Geometric Unification In F-theory

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Based on work with C. Vafa, as well as:

C. Beasley, V. Bouchard, S. Cecotti, M. Cheng, G.L. Kane, J. Marsano

N. Saulina, S. Schäfer-Nameki, J. Seo, J. Shao, A. Tavanfar

See Also:

(In Various Combinations)

Donagi & Wijnholt

Hayashi, Kawano, Tatar, Toda, Tsuchiya, Watari, Yamazaki

Blumenhagen, Braun, Grimm, Jurke, Weigand

Marsano, Saulina, Schäfer-Nameki

Aparicio, Cerdeño, Font, Ibáñez

Randall, Simmons-Duffin

Chen, Chung, Jiang, Li, Nanopoulos, Xie,

$+\cdots$

Outline

- Motivation
- F-theory GUTs
- Flavor
- The Point of E_8

Motivation

Standard Model/MSSM $\subset$ Strings?

What is possible in string constructions?

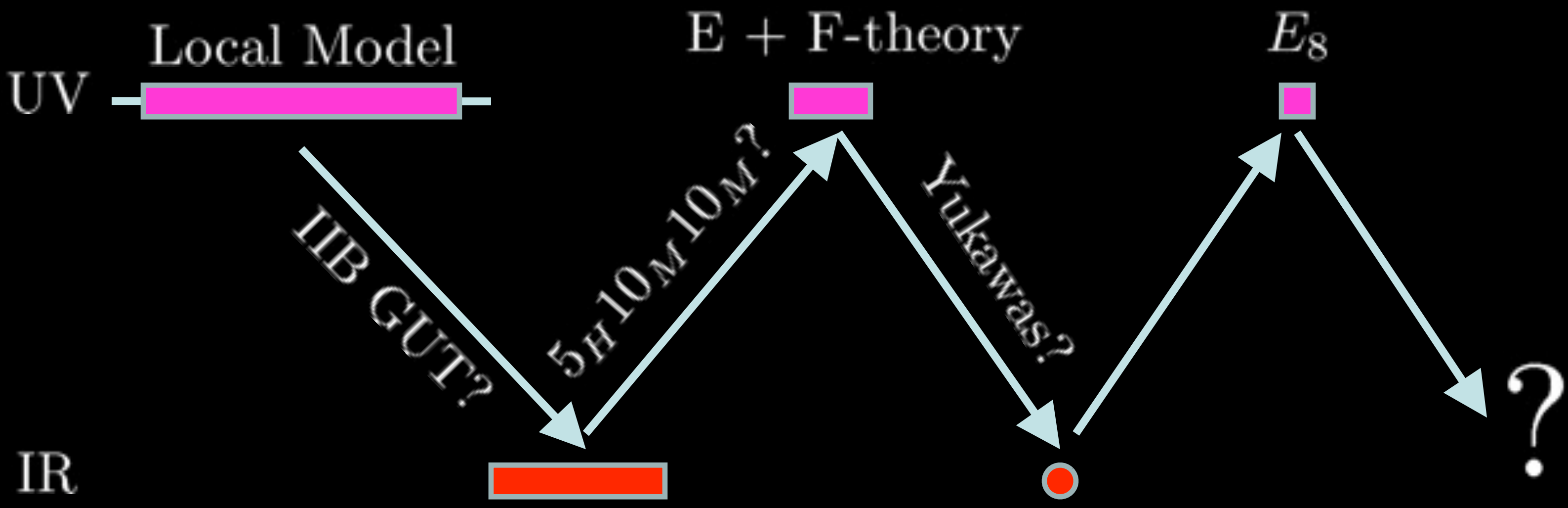
Hybrid Strategy:

Top Down: Specify All Details in UV (Global Models)

Where to look first?

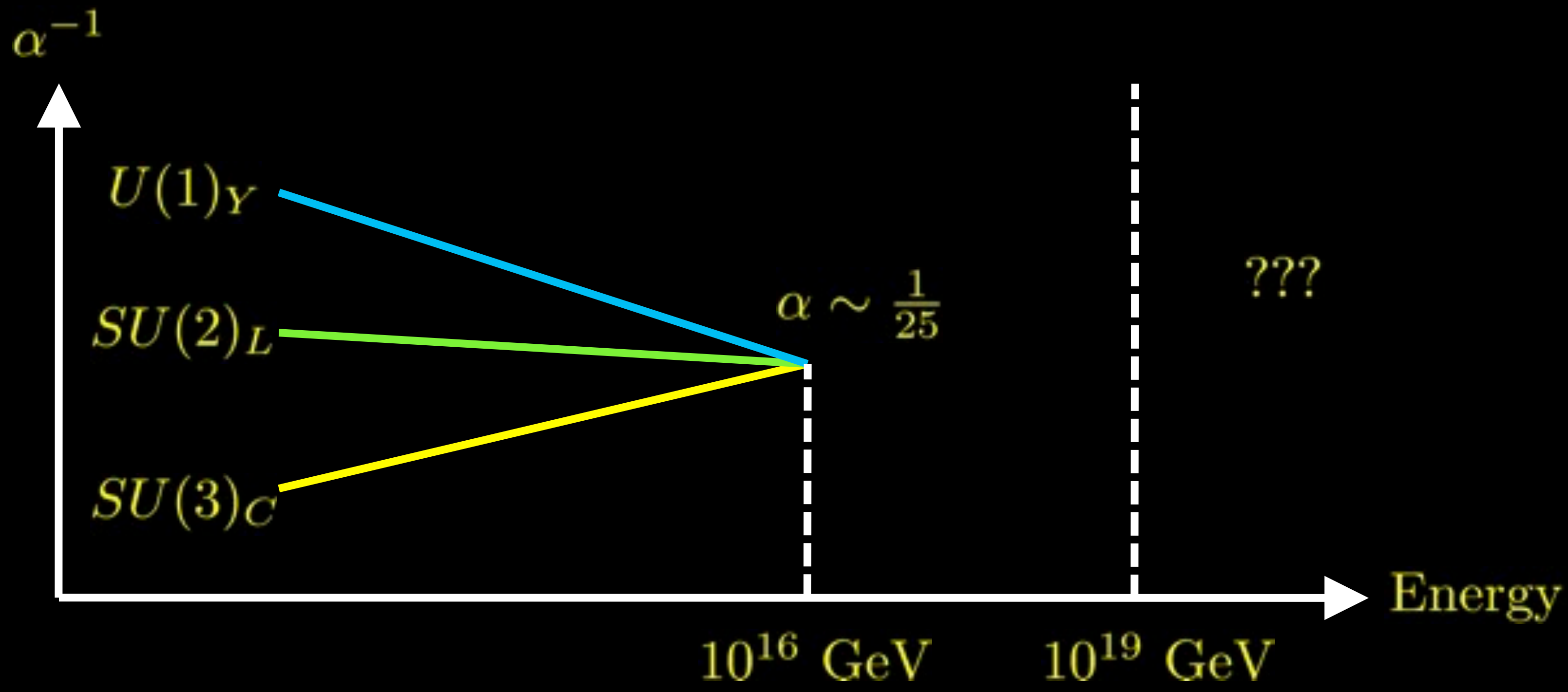
Bottom Up: Decouple some of UV (Local Models)

Too flexible?



