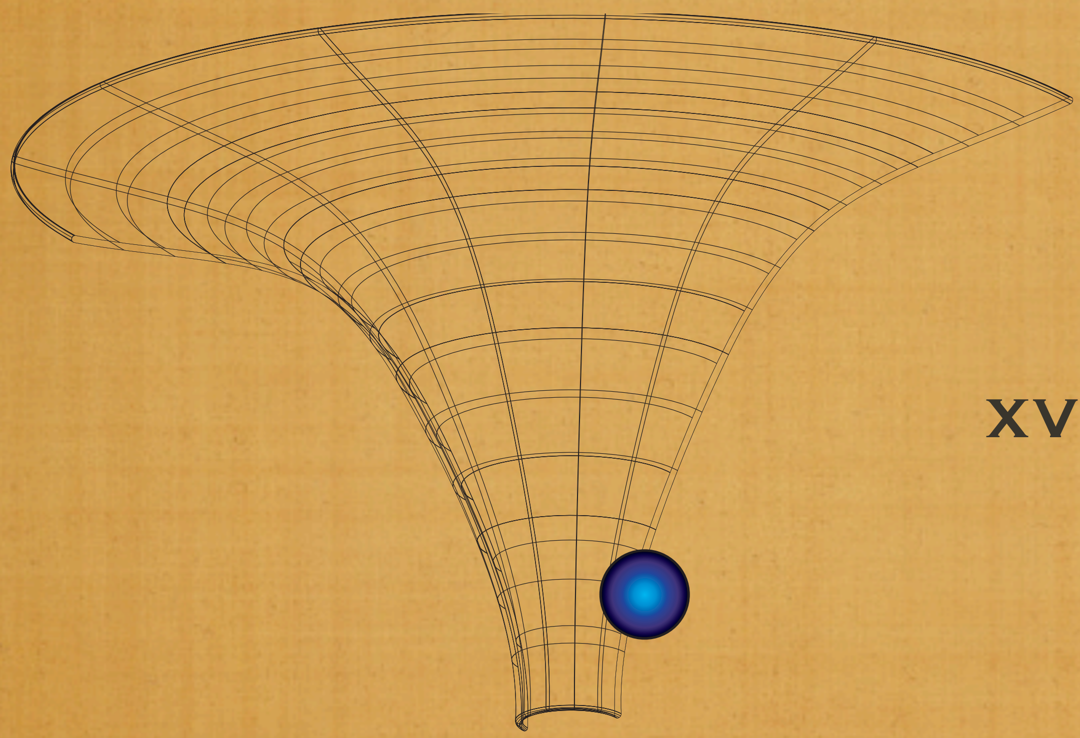
Semiclassical methods In SCFT's and emergent geometry



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CFT/ADS

- The ads/Cft correspondence has revolutionized how we think about quantum gravity and strongly coupled field theories.
- Because the system is more classical in the AdS setup, this side of the correspondence usually receives more attention: we need to solve supergravity equations of motion.
- The CFT will get all the attention in this talk: we will try to derive ADS.

Outline

- superconformal field theories 101
- classical BPS states and the chiral ring.
- Monopole operators and the moduli space of vacua of 3d field theories
- quenched wave functions and geometry of eigenvalue distributions
- Emergent geometry: locality, metric

SCFT 101

- Conformal field theories are characterized by having a larger symmetry than Lorentzian.
- They admit rescalings of the metrics.
- These rescalings can be generalized to requiring Weyl covariance.

$$g_{\mu\nu}(x) \to \exp(2\sigma(x))g_{\mu\nu}(x)$$

Conformal field theories have infrared problems that make the definition of an S-matrix problematic.

Instead, for Euclidean conformal field theories one usually considers the correlations of local operator insertions.

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\ldots\rangle$$

The collection of these numbers determines the theory.

