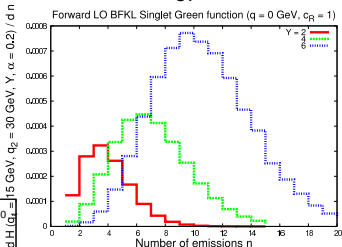
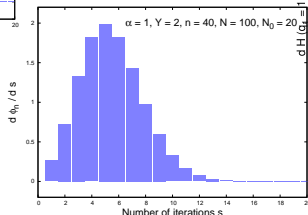
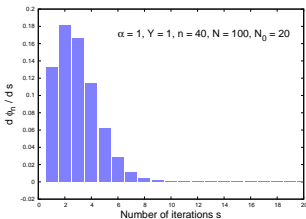
2. BFKL as a matrix Random walk

The Same "Virtuality" Distribution: $\vec{\phi} \equiv (\phi_1, \dots, \phi_N)$

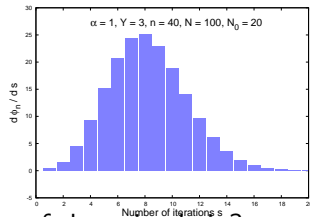


2. BFKL as a matrix Random walk

The Same "Multiplicity" Distribution: $\vec{\phi}_n \rightarrow c_{as}^{(N)} \sum_{s=0}^{\infty} \frac{(\alpha Y \lambda_{as}^{(N)})^s}{s!} \psi_{as,n}^{(N)}$



(Chachamis, ASV)



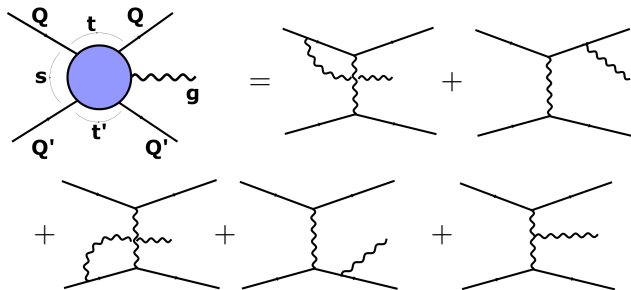
- Is this connected to the coherent state pictures of the spin chain?

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with Serna Campillo & Vazquez-Mozo (Salamanca)
JHEP 03 (2012) 005
- 4 *N*-SUGRA - Double Logs in Energy

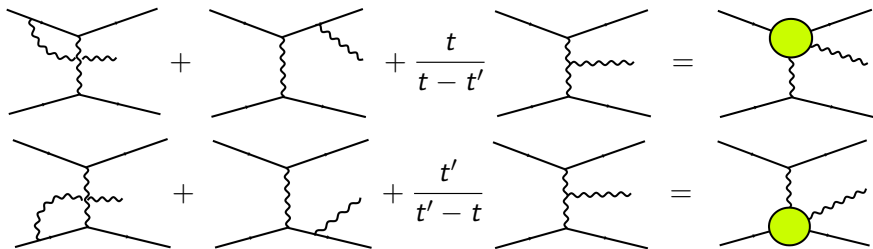
3. Five-point Amplitude in Einstein-Hilbert gravity

A Simple Review Calculation in QCD:

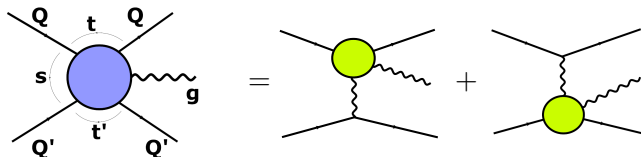


3. Five-point Amplitude in Einstein-Hilbert gravity

Nice Trick:



Exact Amplitude is the Sum of Two Gauge Invariant Effective Vertices:



3. Five-point Amplitude in Einstein-Hilbert gravity

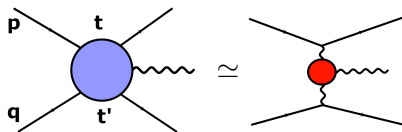
$$t = k_1^2, t' = k_2^2$$

Sudakov expansion

$$k_1 = \alpha_1 p + \beta_1 q + k_1^\perp \quad k_2 = \alpha_2 p + \beta_2 q + k_2^\perp.$$

Multi-Regge kinematics (MRK):

$$1 \gg |\alpha_1| \gg |\alpha_2| = -\frac{t'}{s} \quad 1 \gg |\beta_2| \gg |\beta_1| = -\frac{t}{s}$$

