Deutsches Elektronen-Synchrotron
in der Helmholtz-Gemeinschaft

DESY

From Symplectic Fermions to Sigma Models on $CP^{N-1|N}$

ETH, May 2009

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based on work w. C. Candu, C.Creutzig, V. Mitev, T Quella, H. Saleur; 2 papers *in preparation*

Superspace Sigma Models

Aim: Study non-linear sigma models with target space supersymmetry **not world-sheet**

Strings in AdS backgrounds [pure spinor]

c = 0 CFTs; symmetry e.g. PSU(2,2|4) ...

Focus on scale invariant QFT, i.e. 2D CFT

Properties: Weird: logarithmic Conformal Field Theories!

Remarkable: Many families with cont. varying exponents

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Focus on scale invariant QFT, i.e. 2D CFT

Properties: Remarkable: logarithmic CFTs! ..[VS,Saleur]

Weird: Many families w continuously varying exponents

Examples: Super-Cosets

[Candu] [thesis]

Families$_v$ w. compact form, w.o. H-flux:

volume ↗ cpct symmetric superspaces

G/G^{Z_2}

$$\frac{OSP(2S+2|2S)}{OSP(2S+1|2S)} \rightarrow S^{2S+1|2S}$$

c = 1

$$\frac{OSP(2S+2|2S)}{OSP(2S+2-n|2S) \times SO(n)}$$

$$\frac{U(N|N)}{U(N-1|N) \times U(1)} \rightarrow CP^{N-1|N}$$

c = -2

$$\frac{U(N|N)}{U(N-n|N) \times U(n)}$$

⋮

- note: $c^v\,(GL(N|N)) = 0 = c^v\,(OSP(2S+2|2S))$

From AdS/CFT to Ghost

Dual of weakly coupled N=4 SYM theory ?

Candidate: Chiral field on $CP^{3|4}$ [Witten 04]

- **Has superconformal symmetry PSU(2,2|4)**
- **Has compact base ↔ strongly curved AdS**

Symplectic fermions are simplest member

$$CP^{0|1} = U(1|1)/U(1) \times U(1)$$

→ Study $CP^{N-1|N}$ as generalization of SF

formulas mostly N=2

Plan of Talk

M
O
D
E
L
S

- Symplectic Fermions – a Brief Review
- The Chiral Field on Superspace $CP^{1|2}$

  Bulk and twisted Neumann boundary conditions

M
E
T
H
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S

- Properties of the Chiral Field on $CP^{1|2}$

  Exact boundary partition functions

  Quasi-abelian evolution; Lattice models;

  Symplectic fermions as cohomology

- Conclusion & Some Open Problems

II Symplectic Fermions: The Bulk

$$S \sim \int d^2 z \, (\partial \eta^+ \bar{\partial} \eta^- + \partial \eta^- \bar{\partial} \eta^+)$$

Only global u(1) sym. is manifest

Has affine psu(1|1) sym: $F^{\pm}(z) = \partial \eta^{\pm}(z, \bar{z})$

$$[\, F_n^+ \, , \, F_m^- \,]_+ = n \, \delta_{n+m,0}$$

Action rewritten in terms of currents: $\begin{pmatrix} & F^+ \\ F^- & \end{pmatrix}$

$$S \sim \int d^2 z \, (F^+ \bar{F}^- + F^- \bar{F}^+)$$

U(1) x U(1)

Possible modification: θ-angle

trivial in bulk

$$S \sim \int d^2 z \, ((1+\theta) F^+ \bar{F}^- + (1-\theta) F^- \bar{F}^+)$$

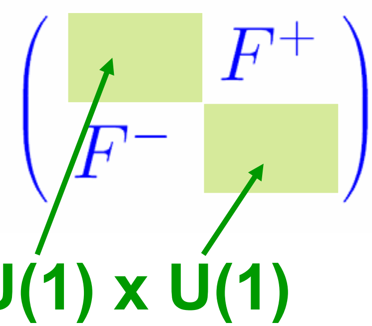
II Symplectic Fermions: The Bulk

$$S \sim \int d^2 z \, (\partial\eta^+ \bar\partial\eta^- + \partial\eta^- \bar\partial\eta^+) + \theta \, (\partial\eta^+ \bar\partial\eta^- - \partial\eta^- \bar\partial\eta^+)$$

Has affine psu(1|1) sym: $F^\pm(z) = \partial\eta^\pm(z, \bar z)$

$$[\, F_n^+ \, , \, F_m^- \,]_+ = n \, \delta_{n+m,0}$$

Action rewritten in terms of currents:



$$S \sim \int d^2 z \, (F^+ \bar F^- + F^- \bar F^+)$$

Possible modification: θ-angle **trivial in bulk**

$$S \sim \int d^2 z \, ((1+\theta) F^+ \bar F^- + (1-\theta) F^- \bar F^+)$$

II Symplectic Fermions: Boundary

$$S \sim S_{\text{bulk}} + \vartheta \int dx \eta^{+} \partial_x \eta^{-}$$ ← **boundary term**