Superconformal algebra

Dimension

$$\begin{array}{lcccc} -1 & & K_\mu & & \\ \frac{-1}{2} & & S^i_\alpha & & \\ 0 & M_{\mu\nu} & \Delta & R_{ij} \\ \frac{1}{2} & & Q^j_\alpha & & \\ 1 & & P_\mu & & \end{array}$$

In d=4, R-charge is U(N) or SU(4)

In d=3 R-charge is SO(N)

The list of operators is classified by representations of this algebra: discrete, labeled by scaling dimension

These are the most important commutation relations

$$\{Q^i_\alpha, S^{j\beta}\} = a\delta^{ij}\frac{1}{2}M_{\mu\nu}\sigma^{\mu\nu\beta}{}_\alpha + b\delta^{ij}\Delta\delta^\beta_\alpha + cR^{ij}\delta^\alpha_\beta$$

If N=1 SUSY in d=4, or N=2 SUSY in d=3, we can use the standard superspace

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot\beta}\bar\theta^{\dot\beta}\partial_\mu = Q_\alpha + 2\sigma^\mu_{\alpha\dot\beta}\bar\theta P_\mu$$

Supersymmetric vacua are annihilated by P and Q, but can break conformal invariance.

Easy to show that

$$\langle 0|D_\alpha \mathcal{O}(x,\theta,\bar\theta)|0\rangle = \langle 0|[Q_\alpha + 2\sigma^\mu_{\dot\alpha\dot\beta}\bar\theta P_\mu, \mathcal{O}(x,\theta,\bar\theta)]|0\rangle = 0 = D_\alpha\langle 0|\mathcal{O}(x,\theta,\bar\theta)|0\rangle$$

Vacuum vevs are both chiral and antichiral on-shell superfields.

Off-shell chiral operators form a ring under OPE on any SUSY vacuum.

Chiral operators are lowest component of chiral (composite) superfields.

This ring is called the chiral ring

Holomorphy: chiral ring vevs completely characterize all SUSY vacua (order parameters).

Operator-state correspondence

Assume you have added an operator at the origin in an euclidean CFT

$$ds^2 = r^2\left(\frac{dr^2}{r^2} + d\Omega^2\right)$$

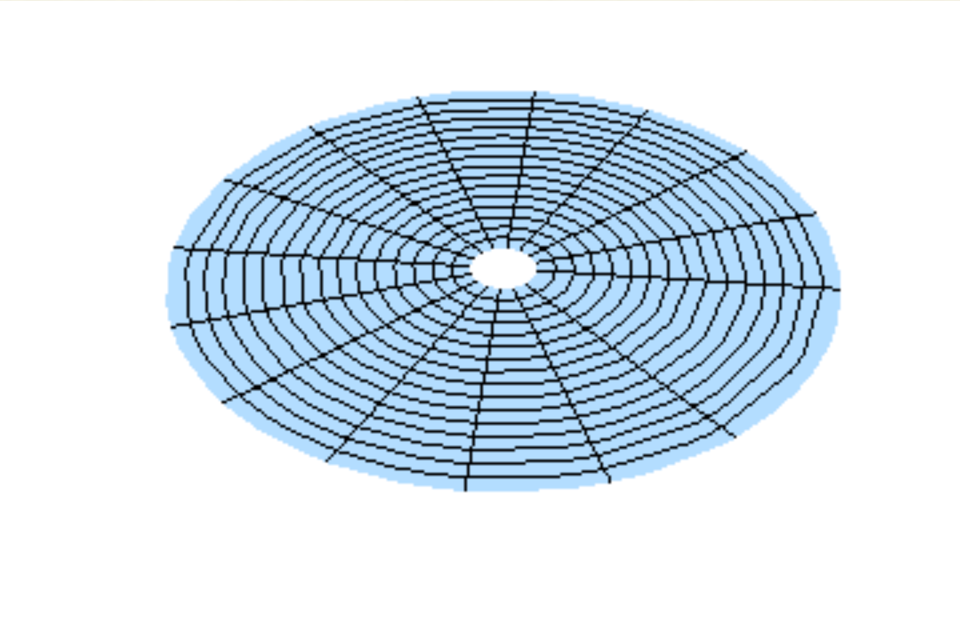
Conformally Weyl rescale to remove origin.