Simplifying Assumptions:

1) Low energy supersymmetry & Unification:



2) $M_{GUT}/M_{pl} \ll 1$

Assumption 1: GUTs

$$SU(5)_{GUT} \supset SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$10_M = \begin{bmatrix} 0 & U & U & Q & Q \\ -U & 0 & U & Q & Q \\ -U & -U & 0 & Q & Q \\ -Q & -Q & -Q & 0 & E \\ -Q & -Q & -Q & -E & 0 \end{bmatrix} \qquad 5_H = \begin{bmatrix} T_u \\ T_u \\ T_u \\ H_u \\ H_u \end{bmatrix}$$

$$\overline{5}_M = \begin{bmatrix} D & D & D & L & L \end{bmatrix}$$

$$\overline{5}_H = \begin{bmatrix} T_d & T_d & T_d & H_d & H_d \end{bmatrix}$$

$L_{GUT} \supset 5_H \times 10_M \times 10_M \Rightarrow t$ quark mass

$L_{GUT} \supset \overline{5}_H \times \overline{5}_M \times 10_M \Rightarrow b$ quark & τ lepton mass

Stringy GUTs Need E_N

Stringy Matter From Adjoint Breaking:

$E_8 \supset E_6 \times SU(3) \qquad 248 \to (27, 3) + \cdots$

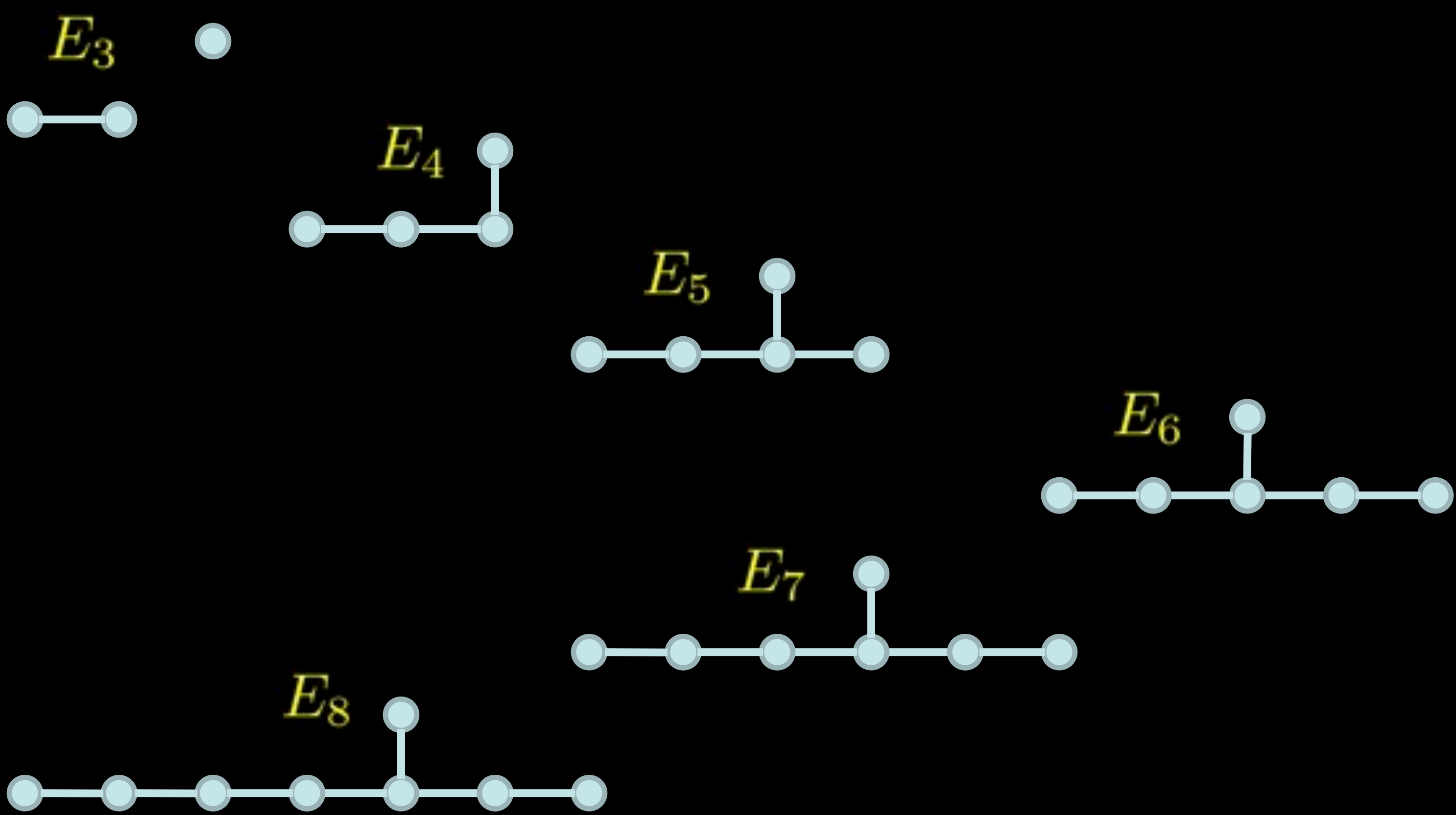
$\mathrm{adj}(E_{6,7,8}) \ni 16$ of SO(10) $\notin \mathrm{adj}(U, USp, SO)$

Interaction Terms Embed in Bigger Groups Too:

$E_8 \supset SU(5) \times SU(5) \qquad 248^3 \to 5 \times 10 \times 10$ of $SU(5)$

Pert. forbidden in $U(5)$ D-brane construction

GUTs and E_N



How much E is necessary? How much is aesthetics?

Assumption 2: $M_{GUT}/M_{pl} \ll 1$

10D Gravity: $R^{3,1} \times \mathcal{M}_6 \Rightarrow G^{4D}_{Newton} \sim \frac{1}{Vol(\mathcal{M}_6)}$

4D Gravity decouples when $Vol(\mathcal{M}_6) \to \infty$

Gauge Theory on $R^{3,1} \times \mathcal{M}_k \subset R^{3,1} \times \mathcal{M}_6$:

$\Rightarrow g^2_{YM} \sim \frac{1}{Vol(\mathcal{M}_k)} \Rightarrow Vol(\mathcal{M}_k) \not\to \infty$

Local Flexibility

Local Model suggests GUT from p - brane, $p = 3, 4, 5, 6, 7$

$\Rightarrow$ Type II strings

E-type Structure: $g_s \rightarrow O(1)$

F-theory branes: 3-branes & 7-branes

E-type and 4d Chiral Matter $\Rightarrow$ 7-branes

1) $\exists$ a GUT

E-type Structures

2) $\exists$ Decoupling Limit

IIB 7-branes

F-theory?

