Plan
- linear algebra
- Integrability in AdS_3/CFT_2

CAVEAT: Missing massless modes
- spin chains from BA
- Reducible spin chains
- Degenerate magnons & mmm

Introduction

- AdS_3/CFT_2 correspondence w/ 16 susys comes in 2 varieties
  $$Sl(2) + 16\,\text{susy} \longrightarrow psu(1,1|2) \quad \text{OR} \quad d(2,1;\alpha)$$

- On AdS side ∃ two sugra solns
  $$AdS_3 \times S^3 \times T^4 \quad \text{OR} \quad AdS_3 \times S^3 \times S^3 \times S^1$$
  I will call these $\underline{T^4}$ OR $\underline{S^1}$ in talk

- For T^4 we know CFT_2 it is (susy) $Sym^N(T^4)$
  Physically this is ∵ $AdS_3 \times S^3 \times T^4$ is near horizon limit of D1+D5 system. the gauge th there is SYM in $d=1+1$ this theory is not conformal but flows to (susy) $Sym^N(T^4)$

- For S^1 we do not know the CFT_2 !

- Integrability potentially v useful in T^4 model
  ∵ it allows us to compute directly in RR flux bkd
  we do not need to do S-duality

- But we do not know how to get spin chain directly from $Sym^N(T^4)$
- $Sym^N(T^4)$ is v interesting theory
  - black holes
  - instanton moduli space

Linear algebra

$d(2,1;\alpha)$ <u>bosons</u> $sl(2)\times SU(2)\times SU(2)$

<u>fermions</u> $Q_{\beta\beta'\beta''}$ trispinors

$$\{Q_{\beta\beta'\beta''}, Q_{\gamma\gamma'\gamma''}\} = (\varepsilon\sigma^i)_{\beta\gamma}\,\varepsilon_{\beta'\gamma'}\varepsilon_{\beta''\gamma''}\,J^i + \sin^2\phi\,\varepsilon_{\beta\gamma}(\varepsilon\sigma^{i'})_{\beta'\gamma'}\varepsilon_{\beta''\gamma''}\,J^{i'} + \cos^2\phi\,\varepsilon_{\beta\gamma}\varepsilon_{\beta'\gamma'}(\varepsilon\sigma^{i''})_{\beta''\gamma''}\,J^{i''}$$

$\boxed{\alpha \equiv \sin^2\phi}$ Special values $\alpha = 0, \pi/2$ $psu(1,1|2)$

$\alpha = \pi/4$ $osp(4|2) \sim d(2,1)$

Short representations & subalgebras

$d(2,1;\alpha)$

$$\begin{array}{ll} |\phi^{(0)}_{\beta'}\rangle & \\ & |\psi^{(0)}_{\beta''}\rangle \\ |\phi^{(1)}_{\beta'}\rangle & \\ & |\psi^{(1)}_{\beta''}\rangle \\ |\phi^{(2)}_{\beta'}\rangle & \\ & |\psi^{(2)}_{\beta''}\rangle \end{array}$$

Q, J^i

$sl(2|1)$

$$\begin{array}{ll} |\phi^{(0)}\rangle & \\ & |\psi^{(0)}\rangle \\ |\phi^{(1)}\rangle & \\ & |\psi^{(1)}\rangle \\ |\phi^{(2)}\rangle & \\ & |\psi^{(2)}\rangle \end{array}$$

Q, J^i

$sl(2)$

$$\begin{array}{l} |\phi^{(0)}\rangle \\ |\phi^{(1)}\rangle \\ |\phi^{(2)}\rangle \end{array}$$

J^i

weights $(-\frac{\alpha}{2}; \frac{1}{2}; 0)$ $(-\alpha/2; \alpha/2)$ $-\alpha/2$

- at $\alpha = 1$ this becomes a $(-1/2; 1/2)$ rep of $psu(1,1|2)$
- at $\alpha = 0$ this representation is <u>not irreducible</u>

Overview of AdS_3/CFT_2 integrability

$$AdS_3 \times S^3 \times S^3 \times S^1 \sim \frac{SO(2,2)}{SO(1,2)} \times \left(\frac{SO(4)}{SO(3)}\right)^2 \times U(1)$$