$$t = log(r)$$

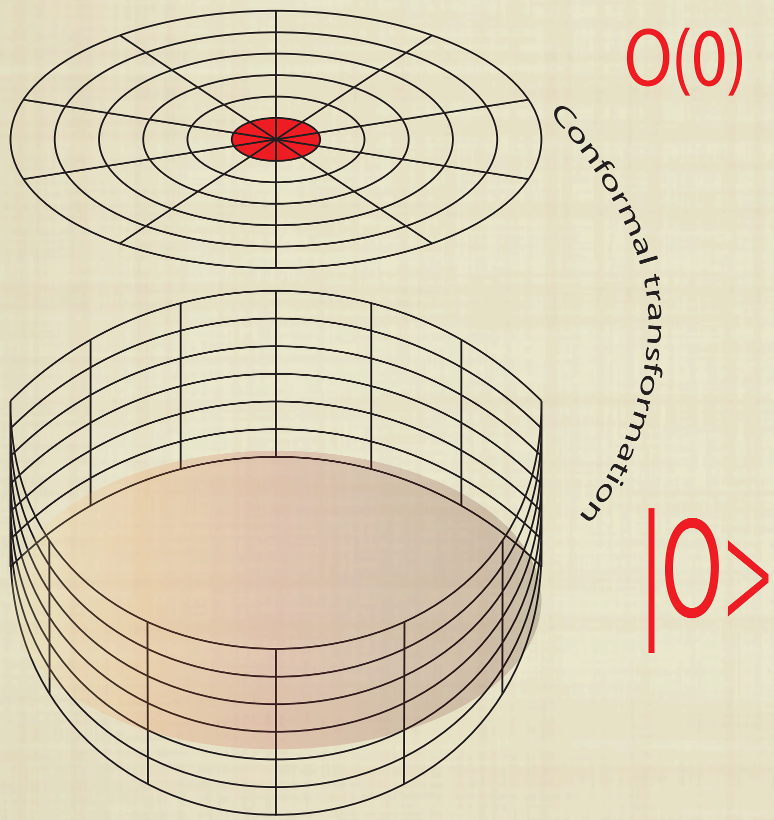
$$dt^2 + d\Omega^2$$

How do we know we inserted an operator?

The origin is characterized now by the infinite 'past'.The presence of the operator becomes a boundary condition in the time coordinate.



In Lorentzian systems a time boundary condition is an initial condition: to an operator one can associate a state in the theory.



$$\mathcal{O}(0) \sim |\mathcal{O}\rangle$$

Weyl Covariance requires that Hamiltonian in radial time is scaling dimension

Dictionary between states and operators

States	Operators
Angular momentum	spin
Energy	dimension
R-charge	R-charge

Unitarity on the cylinder

$S \simeq Q^\dagger$
$K \simeq P^\dagger$ ⟵ Q,P raise energy (dimension)
K,S lower energy

All representations are characterized by
a lowest energy state (superprimary)

Annihilated by S,K

Commutation relations + unitarity Gives BPS bound

$$\{Q, S\} = H \pm R \pm L_z \geq 0$$

Chiral ring states are equivalent to states such that

$$H = R$$

Saturate BPS inequality.

Classical states

Symmetries of cylinder make hamiltonian methods very useful.

Instead of considering quantum BPS states, one can consider classical states that saturate the BPS inequality (These are bosonic)

Coherent states in quantum theory: superposition of quantum states with different energies.

BPS equations

Two cases:

4d SCFT

$$H \simeq F_{\mu\nu}^2 + \Pi^2 + |\nabla\phi|^2 + |\phi|^2 + V(\phi)$$

3d SCFT

Conformal coupling to metric on cylinder

$$H \simeq |\Pi^2| + |\nabla\phi|^2 + \frac{1}{4}|\phi|^2 + V(\phi) \quad + \cancel{F_{\mu\nu}^2}$$

Gauge dynamics is first order (Chern simons) Schwarz: hep-th/0411077

$R \sim \phi\Pi - \bar{\phi}\bar{\Pi}$

With some normalization

$H - R = \text{Sum of squares}$

4D

$$\dot{\phi} = \pm i\phi$$

First order equations

$$\nabla\phi = 0$$

Field is constant on sphere

$$F_{\mu\nu} = 0$$

Glue is trivial

$$D = 0$$

$$F = 0$$

Vacuum equations of moduli space.

