15-th European Workshop on String Theory. Zürich, 11 September 2009.

Gravity duals of unquenched quark-gluon plasmas

Francesco Bigazzi (Université Libre de Bruxelles)

based on forthcoming paper with

Aldo L. Cotrone (KU, Leuven), Angel Paredes (U. Barcelona),
Alfonso V. Ramallo, Javier Mas, Javier Tarrio (U. Santiago de Compostela)

Motivations

- New state of matter discovered at RHIC: a strongly coupled plasma of quarks and gluons. Liquid with very low viscosity.
- A challenge for theoretical physics

- Lattice QCD ok for equilibrium properties, not so well suited for perturbations, transport properties, interactions with hard probes, finite quark densities…

- Gauge/gravity duality provides a remarkable framework to address those problems at least for certain classes of strongly coupled non abelian plasmas still quite different from real world QCD.
- Despite this: not so bad quantitative matching with sQGP properties (e.g. η/s)! This encourages exploring those models.

Motivations

- Prototypes: planar strongly coupled thermal quivers on Nc D3-branes at CY3 cones. They are $\mathcal{N}$=1 supersymmetric and conformal theories at T=0.
- Dual description (T≠0) : IIB on AdS_5 (black hole) x X_5, constant dilaton and F5 RR flux.
- X5: Sasaki-Einstein base of the cone

- Example 1: $X_5=S^5 \leftrightarrow \mathcal{N}=4$ SU(Nc) SYM
- Example 2: X_5 = T^(1,1) $\leftrightarrow \mathcal{N}$=1 SU(Nc)xSU(Nc) Klebanov-Witten quiver
- Infinite classes of known dual pairs more

Motivations

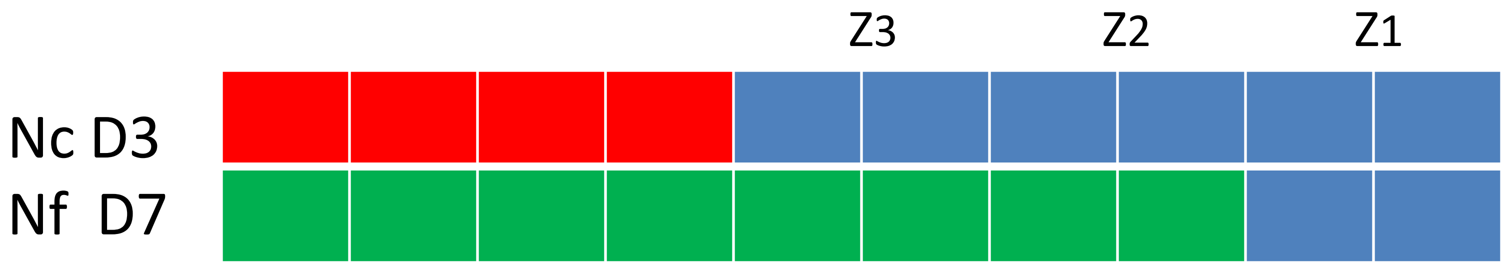
- Evidently many differences from real world QCD
- In particular they do not have matter fields in the fundamental: no quarks
- Flavors can be added by means of D7-branes
- To account for vacuum polarization effects due to dynamical flavors , i.e. to go beyond the so-called quenched approximation , we need to account for the backreaction of the flavor branes on the background.
- Not an easy task in the thermal case. In fact most of the known results concern the quenched approximation where the flavor branes are treated as probes.

Plan of the talk

For D3-D7 quark-gluon plasmas with D3 at generic CY cone over X_5, and D7 corresponding to massless flavors with flavor symmetry group = product of abelian factors, I will

- Present dual non-extremal backreacted gravity solutions. Regular at the horizon. Analytically given in a perturbative expansion.
- Study thermodynamics
- Study energy loss of partons in D7-D3 plasmas

Simplest example: Nc D3 + Nf D7 in flat space ($X_5=S^5$) at T=0



- At Nf=0 a SCFT: $\mathcal{N}$=4 SU(Nc) SYM (SU(4)R). In $\mathcal{N}$=1 components:

$$W_0 = \Phi_1[\Phi_2, \Phi_3] \qquad SU(3) \times U(1)_R \text{ symmetry}$$

- Add Nf D7-branes wrapping non compact 4manif (ex. $Z_1 = \mu_1$)

D3-D7 strings↔ fundamental hypers. SU(Nf) flavor symmetry

$$W_2 = W_0 + \tilde{q}_i(\Phi_1 - m_1)q_i$$

Break global symmetry, conformal invariance and susy

$\mathcal{N}$=4 → $\mathcal{N}$=2 , b0 = (3Nc - 3Nc) - Nf → UV Landau pole

We will consider $\mathcal{N}$=1 setups. They will inherit this UV behavior.
We will focus on IR physics well below the Landau Pole.