Supergeometry
in suitable κ gauge

$$\frac{d(2,1;\alpha)^2}{SO(1,2) \times SO(3)^2} \times U(1)$$

$$\frac{1}{R^2_{AdS_3}} = \frac{1}{R^2_{S^3_1}} + \frac{1}{R^2_{S^3_2}} \qquad \sin^2\phi \equiv \frac{R_{AdS_3}{}^2}{R^2_{S^3_1}} \equiv \alpha$$

Similarly

$$AdS_3 \times S^3 \times T^4 \sim \frac{SO(2,2)}{SO(1,2)} \times \frac{SO(4)}{SO(3)} \times T^4$$

Sugeometry
in suitable κ gauge

$$\frac{PSU(1,1|2)^2}{SO(1,2) \times SO(3)} \times T^4$$

$$\frac{1}{R^2_{AdS_3}} = \frac{1}{R^2_{S^3}}$$

- Both cosets have $\mathbb{Z}_4$ automorphism
- GS action in suitable κ gauge is just MT coset action w/ extra S^1 or T^4
- $\mathbb{Z}_4$ structure gives Lax connections ⟺ EoMs
- From this write finite gap equations
- Obtain all-loop BAs which in continuum limit becomes f.g.e.
- Extract from these weak coupling BAs
- Construct integrable spin chains solved by the w.c. BA

Missing massless modes

- Plane wave limit

$\underline{S^1}$ 20 susys

spectrum	2	+ 2	+ 2	+ 2 (b/f)
m^2	1	α	$1-\alpha$	0

$\underline{T^4}$ 24 susys

spectrum	4	+ 4 (b/f)
m^2	1	0

- number of massless modes jumps for $\alpha = 0, 1$

- Finite gap equations do not see the massless states!

- So spin chains do not see the massless modes!

For example a magnon state schematically

$$|m\,mag(p)\rangle = \sum_k e^{ipK}\, \psi^\dagger_{(x)}\, |vac\rangle$$

where $\psi^+ |\phi^{(0)}\rangle \equiv |\psi^{(0)}\rangle$ has energy $\sqrt{\alpha + h^2(\alpha)\sin^2 p/2}$