Complete solution: initial condition is one point in moduli space

DB: hep-th/0507203, 0710.2086
Grant,Grassi,Kim,Minwalla, 0803.4183

Notice that momenta are linear in fields for BPS solutions.

$$\Pi_\phi \simeq \dot{\phi} \simeq \bar{\phi}$$

Quantization on BPS configurations moduli space gets quantized: Pull-back of Poisson structure to BPS configurations is Kähler form

Chiral field Poisson brackets commute

Anti-chiral fields are canonical conjugate

Holomorphic polarization

$$\psi(\phi) = P(\phi)\psi_0$$

Specialize to N=4 SYM

$$[\phi_i, \phi_j] = 0 = [\phi_i, \bar{\phi}_j]$$

Fields are commuting matrices: diagonalized by gauge transformations

N particles on $\mathbb{C}^3$

P invariant under permutation of eigenvalues:
remnant discrete gauge transformation.

Same answer as perturbation theory

3D: non-perturbative

$\dot{\phi} = \pm \frac{i}{2}\phi$ First order

$\nabla \phi = 0$ Spherically invariant

Potential is sum of squares, must vanish:
classical point in moduli space.

Covariantly constant bifundamental scalars requires that gauge flux for the two gauge groups is the same

$$F^1_{\theta\phi}\phi - \phi F^2_{\theta\phi} = 0$$

Non-trivial gauss' law constraint

$$\frac{\kappa\Phi}{2\pi} = Q_{\text{gauge}}$$

Gauge field configurations can be non-trivial: one is allowed spherically invariant magnetic flux. This carries also electric charge, cancelled by matter.

Borokhov-Kapustin-Wu: hep-th/0206054

Magnetic flux is already quantized at the classical level!

Attiyah-Bott, 1982

These configurations are magnetic monopole operators

Non-perturbative: quantization of flux.

ABJM model

Aharony, Bergmann, Jafferis, Maldacena 0806.1218

$$U(N)_k \times U(M)_{-k}$$

$$A^{1,2}(N, \bar{N})$$
$$B^{1,2}(\bar{N}, N)$$

Vector superfields are auxiliary

$$V_\mu, \sigma, \psi, D$$

N=2 Superspace formulation

Benna, Klebanov, Klose, Smedback 0806.1519

Superpotential: same as Klebanov-Witten conifold

Also a potential term of the form

$$||[\sigma, A]||^2 + ||[\sigma, B]||^2$$

The equations of motion of D are

$$k\sigma_1 + A\bar{A} - \bar{B}B = 0$$
$$-k\sigma_2 + B\bar{B} - \bar{A}A = 0$$

These relax D-term constraints relative to four dimensional field theory with same superpotential.

Full moduli space for single brane is four-complex dimensional.

One can check that moduli space is essentially N particles on $\mathbb{C}^4$

Some extra topological subtleties

Parametrized by unconstrained diagonal values of A,B

Precise monopole spectrum: holomorphic quantization

$$(A^1)^{m_1}(A^2)^{m_2}(B^1)^{n_1}(B^2)^{n_2}$$

Gauss' constraint reads

$$kn = m_1 + m_2 - n_1 - n_2$$

For each eigenvalue

Naively gives the holomorphic coordinate ring of

$$Sym^N \mathbb{C}^4/\mathbb{Z}_k$$

ABJM,
D.B, Trancanelli, 0808.2503

There is a catch:

Only differences of fluxes between gauge groups need to be integer: topological consistency of A,B fields. Are only charged under difference of fluxes.

We can have fractional flux on all eigenvalues simultaneously: only for U(N)xU(N) theory

$$\mathbb{Z}_k \rightarrow \mathcal{M}$$

$$\downarrow$$

$$Sym^N(\mathbb{C}^4/\mathbb{Z}_k)$$

D.B.,J. Park: 0906.3817
C.S. Park 0810.1075
Kim, Madhu: 0906.4751

The extra elements of chiral ring carry a discrete charge: the amount of fractional flux.

In the AdS dual, this charge is a non-trivial homology torsion cycle corresponding to D4 branes wrapped on $\mathbb{CP}^2$

ABJM orbifolds

Douglas-moore procedure on quiver.

Abelian case: BKKS, Imamura,Martelli-Sparks, Terashima,Yagi, ...