Roadmap

- Motivation

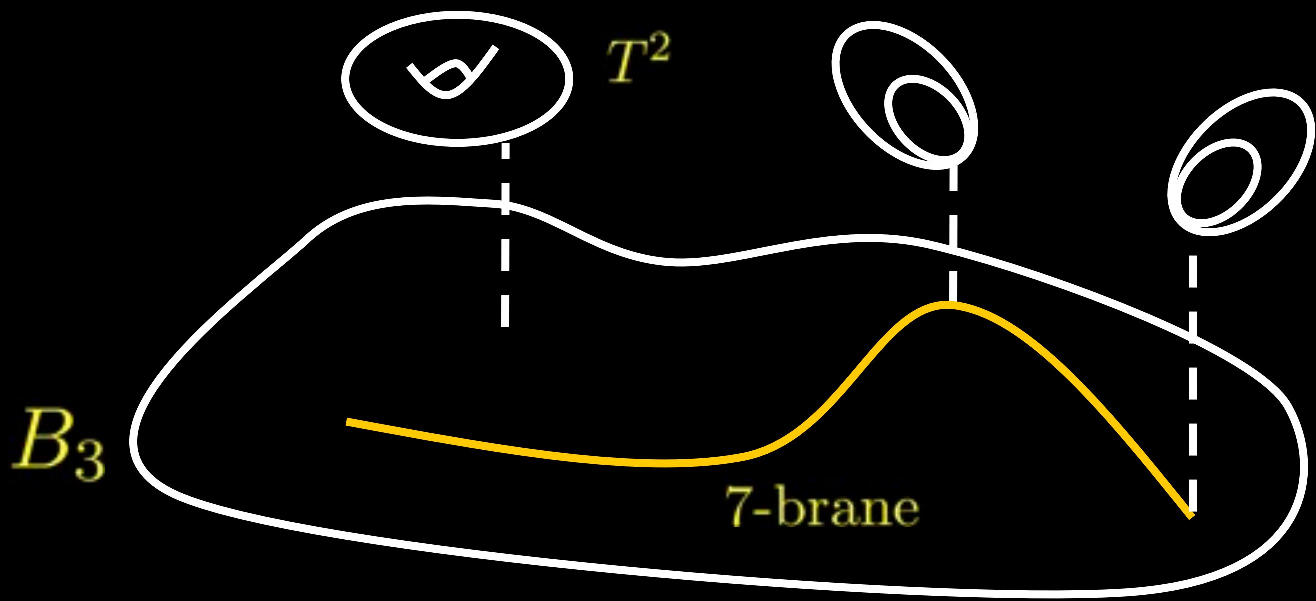
- F-theory GUTs

F-theory Review I

F-theory = Strongly Coupled Formulation of IIB in 12d

$\tau_{IIB} = C_0 + ie^{-\phi}$ is cplx str. of a T^2

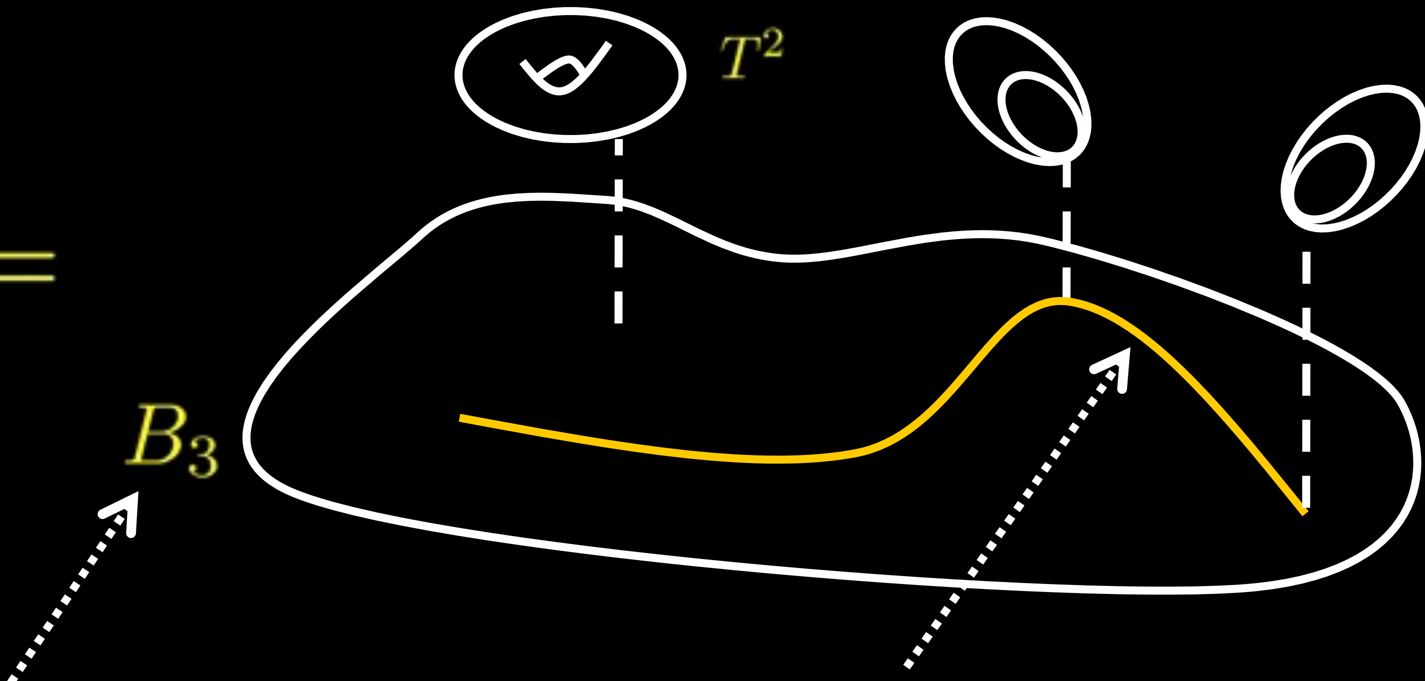
This T^2 pinches off near 7-branes:



F-theory Review II

4d $\mathcal{N} = 1 \Rightarrow$ F / $R^{3,1} \times$ Elliptic CY_4

$CY_4 =$



Local Model $\Rightarrow Vol(B_3) \rightarrow \infty$

$CY_4 \sim S \times C^2/\Gamma_{ADE} \Rightarrow$ 7-brane on S

Geometry $\Rightarrow$ Gauge Theory

$F - th/R^{3,1} \times S \times C^2/\Gamma_{ADE} \Rightarrow 8d$ SYM w/gp G_{ADE}

Example: 8d $SU(N)$ at $z = 0$ from $y^2 = x^2 + z^N$

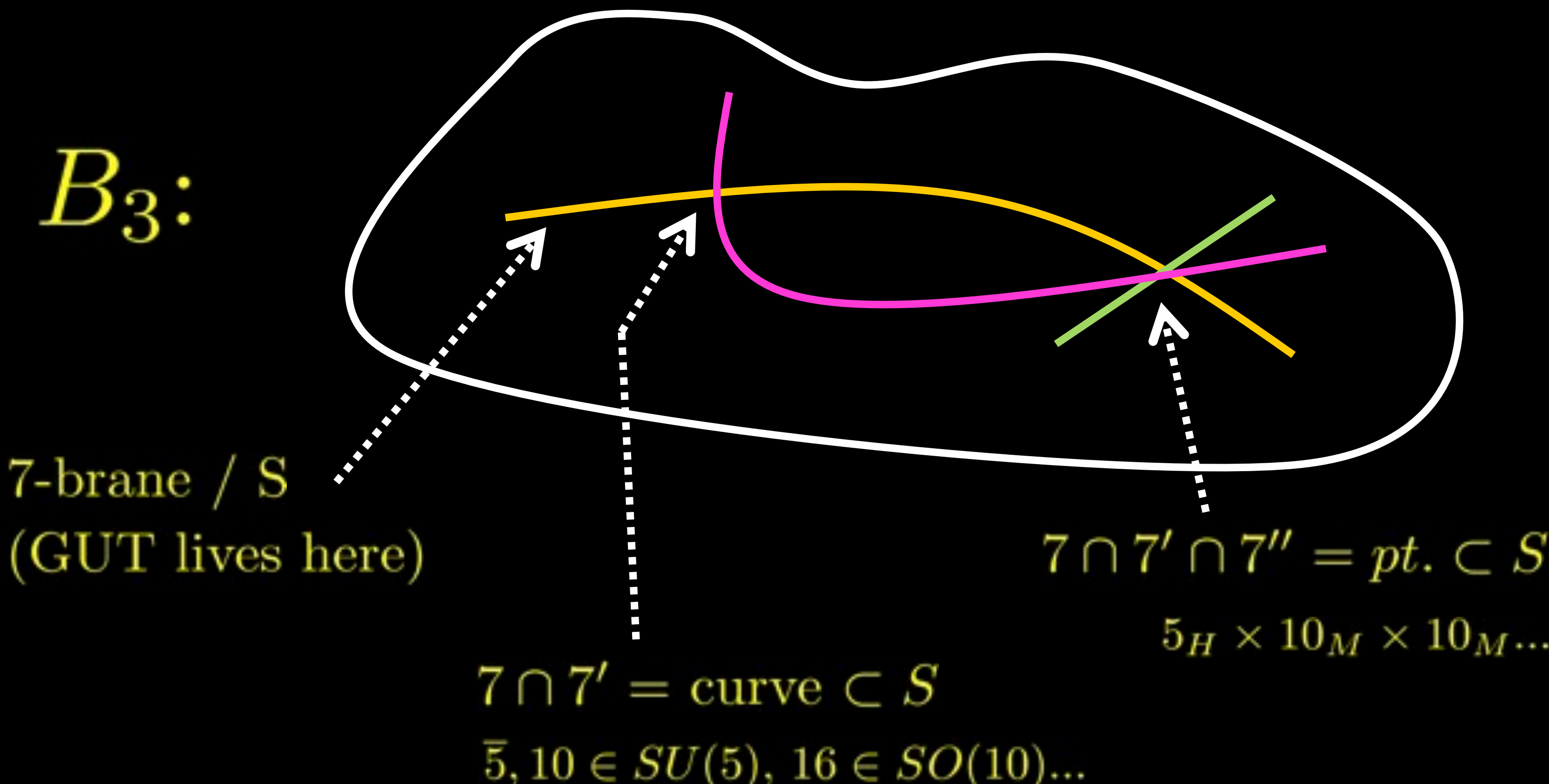
$10d \Rightarrow$ Gravity (decoupled in 4D)