We will take Nf>>1 D7-branes homogeneously smeared over transverse space, to preserve original symmetries and $\mathcal{N}$=1 susy (at T=0).

In sugra: density distribution form Ω instead of delta functions. Ordinary differential equations in a radial variable instead of partial diff. equations
[F.B., Casero, Cotrone, Kiritsis, Paredes 05; Casero, Nunez, Paredes 06]

Generalized embedding $$\sum_{i=1}^{3} a_i Z_i = \mu\,, \quad \sum_{i=1}^{3} |a_i|^2 = 1$$

$$W = \Phi_1[\Phi_2, \Phi_3] + \tilde{q}(a_1\Phi_1 + a_2\Phi_2 + a_3\Phi_3 - m)q$$

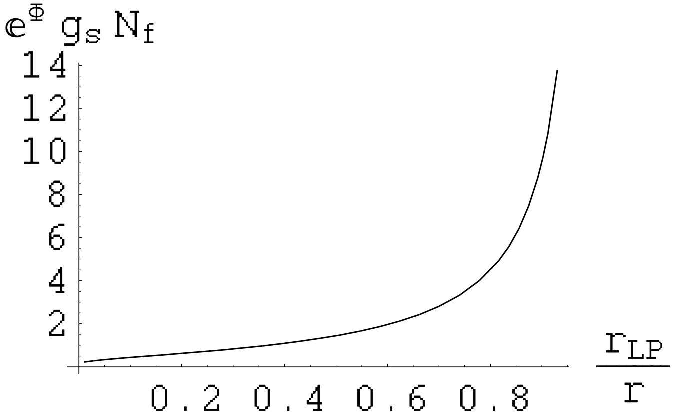
Sum over flavors and integrate over ai in W. Flavor symmetry: U(1)^Nf .

Let's consider the massless case m=0 (i.e. μ=0): D7 reach the origin. Massless susy embeddings also solve D7 worldvolume equations in non-extremal case.

Some guess for a “perturbative” dual sugra solution

$$\beta\left(\frac{1}{g_{YM}^2}\right) \sim -N_f \quad \rightarrow \quad \frac{1}{g_{YM}^2} \sim N_f \log \frac{\Lambda_{LP}}{E}$$

$$g_{YM}^2 \sim g_s e^{\Phi}, \quad \frac{\Lambda_{LP}}{E} \sim \frac{r_{LP}}{r} \quad \rightarrow \quad e^{\Phi} \sim \frac{1}{g_s N_f \log \frac{r_{LP}}{r}}$$



Let’s introduce an arbitrary scale $0 < r_* < r_{LP}$

$$e^{\Phi} \sim \frac{e^{\Phi_*}}{1 + \epsilon_* \log \frac{r_*}{r}}, \quad \Phi_* \equiv \Phi(r_*)$$

$$\epsilon_* \equiv g_s N_f e^{\Phi_*} \sim \lambda_* \frac{N_f}{N_c} \sim \frac{1}{\log \frac{r_{LP}}{r_*}}, \quad \lambda \equiv g_{YM}^2 N_c$$

Focus on a region $r_0 \leq r \ll r_* \ll r_{LP}$

$$e^{\Phi} \approx e^{\Phi_*}\left[1 + \epsilon_* \log \frac{r}{r_*} + \frac{1}{2}\epsilon_*^2 \log^2 \frac{r}{r_*} + \dots\right]$$

$$\epsilon_* \ll 1, \quad \epsilon_* \left|\log \frac{r}{r_*}\right| \ll 1$$

The dual backreacted solutions

$$S = S_{IIB} + S_{fl}$$

$$S_{IIB} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g_{10}} \left[R - \frac{1}{2} \partial_M \Phi \partial^M \Phi - \frac{1}{2} e^{2\Phi} F_{(1)}^2 - \frac{1}{2} \frac{1}{5!} F_{(5)}^2 \right]$$

$$S_{fl} = S_{WZ} + S_{DBI}$$

$$S_{WZ} = T_7 \sum_{N_f} \int_{\mathcal{M}_8} \hat{C}_8 \rightarrow S_{WZ}^{smear} = T_7 \int_{\mathcal{M}_{10}} \Omega \wedge C_8$$