Implies twisted Neumann boundary conditions:

$$\partial_y \eta^{\pm} = \pm \Theta \partial_x \eta^{\pm} \qquad \Theta \sim \theta + \vartheta$$

with currents: $\begin{pmatrix} F^{+} \\ F^{-} \end{pmatrix} = \underbrace{\frac{1+B}{1-B}}_{A} \begin{pmatrix} \bar{F}^{+} \\ \bar{F}^{-} \end{pmatrix}$ $\quad B = \begin{pmatrix} 0 & \Theta \\ -\Theta & 0 \end{pmatrix}$

Results: Θ x Θ 4 ground states **In P_0 of u(1|1)**

Θ_1 x Θ_2 Ground states: $\lambda(\Theta_1,\Theta_2) = \lambda(\mathrm{tr}(A_1 A_2^{-1}))$

↖ **twist fields**

[Creutzig,Quella,VS] [Creutzig,Roenne]

II The Chiral Field on $CP^{N-1|N}$

$$CP^{N-1|N} = \left\{(Z_\alpha) = (z_\alpha, \eta_\alpha) | \varrho^2 = \bar{Z}_\alpha Z^\alpha = 1\right\} / U(1)$$

z_α , η_α →
ωz_α , $\omega \eta_\alpha$ = $U(N|N)/U(N-1|N) \times U(1)$

→ 2 parameter family of 2D CFTs;c = -2

$$\mathcal{S}_{\rm CP} = \frac{R^2}{2\pi}\int_{\varrho^2=1} d^2z \left(D\bar{Z}_\alpha \bar{D} Z^\alpha + D Z_\alpha \bar{D}\bar{Z}^\alpha\right)$$

$$+\frac{i\theta}{2\pi}\int d^2z \left(D\bar{a} - \bar{D}a\right)$$

a - non-dynamical gauge field

D = ∂ - ia

θ term non-trivial; θ=θ+2π

Non-abelian extension of symplectic fermion

II Boundary Conditions for $CP^{N-1|N}$

Boundary condition in σ-Model on target X is

hypersurface Y + bundle with connection A

Dirichlet BC ⊥ to Y; Neumann BC || to Y; $\int_{\partial\Sigma} A$

U(N|N) symmetric boundary cond. for $CP^{N-1|N}$

line bundles on $Y = CP^{N-1|N}$ with monopole A_μ

Spectrum of sections e.g. N=2; μ = 0 **μ integer**

~ (1+14+1) + 48 + 80 + ... + (2n+1) x 16 +

atypicals **typicals of U(2|2)** **superspherical harmonics**

III Boundary conditions & Spectra

Count boundary condition changing operators

at R = ∞ (free field theory)   **Built from $Z_\alpha, \bar{Z}_\alpha, \partial_x Z_\alpha$, ...**

monopole numbers   **Euler fct**   **fermionic contr.**

$$Z^{\mathrm{CP};\nu\mu}_{R=\infty} = q^{\frac{1}{12}}\phi(q)\oint\frac{du}{u^{\mu-\nu}}\phi(q)\lim_{t\to 1}(1-t^2)\prod_{n=0}^{\infty}\prod_{\alpha\beta=\pm\frac{1}{2}}\frac{1+y^\alpha(zu^{-1})^\beta q^n}{1-x^\alpha(zu)^\beta q^n}$$

U(1) gauging   **constraint**   **bosonic contr.**

$$= \sum_\Lambda \psi^{\mathrm{CP}}_{\Lambda;\nu\mu}(q)\,\chi_\Lambda(x,y)$$

← **u(2|2) characters**

~ $\chi_{(1+14+1)} + \chi_{48} + \chi_{80} + \ldots q\,\chi_{ad} + \ldots$ **for N=2 and μ = ν**

III Result: Spectrum at finite R

Branching fcts from R = ∞

universal indep of Λ

Characters of u(2|2) reps

$$Z_R^{\mathrm{CP};\nu\mu} = q^{\frac{1}{2}\lambda(\lambda-1)} \sum_\Lambda \psi^{\mathrm{CP}}_{\Lambda;\nu\mu}(q)\, q^{f_{\nu\mu}(R,\theta) C^{(2)}_\Lambda}\, \chi_\Lambda(x,y)$$

$\lambda = \lambda(\Theta_1,\Theta_2)$ from symplectic Fermions

Casimir of U(2|2)

$\Theta = \theta + 2\pi\mu$ real

Quasi-abelian Evolution of Weights

Free Boson:

$$\Delta^R_\Phi = \Delta^0_\Phi + f(R) g^2_\Phi$$

at $R=R_0$ — universal — U(1) charge

In boundary theory

bulk more involved

<u>Prop.:</u> Boundary spectra of $CP^{1|2}$ chiral field :

$$\Delta^R_\Phi = \Delta^0_\Phi + f(R) C^{(2)}_\Phi$$

← **quadratic Casimir**

Deformation of conf. weights is `quasi-abelian'