$8d : 7$ on $R^{3,1}\times$ Cplx. Surface $\Rightarrow$ Gauge Group

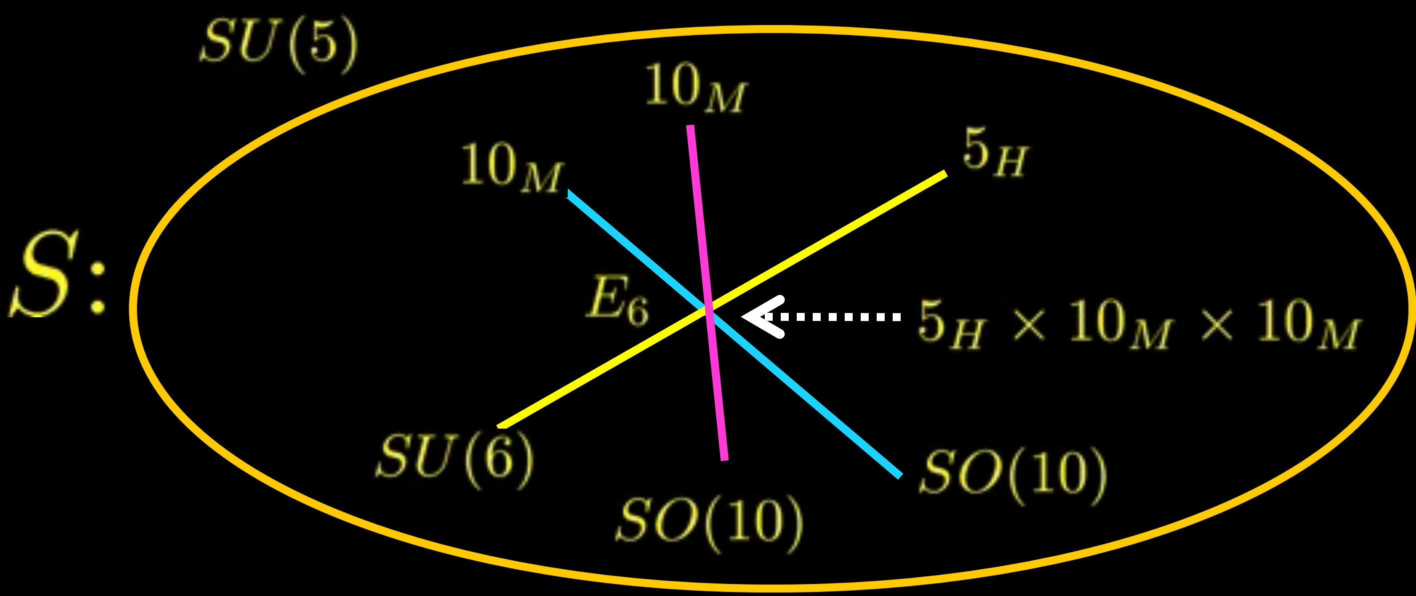
$6d : 7 \cap 7'$ on $R^{3,1}\times$ Cplx. Curve $\Rightarrow$ 6D Matter

$4d : 7 \cap 7' \cap 7''$ on $R^{3,1}\times$ pt. $\Rightarrow \int d^2\theta ABC$ or $\int d^4\theta \frac{A^\dagger BC}{\Lambda_{UV}}$

F-theory GUTs



Example



$$E_6 \supset SU(5) \times U(1) \times U(1)$$

$$78 \to 5_{+6,0} + 10_{-3,+1} + 10_{-3,-1} + \cdots$$

$$78^3 \to 5_{+6,0} \times 10_{-3,+1} \times 10_{-3,-1}$$

4d Spectrum

$$G_S \xrightarrow{\text{instanton}} \Gamma_S \times \cancel{H}_S$$

4d matter $\Longleftrightarrow$ zero modes in instanton background

S Modes: $\overline{\mathcal{D}}_A \Psi = 0$

Σ Modes: $\overline{\mathcal{D}}_{A+A'} \sigma = 0$

$\Rightarrow$ Index Computation

$$\int_M ch(V) Td(M)$$

Beasley JJH Vafa I ’08
Donagi Wijnholt I ‘08

Minimal Spectrum

Beasley JJH Vafa II '08

$$G_S = SU(5) \xrightarrow{U(1)_Y \text{ flux}} SU(3) \times SU(2) \times U(1)_Y$$

No bulk exotics $\Rightarrow$ unique internal flux

Higgs: $\int_{\Sigma_H} F_{U(1)_Y} \neq 0$:

$$\begin{array}{l} \overline{5}_H = \\ 5_H = \end{array} \boxed{\begin{array}{ccc} T_d & T_d & T_d \\ T_u & T_u & T_u \end{array}} \boxed{\begin{array}{cc} H_d & H_d \\ H_u & H_u \end{array}}$$

out   in

Matter: $\int_{\Sigma_M} F_{U(1)_Y} = 0$

Matter: $\int_{\Sigma_M} F_{U(1)_\perp} = 3$

$$\boxed{\begin{array}{c} 3 \times \overline{5}_M \\ 3 \times 10_M \end{array}}$$

in

Roadmap

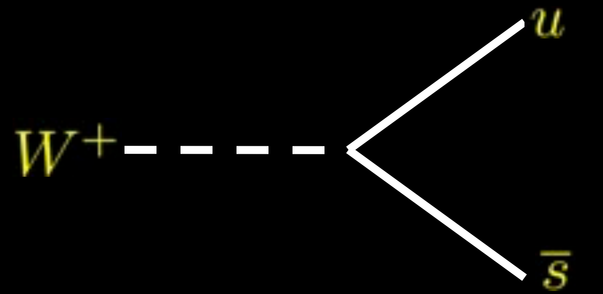
- F-theory GUTs
- Flavor

SM/MSSM Flavor

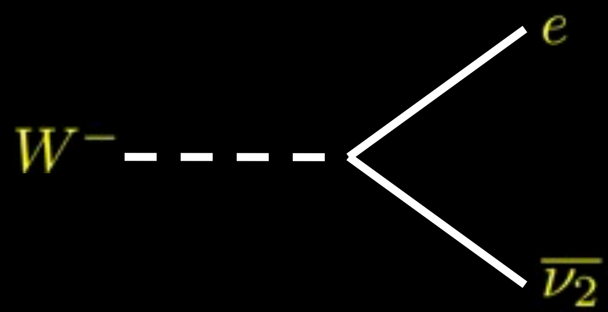
$$W \supset m_u^{ij} \cdot U_L^i U_R^j + m_d^{ij} \cdot D_L^i D_R^j + m_l^{ij} \cdot E_L^i E_R^j + m_\nu^{ij} \cdot N_L^i N^j$$

Diagonalize: $V_L \cdot m \cdot V_R^\dagger = \mathrm{diag}(\widetilde{m}_1, \widetilde{m}_2, \widetilde{m}_3)$

$$V_{CKM}^{(quark)} = V_u^L \cdot V_d^{L\dagger}$$



$$V_{PMNS}^{(lepton)} = V_l^L \cdot V_\nu^{L\dagger}$$



Quark Wishlist

Two Light Generations

Hierarchical CKM Matrix:

$$|V_{CKM}| \sim \begin{bmatrix} 1 & \varepsilon & \varepsilon^3 \\ \varepsilon & 1 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & 1 \end{bmatrix} \sim \begin{bmatrix} 0.97 & 0.23 & 0.004 \\ 0.23 & 0.97 & 0.04 \\ 0.008 & 0.04 & 0.99 \end{bmatrix}$$

Qualitative Ingredients

3 10_M's on Σ_{10} curve(s)

3 $\overline{5}_M$'s on $\Sigma_{\overline{5}}$ curve(s)

$\boxed{+\ \text{Flux}}$

$5_H \times 10_M \times 10_M$ point(s)

$\overline{5}_H \times \overline{5}_M \times 10_M$ point(s)

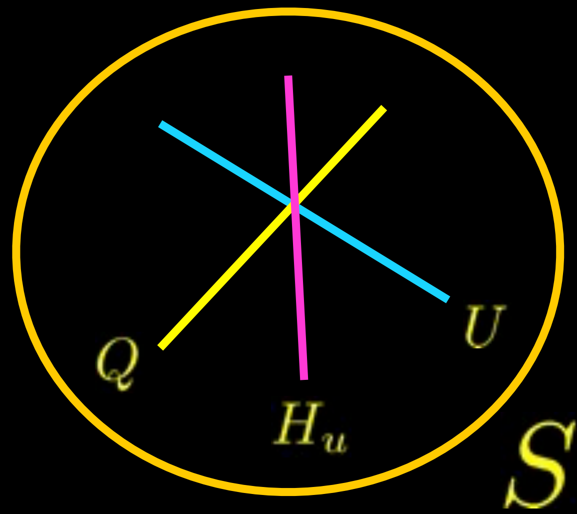
Simplest Case: 1 curve & point of each type

What Yukawas Do We Get?