can be constructed in spinchain for $\alpha \neq 0$

but is not there for naive $\alpha = 0$ spinchain

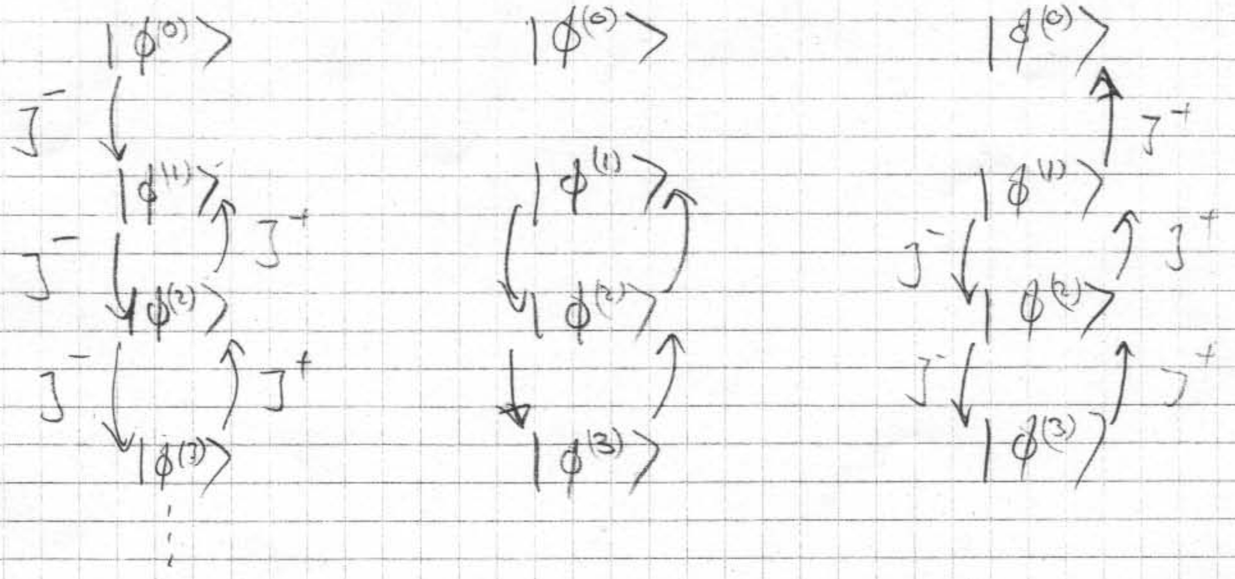
Naive spin chains

- We focus on left moving sector
- T^4 model homogeneous spinchain in $\left(-\frac{1}{2}; \frac{1}{2}\right)$ rep
- S^1 model alternating spinchain $\left(-\frac{\alpha}{2}; \frac{1}{2}; 0\right) \times \left(-\frac{1-\alpha}{2}; 0; \frac{1}{2}\right)$
- Both models have an R-matrix
- In the $sl(2|1)$ subsector we have an explicit Yangian derivation of this R-matrix based on Drinfeld's second realisation
- Observation $\alpha \to 0$ R-matrix nonsingular but "bigger" than T^4 R-matrix
- So integrability survives $\alpha \to 0$ limit & gives bigger spinchain than hom T^4 s.c.

Reducible alternating spinchains

- $\alpha \to 0$ representation theory

$sl(2)$



Verma module $\qquad$ $0 \oplus -\frac{1}{2}$ $\qquad$ "dual" Verma module

- We have a Yangian and R-matrix for all these representations
- The Vm and dVm are non-unitary reps
- the monodromy matrix for Vm and dVm has Jordan blocks (the R matrix does not)

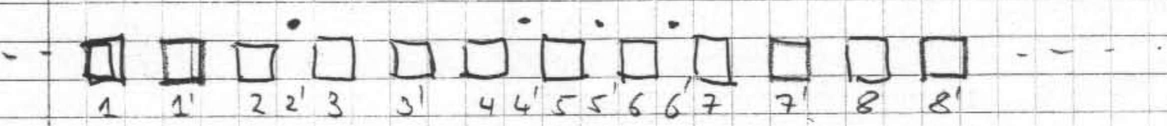
- Conclusion: $\alpha \to 0$ spinchain is a $PSU(1,1|2)$ <u>reducible alternating chain</u> $\quad (-\tfrac{1}{2}, \tfrac{1}{2}) \times \left[0^{\oplus 2} \oplus (-\tfrac{1}{2}, \tfrac{1}{2})\right]$

Locality in the reducible alternating chain

- Since we know the R matrix, we can construct the monodromy and transfer matrices.

- It turns out that these depend only on the states in the $(-\frac{1}{2},\frac{1}{2})$ irreps and ignore all the singlets

- So a spinchain state looks like



□ "bosonic" $(-\frac{1}{2},\frac{1}{2})$

□ "fermionic" $(-\frac{1}{2},\frac{1}{2})$

• $0 \oplus 0$

Transfer matrix (& Hamiltonian) is LOCAL wrt

$(\ldots 1, 1' 2 3 3' 4 5 6 7 7' 8 8' \ldots)$

and apart from $\overset{b/f}{\text{statistics}}$ takes form of conventional $psu(1,1|2)$ homogeneous $(-\frac{1}{2},\frac{1}{2})$ chain

Degenerate Magnons

- the red. alt chain has <u>many</u> groundstates

For example

$$|m\,\text{mag}(p)\rangle_{\alpha\to 0}$$

is a groundstate (for all p) since $|\phi^0\rangle$ is singlet "vacuum"

$|\psi^0\rangle$ is $(-\frac{1}{2},\frac{1}{2})$ "vacuum"

- But this 1-loop reasoning is incomplete

Recall this state's Energy is

$$\sqrt{\alpha + 4h^2\sin^2 p/2}$$

So at $\alpha \to 0$ it is

$$2\,|h \sin p/2|$$

- So only the $p=0$ state is a groundstate
- It is the extra missing massless state!
- All other degenerate magnons have their energy lifted