Careful study along same lines shows

$$\mathbb{C}^4/\mathbb{Z}_{kn}\times\mathbb{Z}_n$$

Non-abelian case: D.B, Romo

$$\mathbb{C}^4/\mathbb{Z}_{k|\Gamma|}\times\Gamma$$

Crucial that Chern Simons levels are proportional to dimension of irreps of Γ

Match to AdS

- Standard bulk brane monopole is D0-brane
- Branes fractionate at singularities
- Fractional brane charges are mapped to gauge flux on each U(N) (first Chern classes)
- Fractional brane R-charge requires flux on shrunken cycles: the Hopf fiber is non-trivially fibered. (See also Aganagic 0905.3415)

Quenched wave functions

Ground state wave function ψ_0

other degrees of freedom?

Strong coupling

What can be computed?

Some things to notice

Description of BPS states is valid classically for any value of the coupling constant different than zero.

Should be valid at strong coupling too.

Provides a route to understand some aspects of strong coupling physics.

A quenched approximation

Look at spherically invariant configurations first (those that are relevant for BPS chiral ring states). These are only made out of s-wave modes of scalars on the sphere.

Dimensionally reduce to scalars.

$$S_{sc} = \int dt\, \mathrm{tr} \left(\sum_{a=1}^{6} \frac{1}{2}(D_t X^a)^2 - \frac{1}{2}(X^a)^2 - \sum_{a,b=1}^{6} \frac{1}{8\pi^2} g_{YM}^2 [X^a, X^b][X^b, X^a] \right)$$

$$N^2 \qquad N^2 \qquad \lambda N^2$$

Naive estimate: Eigenvalues are of order

$$\sqrt{N}$$

Potential dominates

Natural assumption:

Physics is dominated by minimum of potential.
We then expand around those configurations.

Produces an effective model of gauged commuting matrix quantum mechanics.
Off-diagonal elements are `heavy'.

One can use gauge transformations to diagonalize matrices.

One can compute an effective Hamiltonian
by calculating the induced measure on the eigenvalues and
getting the correct Laplacian.

$$\mu^2 = \prod_{i<j} |\vec{x}_i - \vec{x}_j|^2$$

$$H = \sum_i -\frac{1}{2\mu^2} \nabla_i \mu^2 \nabla_i + \frac{1}{2} |\vec{x}_i|^2$$

DB, hep-th/0507203

The problem reduces to a system of N bosons in six dimensions, with a non-trivial interaction induced by the measure and a confining harmonic oscillator potential.

$$H = \sum_i -\frac{1}{2\mu^2}\nabla_i \mu^2 \nabla_i + \frac{1}{2}|\vec{x}_i|^2$$

Conformal coupling of scalars to sphere

Solve the Schrodinger equation

Wave function of the "Universe"

$$\psi_0 \sim \exp(-\sum \vec{x}_i^2/2)$$

$$\hat{\psi} = \mu\psi$$

Probability density

$$|\hat{\psi}_0^2| \sim \mu^2 \exp(-\sum x_i^2) = \exp\left(-\sum \vec{x}_i^2 + 2\sum_{i<j} \log|\vec{x}_i - \vec{x}_j|\right)$$

Eigenvalue gas

Similar to a Boltzman gas of N Bosons in 6d with a confining potential and logarithmic repulsive interactions.

$$|\hat{\psi}_0^2| \sim \mu^2 \exp(-\sum x_i^2) = \exp\left(-\sum \vec{x}_i^2 + 2\sum_{i<j} \log|\vec{x}_i - \vec{x}_j|\right)$$

$$\exp(-\beta \tilde{H})$$

Go to collective coordinate description: joint eigenvalue density distribution.