Quark Yukawas:

$R^{3,1}$: $W \supset \lambda_u^{ij} \cdot Q^i U^j H_u + \cdots$

$\mathcal{M}_6$: $\overline{\mathcal{D}}\Psi = 0$: $\Psi_Q^i, \, \Psi_U^i, \, \Psi_{H_u}, \cdots$



$$\lambda_u^{ij} = \underbrace{\Psi_Q^i(p)\Psi_U^j(p)\Psi_{H_u}}(p) + \cdots$$

(outer product)

See Beasley JJH Vafa II ‘08
And Hayashi et al. ‘09

$$\Rightarrow \begin{bmatrix} m_u & & \\ & m_c & \\ & & m_t \end{bmatrix} = \begin{bmatrix} 0 & & \\ & 0 & \\ & & m \end{bmatrix}$$

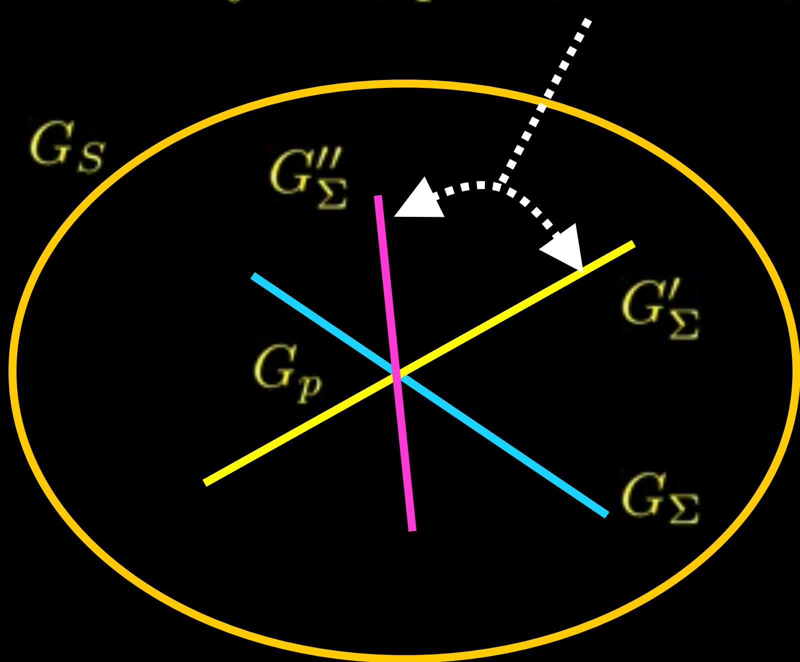
How Many Curves Are Touching?

Local Higgsing: $G_p \rightarrow G_S \times G_{def}$

Parameterized by Cartan(G_{def}) mod $\mathfrak{S}_{mono} \subset Weyl(G_{def})$

"Monodromy Group" identifies curves in the geometry

S:



Example:

$$y^2 = x^2 + z^2 + \alpha z + \beta$$

$$y^2 = x^2 + (z - t_+)(z - t_-)$$

Mono $\Rightarrow$ less factorization

More On Monodromy

Monodromy Important For Yukawas:

$$\lambda_{ij} \cdot 5_H \times 10_M^{(i)} \times 10_M^{(j)}$$

No Monodromy: $\lambda^{ij} = \begin{bmatrix} 0 & A & B \\ A' & 0 & C \\ B' & C' & 0 \end{bmatrix} \Rightarrow \geq 2$ heavy gens

$\Sigma_{(i)} \overset{\mathfrak{S}_{mono}}{\longleftrightarrow} \Sigma_{(j)} \Rightarrow 1$ heavy gen allowed

Hayashi et al. '09

$$+\cdots ?$$

Cecotti, Cheng, JJH, Vafa
To Appear

Geometry $\Rightarrow$ Rank 1

$+$ H-flux $\Rightarrow$ Rank 3

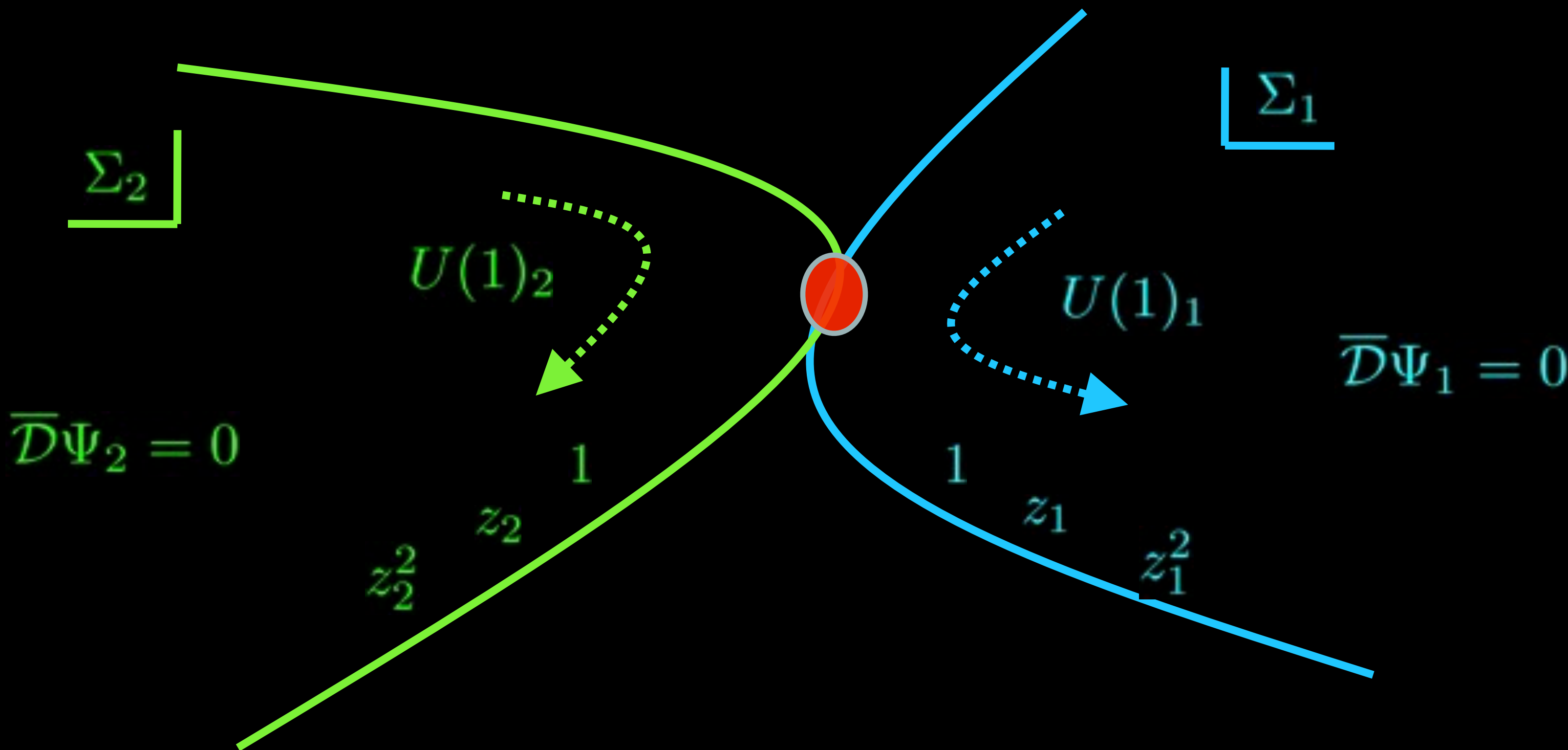
H-flux induces deformation in superpotential:

$$W = \int Tr(\overline{\partial} A^{0,1} + A^{0,1} * A^{0,1}) * \phi^{2,0}$$

Non-Comm. Deformation

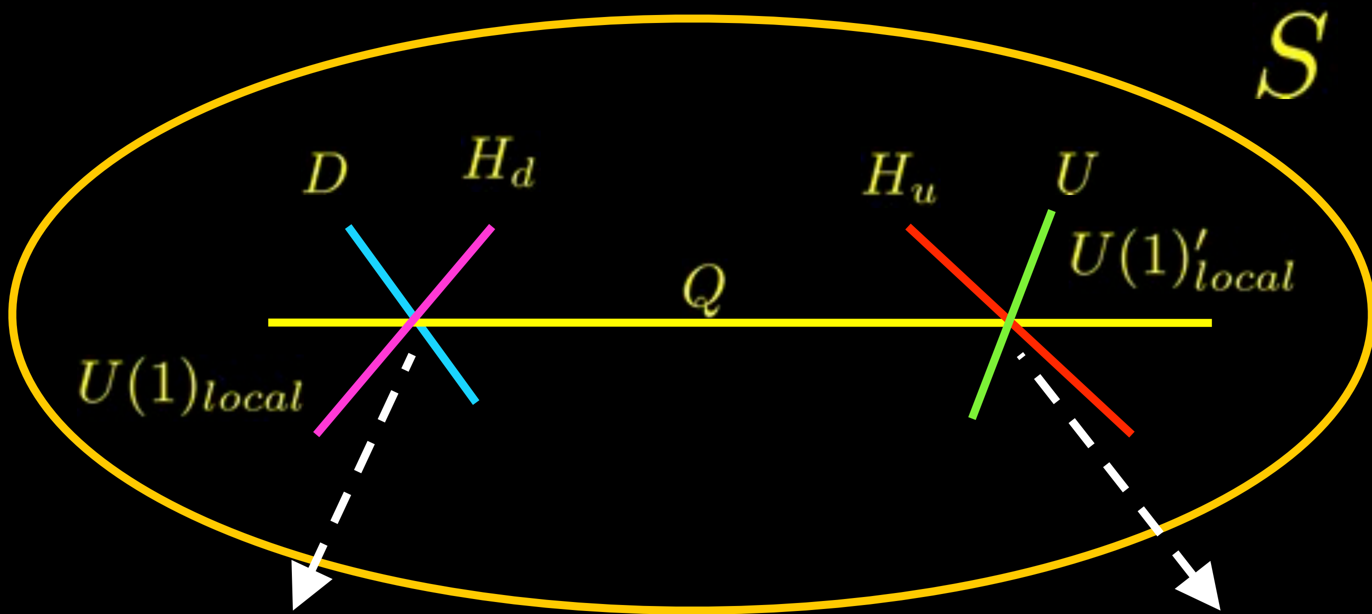
Main Idea

$U(1)$ Froggatt-Nielsen Geometrized:



Fluxes violate $U(1)$ selection rule $\Rightarrow$ hierarchical corrections

$U(1)$ Selection Rules

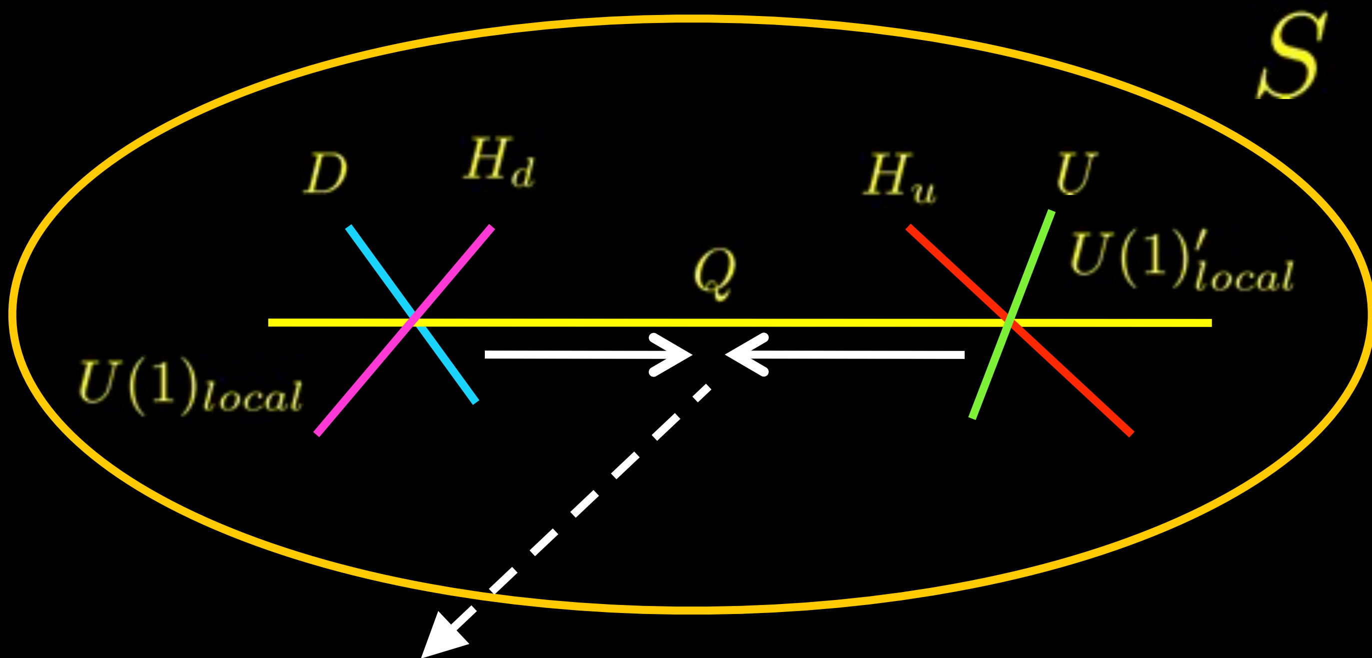


$$\lambda_d \sim \begin{bmatrix} \varepsilon_D^5 & \varepsilon_D^4 & \varepsilon_D^3 \\ \varepsilon_D^4 & \varepsilon_D^3 & \varepsilon_D^2 \\ \varepsilon_D^3 & \varepsilon_D^2 & 1 \end{bmatrix} \qquad T_L \cdot \lambda_u \cdot T_R^\dagger \sim \begin{bmatrix} \varepsilon_U^8 & \varepsilon_U^6 & \varepsilon_U^4 \\ \varepsilon_U^6 & \varepsilon_U^4 & \varepsilon_U^2 \\ \varepsilon_U^4 & \varepsilon_U^2 & 1 \end{bmatrix}$$

IF $U(1)_{local} \neq U(1)'_{local}$: No CKM Hierarchy

$$p_{down} \rightarrow p_{up}$$

$U(1)_{local} \rightarrow U(1)'_{local} \Rightarrow$ CKM Hierarchy



$$|V_{CKM}| \sim \begin{bmatrix} 1 & \varepsilon & \varepsilon^3 \\ \varepsilon & 1 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & 1 \end{bmatrix}$$

$E_6 : 5_H 10_M 10_M$

E_7

$SO(12) : \overline{5}_H \overline{5}_M 10_M$

CKM Matrix

$$|V_{CKM}| \sim \begin{bmatrix} 1 & \varepsilon & \varepsilon^3 \\ \varepsilon & 1 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & 1 \end{bmatrix} \qquad \varepsilon^2 \sim Flux^2/M_*^4$$

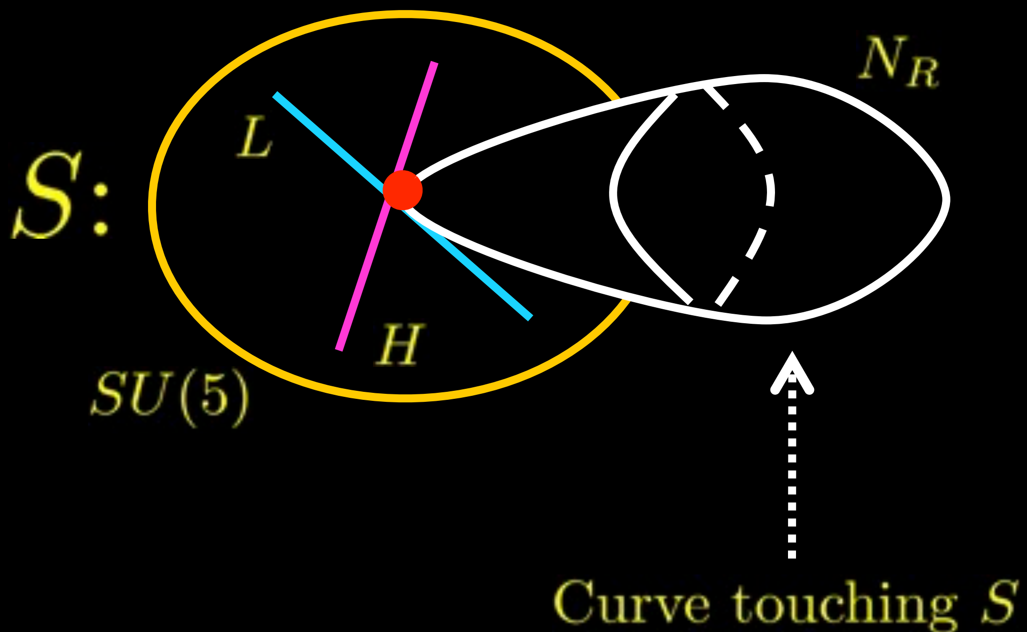
$$\sim Vol(S)^{-1}/M_*^4 \sim \alpha_{GUT}$$