$$S_{DBI} = -T_7 \sum_{N_f} \int_{\mathcal{M}_8} d^8\xi \, e^{\Phi} \sqrt{-\det(\hat{g}_8)} \quad \longrightarrow$$

$$S_{DBI}^{smear} = -T_7 \int d^{10}x \sqrt{-g_{10}} e^{\Phi} \sum_i \sqrt{\frac{1}{2} \Omega_{MN}^{(i)} \Omega_{PQ}^{(i)} g^{MN} g^{PQ}}$$

$$\Omega \equiv -g_s \sum_i \Omega^{(i)}$$

Flavored, non-extremal solution: generic X5 case

[X5 Sasaki-Einstein: $ds^2_{X_5} = ds^2_{KE} + (d\tau + A_{KE})^2, \quad dA_{KE} = 2J_{KE}$]

$$ds^2_{10} = h^{-\frac{1}{2}}\left[-b\,dt^2 + d\vec{x}_3^2\right] + h^{\frac{1}{2}}\left[S^8F^2b^{-1}\,dr^2 + r^2 ds_5^2\right]$$

$$h = \frac{R^4}{r^4}, \qquad b = 1 - \frac{r_h^4}{r^4}$$

$$ds_5^2 = S^2 ds^2_{KE} + F^2(d\tau + A_{KE})^2, \quad dA_{KE} = 2J_{KE}$$

$$\Phi = \Phi(r), \quad F_{(5)} = Q_c\,(1 + *)\mathrm{vol}(X_5)$$

$$F_{(1)} = Q_f(d\tau + A_{KE}), \quad dF_{(1)} = 2Q_f J_{KE} \equiv -g_s\Omega$$

$$R^4 = \frac{Q_c}{4}, \quad Q_c = \frac{(2\pi)^4 g_s N_c\,\alpha'^2}{Vol(X_5)}, \quad Q_f = \frac{g_s N_f Vol(X_3)}{4Vol(X_5)}$$

The perturbative solution

$$F = 1 - \frac{\epsilon_*}{24}\left(1 + \frac{2r^4 - r_h^4}{6r_*^4 - 3r_h^4}\right) + \frac{\epsilon_*^2}{1152}\left(17 - \frac{94}{9}\frac{2r^4 - r_h^4}{2r_*^4 - r_h^4} + \frac{5}{9}\frac{(2r^4 - r_h^4)^2}{(2r_*^4 - r_h^4)^2} + \right.$$
$$\left. - \frac{8}{9}\frac{r_h^8(r_*^4 - r^4)}{(2r_*^4 - r_h^4)^3} - 48\log\left(\frac{r}{r_*}\right)\right) + O(\epsilon_*^3) \ ,$$

$$S = 1 + \frac{\epsilon_*}{24}\left(1 - \frac{2r^4 - r_h^4}{6r_*^4 - 3r_h^4}\right) + \frac{\epsilon_*^2}{1152}\left(9 - \frac{106}{9}\frac{2r^4 - r_h^4}{2r_*^4 - r_h^4} + \frac{5}{9}\frac{(2r^4 - r_h^4)^2}{(2r_*^4 - r_h^4)^2} + \right.$$
$$\left. - \frac{8}{9}\frac{r_h^8(r_*^4 - r^4)}{(2r_*^4 - r_h^4)^3} + 48\log\left(\frac{r}{r_*}\right)\right) + O(\epsilon_*^3) \ ,$$

$$\Phi = \Phi_* + \epsilon_* \log\frac{r}{r_*} + \frac{\epsilon_*^2}{72}\left(1 - \frac{2r^4 - r_h^4}{2r_*^4 - r_h^4} + 12\log\frac{r}{r_*} + 36\log^2\frac{r}{r_*} + \right.$$
$$\left. + \frac{9}{2}\left(Li_2\left(1 - \frac{r_h^4}{r^4}\right) - Li_2\left(1 - \frac{r_h^4}{r_*^4}\right)\right)\right) + O(\epsilon_*^3)$$

$$\Phi_* \equiv \Phi(r_*), \qquad Li_2(u) \equiv \sum_{n=1}^{\infty}\frac{u^n}{n^2}$$

The perturbative expansion parameter

$$\epsilon_* \equiv Q_f \, e^{\Phi_*} = \frac{Vol(X_3)}{16\pi \, Vol(X_5)} \lambda_* \frac{N_f}{N_c}$$

$$\lambda_* = g_{YM}^2 N_c \equiv 4\pi \, g_s N_c e^{\Phi_*}$$

$$\epsilon \sim g_{YM}^2 \, N_f$$

Weights the internal flavor loop contributions to gluon polarization diagrams

Comments on the solution

- Massless-flavored susy solution (b=1) exactly known . It has a dilaton blowing up at r_{LP} (UV Landau pole) and a (good) singularity in the IR [Benini, Canoura, Cremonesi, Nunez, Ramallo 06].