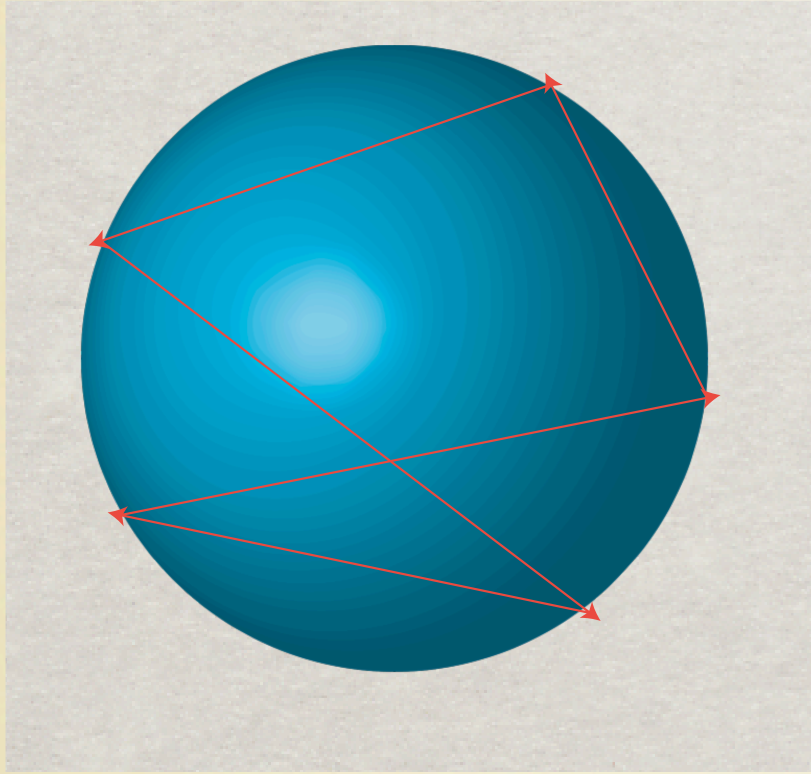
Saddle point approximation:

$$\rho = N \frac{\delta(|\vec{x}| - r_0)}{r_0^{2d-1} Vol(S^{2d-1})}$$

$$r_0 = \sqrt{\frac{N}{2}}$$

D.B., D. Correa, S. Vazquez, hep-th/0509015

This geometric sphere on dynamical variables should be identified with dual sphere on AdS geometry.



Strings are built by exciting off-diagonal modes. The masses end up being related to the distances between eigenvalues: Coulomb branch masses.

LOCALITY!

Can reproduce plane wave limit and energies of simple longer strings (giant magnons) directly from field theory.

D.B., D. Correa, S. Vazquez, hep-th/0509015 JHEP 0602, 048 (2006)

Coulomb branch dynamics means we can also use magnetic excitations for the off-diagonal modes.

Reproduce D-string giant magnon energies and check S-duality.

Distances between eigenvalues again determine spectrum, but now we keep τ finite as N is taken large.

$$\tilde{m}_{ij}^2 = 1 + \frac{h(\lambda)|p - q\tau|^2}{4\pi^2}|\hat{x}_i - \hat{x}_j|^2 \,.$$

S-duality transforms both the 't Hooft coupling and τ. We have correct states to match to S-dual.

(Calculation of masses is due to Sen '94)

$$h(\lambda) = \lambda g(1/\lambda)$$

We find the following functional relation by requiring consistency with S-duality

$$g\left(\frac{y}{|\tau|^2}\right) = g(y)$$

The only function that can do this is constant: non-renormalization theorem for giant magnon dispersion relation.

D.B., D. Trancanelli arXiv:0904.0444

For ABJM:

Reproduce perturbative results by semiclassical methods

D.B., D. Trancanelli arXiv:0808.2503

$h(\lambda)$ Is not constant

no S-duality to bootstrap it

Geometry of M-theory fiber can only be understood non-perturbatively: locality on this circle can not be argued by masses of states.

Conclusion

- It is interesting to study Classical solutions of conformal field theories on sphere: coherent state 'operators'
- Determine chiral ring spectrum including non-perturbative monopole operators
- The best way to understand topology of moduli space in 3d field theories: no guessing
- Fractional flux correction to moduli space

- Suggest a quenched approximation for strong coupling regime
- In 4d theories can reproduce sasaki-Einstein metric*, locality, giant magnons for (p,q)-strings
- 3d geometry is more mysterious and renormalized

- * Extra input- D.B, S. Hartnoll (0711.3026)

Questions

- Can wave functions be studied more systematically? (Corrections)
- How does this self-quenching break down?
- emergent Locality implies one can ask questions about quantum gravity more precisely
- Small black holes? Time warping? AdS locality?
- M-theory still harder: can not avoid discussion of non-perturbative physics.