$$\Downarrow$$

$$|V_{CKM}^{F-th}| \sim \begin{bmatrix} 1 & 0.2 & 0.008 \\ 0.2 & 1 & 0.04 \\ 0.008 & 0.04 & 1 \end{bmatrix}$$

$$|V_{CKM}^{obs}| \sim \begin{bmatrix} 0.97 & 0.23 & 0.004 \\ 0.23 & 0.97 & 0.04 \\ 0.008 & 0.04 & 0.99 \end{bmatrix}$$

Neutrinos



$$H = 7_{GUT} \cap 7'_{\perp}$$

$$L = 7_{GUT} \cap 7''_{\perp}$$

$$\boxed{N_R = 7'_{\perp} \cap 7''_{\perp}}$$

Beasley, JJH, Vafa '09
Bouchard, JJH, Seo Vafa '09
(see also Tatar, Tsuchiya, Watari '09)

KK & Neutrinos

$m_\nu \sim \frac{M_{weak}^2}{\Lambda_{UV}}$, Λ_{UV} Near GUT scale

Integrating out KK Modes Yields Seesaws:

Majorana: $\int d^2\theta \frac{(H_u L)^2}{\Lambda_{UV}} \longrightarrow m_\nu N_L N_L$

$$\langle H \rangle \sim M_w + M_w^2 \theta^2$$

Dirac: $\int d^4\theta \frac{H_d^\dagger L N_R}{\Lambda_{UV}} \longrightarrow m_\nu N_L N_R$

Heavy States & $U(1)$

Integrating out Heavy States $\Rightarrow$ Neutrinos Light

Bouchard, JJH, Seo Vafa '09

$\Psi_{HEAVY} \neq$ Zero Mode $\Rightarrow z, \overline{z}$ both contribute

Bigger $U(1)$ Violation $\Rightarrow$ Less Hierarchy:

$$\lambda_\nu \sim \begin{bmatrix} \varepsilon_N^2 & \varepsilon_N^{3/2} & \varepsilon_N^1 \\ \varepsilon_N^{3/2} & \varepsilon_N^1 & \varepsilon_N^{1/2} \\ \varepsilon_N^1 & \varepsilon_N^{1/2} & 1 \end{bmatrix} \qquad T_L \cdot \lambda_l \cdot T_R^\dagger \sim \begin{bmatrix} \varepsilon_L^8 & \varepsilon_L^6 & \varepsilon_L^4 \\ \varepsilon_L^6 & \varepsilon_L^4 & \varepsilon_L^2 \\ \varepsilon_L^4 & \varepsilon_L^2 & 1 \end{bmatrix}$$

ν Masses

ν_3

$m_{\nu_1} : m_{\nu_2} : m_{\nu_3} \sim \varepsilon_N^2 : \varepsilon_N : 1 \Rightarrow$ "Normal Hierarchy"

ν_2

ν_1

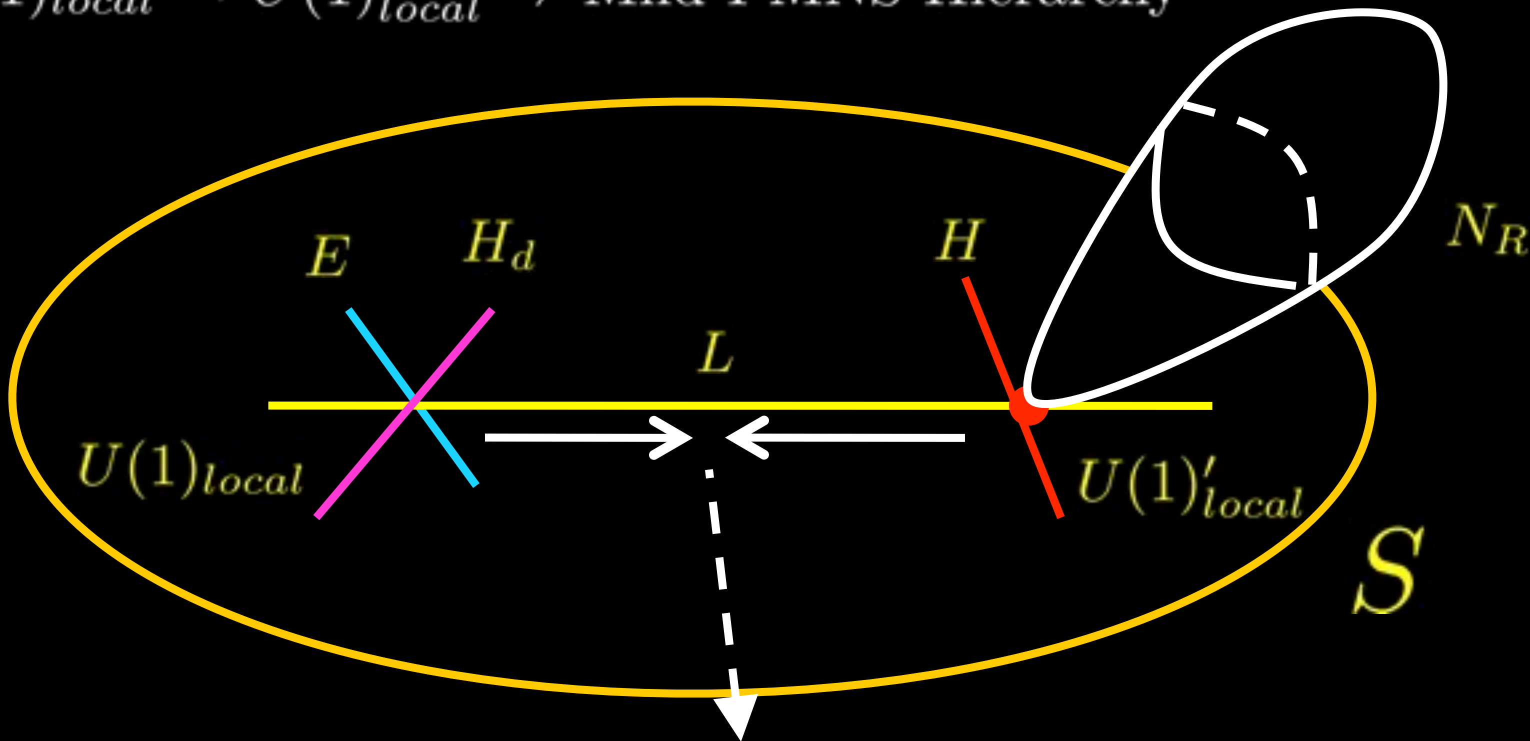
Predict: $\frac{m_{\nu_2}^2 - m_{\nu_1}^2}{m_{\nu_3}^2 - m_{\nu_2}^2} \sim \alpha_{GUT} \sim 0.04$

Close!