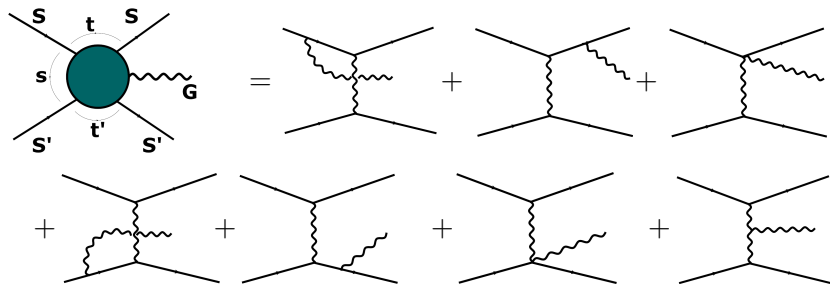


Universal Reggeized g - Reggeized g - g Effective Vertex (Lipatov) in MRK:

$$= ig\eta_{\mu\nu} \left\{ \left(\alpha_1 - \frac{2t}{s\beta_2} \right) p^\nu + \left(\beta_2 - \frac{2t'}{s\alpha_1} \right) q^\nu - \left(k_1^\perp + k_2^\perp \right)^2 \right\}$$

3. Five-point Amplitude in Einstein-Hilbert gravity

The Closest Calculation in Einstein-Hilbert Gravity:



3. Five-point Amplitude in Einstein-Hilbert gravity

Nice Trick is Still Nice but More Tricky:

$$\begin{aligned}
 & \text{Diagram 1} + \text{Diagram 2} + \frac{t}{t-t'} \left\{ \text{Diagram 3} + \text{Diagram 4} \right\} = \text{Diagram 5} \\
 & \text{Diagram 6} + \text{Diagram 7} + \frac{t'}{t'-t} \left\{ \text{Diagram 8} + \text{Diagram 9} \right\} = \text{Diagram 10} \\
 & \frac{t'}{t'-t} \text{Diagram 11} + \frac{t}{t-t'} \text{Diagram 12} = 0
 \end{aligned}$$

The diagrams represent Feynman diagrams for a five-point amplitude in Einstein-Hilbert gravity. Diagrams 1-4 and 6-9 show various tree-level topologies with a wavy internal line. Diagrams 5 and 10 are the sum of these terms, featuring a central orange circle. Diagrams 11 and 12 are the difference terms that cancel out to zero.

Exact Amplitude is the Sum of Two Gauge Invariant Sub-Amplitudes:

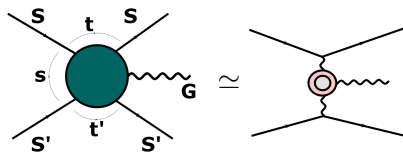
$$\text{Diagram 13} = \text{Diagram 14} + \text{Diagram 15}$$

Diagram 13 is a five-point amplitude with a central teal circle and external legs labeled s, t, s', t', s, t . Diagrams 14 and 15 are the two gauge-invariant sub-amplitudes shown in the previous slide, each with a central orange circle.

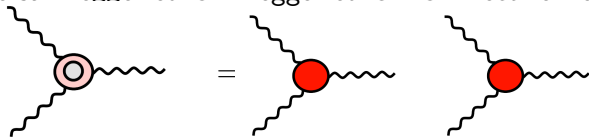
These Sub-Amplitudes are Very Useful in Loop Calculations

3. Five-point Amplitude in Einstein-Hilbert gravity

Using the same Sudakov expansion and Multi-Regge kinematics:



Universal Reggeized G - Reggeized G - G Effective Vertex (Lipatov):



$$+ 4\beta_1\alpha_2 \left\{ \frac{p^\mu p^\nu}{\beta_2^2} + \frac{q^\mu q^\nu}{\alpha_1^2} + \frac{p^\mu q^\nu + q^\mu p^\nu}{\alpha_1\beta_2} \right\}$$

Subtraction Term to Fulfill Steinman Relations (no simultaneous singularities in overlapping channels).

- Is integrability present in gravity?

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with Bartels (Hamburg) & Lipatov (St Petersburg)
arXiv:1208.3423

4. N -SUGRA - Double Logs in Energy

Four-graviton scattering in N -SUGRA

$$\mathcal{A}_{4,(N)} = \mathcal{A}_4^{\text{Born}} \mathcal{M}_{4,(N)}$$

At 1-loop there are three contributions ($\alpha = \text{Newton's constant}/\pi$):

$$\begin{aligned} \mathcal{M}_{4,(N=8)}^{(1)} &= \underbrace{\alpha t \ln\left(\frac{-s}{-t}\right) \ln\left(\frac{-u}{-t}\right)}_{\text{Double Logs}} \\ &+ \underbrace{\alpha \frac{t}{2} \ln\left(\frac{-t}{\lambda^2}\right) \left(\ln\left(\frac{-s}{-t}\right) + \ln\left(\frac{-u}{-t}\right) \right)}_{\text{Trajectory}} \\ &- \underbrace{\alpha \frac{(s-u)}{2} \ln\left(\frac{-t}{\lambda^2}\right) \ln\left(\frac{-s}{-u}\right)}_{\text{Eikonal}} \end{aligned}$$