- Non extremal solution asymptotes to the susy solution at large r
- Setting rh=0, reproduces the susy flavored solution order by order
- Setting Nf=0 reproduces the $AdS_5BH \times X_5$
- Non extremal solution is regular at the horizon

- Terms in powers of r/r*, rh/r* account for UV completion. We will neglect them

Regimes of validity

Hierarchy of scales $r_h \ll r_* \ll r_a < r_{LP}$

$r_* \sim \Lambda_{UV}$: arbitrary UV cutoff scale

r_a scale of UV pathologies in holographic a-function

[F.B. , Cotrone, Paredes, Ramallo 08]

Decoupling of IR region from UV one requires

$$\frac{r_*}{r_{LP}} = e^{-\frac{1}{\epsilon_*}} \ll 1 \quad \Longrightarrow \quad \epsilon_* \ll 1$$

λ>>1, 1<< Nf<< Nc (neglect curvature corrections + smearing)

λ^(-3/2) << ϵ^2 (neglect first curvature corrections)

Thermodynamics

Expansion parameter at the horizon $\epsilon_h \equiv \frac{\lambda_h Vol(X_3)}{16\pi Vol(X_5)} \frac{N_f}{N_c}$

$$\epsilon_h = \epsilon_* \frac{e^{\Phi_h}}{e^{\Phi_*}} = \epsilon_* + \epsilon_*^2 \log \frac{r_h}{r_*} + O(\epsilon_*^3)$$

Temperature

$$T = \frac{2r_h}{2\pi R^2 S_h^4 F_h} = \frac{r_h}{\pi R^2} \left[1 - \frac{1}{8}\epsilon_h - \frac{13}{384}\epsilon_h^2 + O(\epsilon_h^3) \right]$$

$$\frac{d\epsilon_h}{dT} = \frac{\epsilon_h^2}{T} + O(\epsilon_h^3)$$

Effect of broken conformal invariance at quantum level

Entropy density

$$s = \frac{2\pi \ A_8}{\kappa_{(10)}^2 \ V_3} = \frac{\pi^5}{2Vol(X_5)} N_c^2 T^3 \left[1 + \frac{1}{2}\epsilon_h + \frac{7}{24}\epsilon_h^2 + O(\epsilon_h^3) \right]$$

In the literature results on thermodynamics at first order
[Mateos, Myers, Thomson 07]

Energy density (also from dual ADM energy)

$$\varepsilon = \frac{E_{ADM}}{V_3} = \frac{3}{8}\frac{\pi^5}{Vol(X_5)}N_c^2T^4\left[1+\frac{1}{2}\epsilon_h(T)+\frac{1}{3}\epsilon_h(T)^2+O(\epsilon_h(T)^3)\right]$$

Heat capacity (density)

$$c_V = \partial_T\varepsilon = \frac{3}{2}\frac{\pi^5}{Vol(X_5)}N_c^2T^3\left[1+\frac{1}{2}\epsilon_h(T)+\frac{11}{24}\epsilon_h(T)^2+O(\epsilon_h(T)^3)\right]$$

Free energy density (also from dual regularized Eucl. action)

$$\frac{F}{V_3} = -p = \varepsilon - Ts = -\frac{1}{8}\frac{\pi^5}{Vol(X_5)}N_c^2T^4\left[1+\frac{1}{2}\epsilon_h(T)+\frac{1}{6}\epsilon_h(T)^2+O(\epsilon_h(T)^3)\right]$$

$$[s = \partial_T p]$$

Breaking of conformal invariance: a second order effect

Interaction measure

$$(\varepsilon - 3p)/T^4 = [\pi^5 N_c^2/16Vol(X_5)]\epsilon_h(T)^2$$

Speed of sound

$$v_s^2 = \frac{s}{c_V} = \frac{1}{3}\left[1 - \frac{1}{6}\epsilon_h(T)^2 + O(\epsilon_h(T)^3)\right]$$