[Bershadsky et al] [Quella,VS,Creutzig] [Candu, Saleur]

e.g. (1+14+1) remains at Δ=0; 48, 80, ... are lifted

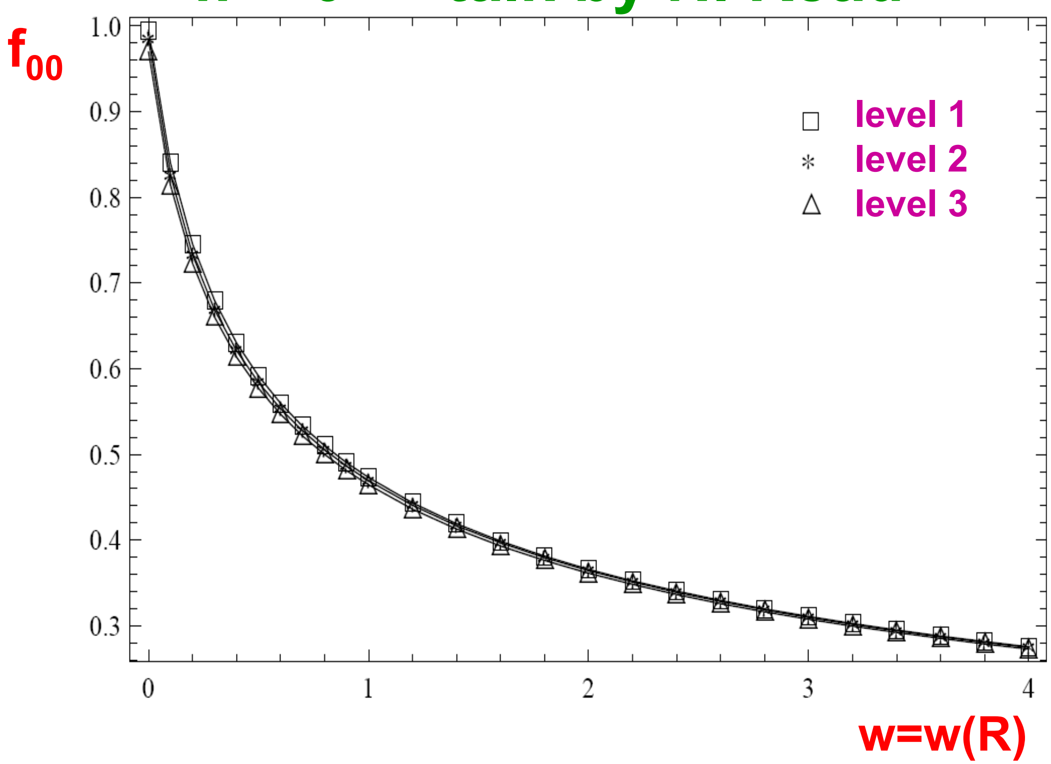
Lattice Models and Numerics

$$H_w = H_w^{(\nu)} - \sum_{j=1}^{2L} E_j - w \sum_{j=1}^{2L-1} P_{j,j+2} + H_w^{(\nu)}$$

boundary term

$$P_{j,j+2} = P_j P_{j+1} P_j$$

w = 0 → talk by N. Read



acts on states space:

$$V_f^{\otimes\nu} \otimes V_f \otimes \bar{V}_f \otimes \cdots \otimes \bar{V}_f \otimes \bar{V}_f^{\otimes\mu}$$

Extract $f_{\nu\mu}(R)$ from

$$\Delta h_k = f_{\nu\mu}(R)(C_{lk} - C_{l0})$$

$l = \mu - \nu$

no sign of instanton effects

SFermions as Cohomology

$$S \sim \int d^2z\,\big((1+\theta)(J^+\bar{J}^- + F_1^+\bar{F}_1^- + F_2^+\bar{F}_2^-) + (1-\theta)(+\leftrightarrow -)\big)$$

U(2|2):

$$\begin{pmatrix} & J^+ & F_1^+ & F_2^+ \\ J^- & & Q & \\ F_1^- & & & \\ F_2^- & & & \end{pmatrix}$$

in 3 of U(1|2)

U(1) x U(1|2)

Result: [Candu,Creutzig,Mitev,VS] $\Xi = F_1^+\bar{J}^- + F_1^+\bar{J}^-$

$$S \sim \int d^2z\,\big((1+\theta)F_2^+\bar{F}_2^- + Q(\Xi) + (1-\theta)(+\leftrightarrow -)\big)$$

Cohomology of Q arises from states in atypical modules with sdim ≠ 0

all multiplets of ground states contribute → $\lambda(\Theta_1,\Theta_2)$ from SF

Conclusions and Open Problems

- Exact Boundary Partition Functions for $CP^{1|2}$

  Techniques: QA evolution; Lattice; Q-Cohomology

- For $S^{2S+1|S}$ there is WZ-point at radius $R = 1$

  osp(2S+2|2s) at level k=1

  [Candu,H.Saleur]
  [Mitev,Quella,VS]

  Is there WZ-point in moduli space of $CP^{N-1|N}$?

  e.g. psu(N|N) at level k=1

  $CP^{0|1} = PSU(1|1)_{k=1}$

- Much generalizes to PCM on PSU(1,1|2)!

  QA evolution, simple subsector

  $AdS_3 \times S^3$

It’s all Logarithmic CFT!