4. N -SUGRA - Double Logs in Energy

In the Regge limit $u \simeq -s$

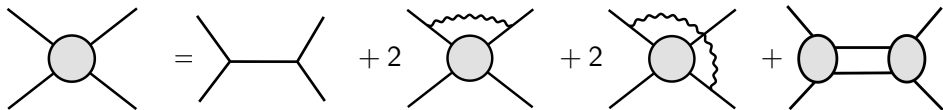
$$\begin{aligned}\mathcal{M}_{4,(N=8)}^{(1)} &\simeq \underbrace{(\alpha t) \ln^2 \left(\frac{s}{-t} \right)}_{\text{Double Logs}} \\ &+ \underbrace{(\alpha t) \ln \left(\frac{-t}{\lambda^2} \right) \ln \left(\frac{s}{-t} \right)}_{\text{Trajectory}} \\ &+ \underbrace{i \pi (\alpha s) \ln \left(\frac{-t}{\lambda^2} \right)}_{\text{Eikonal}}\end{aligned}$$

Can we resum the Double Log contributions to all-orders? Let us use

$$\mathcal{A}_{4,(N)} = \mathcal{A}_4^{\text{Born}} \left(\frac{s}{-t} \right)^{\alpha t \ln \left(\frac{-t}{\lambda^2} \right)} \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{-t} \right)^\omega \frac{f_\omega^{(N)}}{\omega}$$

4. N -SUGRA - Double Logs in Energy

Double Logs are dominated by the contributions



with

- (1) softest virtual gravitons
- (2) pair of softest gravitons/gravitinos in t -channel (Ladder)

The corresponding equation is

$$f_{\omega}^{(N)} = 1 - (\alpha t) \frac{d}{d\omega} \left(\frac{f_{\omega}^{(N)}}{\omega} \right) + (\alpha t) \left(\frac{N-6}{2} \right) \left(\frac{f_{\omega}^{(N)}}{\omega} \right)^2$$

With perturbative solution:

$$f_{\omega}^{(N)} = 1 + (\alpha t) \frac{(N-4)}{2\omega^2} + (\alpha t)^2 \frac{(N-4)(N-3)}{2\omega^4} - (\alpha t)^3 \frac{(N-4)(5N^2 - 26N + 36)}{8\omega^6} + \dots$$

4. N -SUGRA - Double Logs in Energy

Agreement with exact 2-loop results for $N = 4, \dots, 8$ SUGRA obtained with the conjecture that gravity is a double copy of two gauge theories (Boucher-Veronneau, Dixon, Bern, Carrasco, Johansson, Dunbar, Norridge).

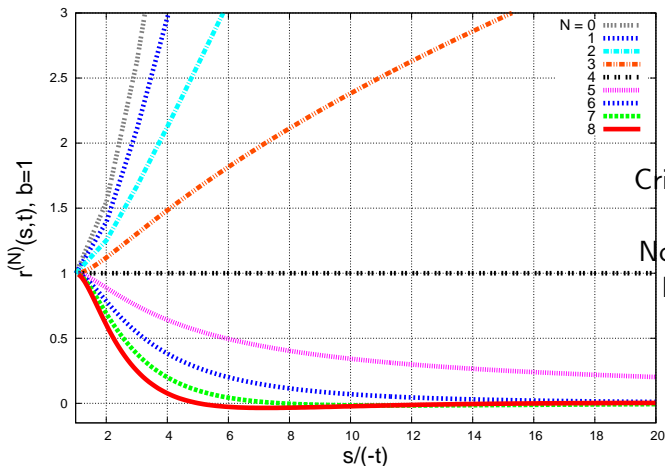
We make **All-Orders** predictions, e.g. $N = 8$:

$$\begin{aligned} \mathcal{A}_{4,(N=8)} &= \mathcal{A}_4^{\text{Born}} \left(\frac{-t}{\lambda^2} \right)^{\alpha t (\ln(\frac{s}{-t}) + i\pi(\frac{s}{t}))} \\ &\times \left\{ 1 + 2 \left(\frac{\alpha t}{2} \right) \ln^2 \left(\frac{s}{-t} \right) + \frac{5}{3} \left(\frac{\alpha t}{2} \right)^2 \ln^4 \left(\frac{s}{-t} \right) \right. \\ &\quad + \frac{37}{45} \left(\frac{\alpha t}{2} \right)^3 \ln^6 \left(\frac{s}{-t} \right) + \frac{353}{1260} \left(\frac{\alpha t}{2} \right)^4 \ln^8 \left(\frac{s}{-t} \right) \\ &\quad \left. + \frac{583}{8100} \left(\frac{\alpha t}{2} \right)^5 \ln^{10} \left(\frac{s}{-t} \right) + \dots \right\} \end{aligned}$$

4. N -SUGRA - Double Logs in Energy

All-orders Resummation:

Solution of a linear Schrödinger eqn as a Parabolic Cylinder function.



$N = 4$ SUGRA:
Critical theory separating
Convergent ($N > 4$)
Non-convergent ($N < 4$)
High Energy behaviour

- Is there a simple representation to all-orders of the full amplitude?

Regge Limit of Gauge Theories & Gravity

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... still many things to be learnt from this limit ...