Bulk viscosity bound [Buchel 07]

$$\frac{\zeta}{\eta} \geq 2\left(\frac{1}{3} - v_s^2\right) \qquad \zeta_{min} = \frac{\pi^4}{72Vol(X_5)} N_c^2 T^3 \epsilon_h(T)^2 + O(\epsilon_h^3)$$

We neglect curvature corrections: shear viscosity given by

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

[Kovtun, Son, Starinets 04]

Jet quenching parameter

Characterize medium-induced suppression of high-p_T jets, due to radiative energy loss of partons moving through the plasma. Non perturbative definition in terms of a certain light-like Wilson loop. Evaluated in dual gravity setup [Liu, Rajagopal, Wiedemann 06]

$$\hat{q}^{-1} = \pi\,\alpha' \int_{r_h}^{r_*} e^{-\frac{\Phi}{2}} \frac{\sqrt{g_{rr}}}{g_{xx}\sqrt{g_{xx}+g_{tt}}}dr$$

$$\hat{q} = \frac{\pi^3\sqrt{\lambda_h}\Gamma(\frac{3}{4})}{\sqrt{Vol(X_5)}\,\Gamma(\frac{5}{4})}T^3\left[1+\frac{1}{8}(2+\pi)\epsilon_h+\gamma\,\epsilon_h^2+O(\epsilon_h^3)\right] \qquad \gamma\approx 0.5565$$

$$\hat{q} = c\sqrt{\lambda_h}\sqrt{\frac{s}{N_c^2}}T^{\frac{3}{2}}\left[1+\frac{\pi}{8}\epsilon_h+(\gamma-\frac{11}{96}-\frac{\pi}{32})\epsilon_h^2\right]$$

Compare unflavored theory ($N_f=0$) with flavored theory taking

- λ_h (or $g^2_{YM,h}$), s and T fixed. Vary N_c accordingly
- ε, $\mathcal{F}_{\bar{Q}Q}$ fixed. Vary T and λ_h accordingly

$\hat{q}$ increases if $N_f > 0$

Extrapolations to RHIC

$$\alpha_s \sim 1/2 \text{ and } N_c = 3 \qquad \text{i.e. } \lambda_h \sim 6\pi$$

$$\epsilon_h \sim \frac{1}{4\pi} N_f \sim 0.24 \text{ for } N_f = 3$$

$$T = 300 \text{ MeV} \qquad \hat{q} \sim 5.3 \ (\text{Gev})^2/\text{fm}$$

($\hat{q} \sim 4.5 \ (\text{Gev})^2/\text{fm}$ for the unflavored N=4 SYM plasma)

RHIC estimate $\hat{q} \sim 5 - 15 \ (\text{Gev})^2/\text{fm}$

Drag force

At strong coupling, energy loss entirely with stringy framework: parton of velocity v modeled by open string (attached to a probe D7-brane) dragged by constant force that keeps velocity fixed [Herzog, Karch, Kovtun, Kozcaz, Yaffe; Gubser 06]

$$x(\hat{\sigma},\hat{\tau}) = r(\hat{\sigma}) + v\,\hat{\tau}$$

$$\frac{dp}{dt} = -\frac{r_h^2}{2\pi\alpha'\,R^2} e^{\frac{\Phi(r_c)}{2}} \frac{v}{\sqrt{1-v^2}} = -\mu\, M_{kin} \frac{v}{\sqrt{1-v^2}} = -\mu\, p$$

$$\mu\, M_{kin} = \frac{\pi^{5/2}}{2} \frac{\sqrt{\lambda_h}}{\sqrt{Vol(X_5)}} T^2 \Big[1 + \frac{1}{8}(2-\log(1-v^2))\epsilon_h + \\ + \frac{1}{384}\left[44 - 20\log(1-v^2) + 9\log^2(1-v^2) + 12 Li_2(v^2)\right]\epsilon_h^2 + O(\epsilon_h^3) \Big]$$

Again: energy loss increased by the fundamentals

Summary

- We have found sugra duals to strongly coupled thermal quivers coupled to Nf>>1 massless flavors
- Analytic solutions include flavor backreaction up to second order in $\epsilon \sim \lambda$Nf/Nc <<1
- We studied thermodynamics of the system and departure from conformality (ϵ^2 effect)
- We analyzed the energy loss of partons moving through the plasmas finding that fundamentals enhance it.

Future directions

- Compute the bulk viscosity (it is an ϵ^2 effect)
- Study other transport coefficients (ex: conductivity)
- Work out massive-flavored thermal solutions
- Study mesonic spectra and phase transitions

Thank you