Observe: $\frac{m_{\nu_2}^2 - m_{\nu_1}^2}{m_{\nu_3}^2 - m_{\nu_2}^2} = \frac{m_{sol}^2}{m_{atm}^2} \sim 0.03$

ν Mixing Hierarchy

$U(1)_{local} \rightarrow U(1)'_{local} \Rightarrow$ Mild PMNS Hierarchy



$$|V_{PMNS}| \sim \begin{bmatrix} U_{e1} & \varepsilon^{1/2} & \varepsilon \\ \varepsilon^{1/2} & U_{\mu 2} & \varepsilon^{1/2} \\ \varepsilon & \varepsilon^{1/2} & U_{\mu 3} \end{bmatrix}$$

PMNS Matrix

Bouchard, JJH, Seo Vafa '09

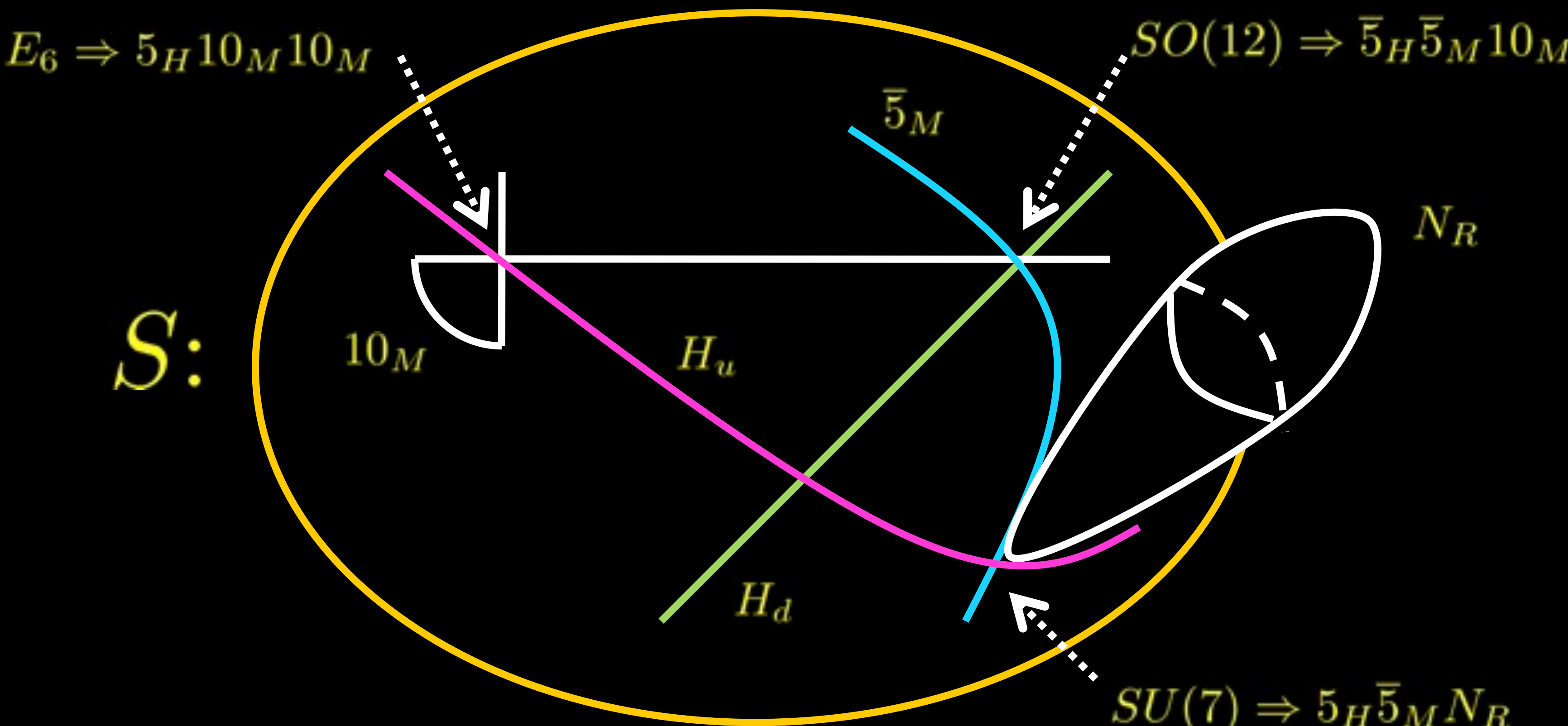
$$\begin{array}{cc} & \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ \left|V_{PMNS}^{F-th}\right| \sim & \begin{bmatrix} 0.87 & 0.45 & \boxed{0.2} \\ 0.45 & 0.77 & 0.45 \\ 0.2 & 0.45 & 0.87 \end{bmatrix} \begin{array}{c} \nu_e \\ \nu_\mu \\ \nu_\tau \end{array} \end{array}$$

$$\left|V_{PMNS}^{obs(3\sigma)}\right| \sim \begin{bmatrix} 0.77 - 0.86 & 0.50 - 0.63 & \boxed{0.00 - 0.22} \\ 0.22 - 0.56 & 0.44 - 0.73 & 0.57 - 0.80 \\ 0.21 - 0.55 & 0.40 - 0.71 & 0.59 - 0.82 \end{bmatrix}$$

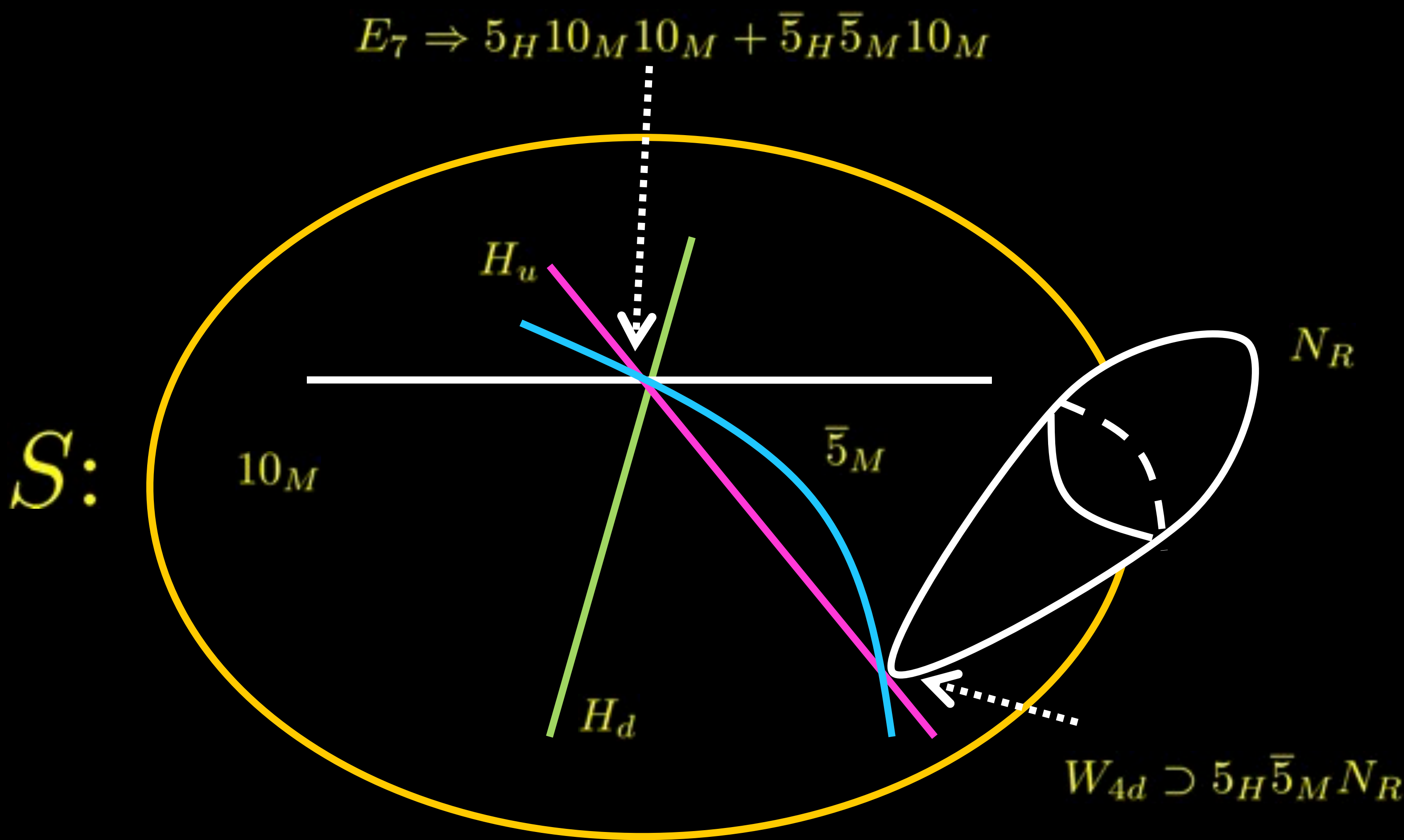
$\Rightarrow$ Predict $V_{PMNS}^{1,3}$ close to current bound

Point Unification

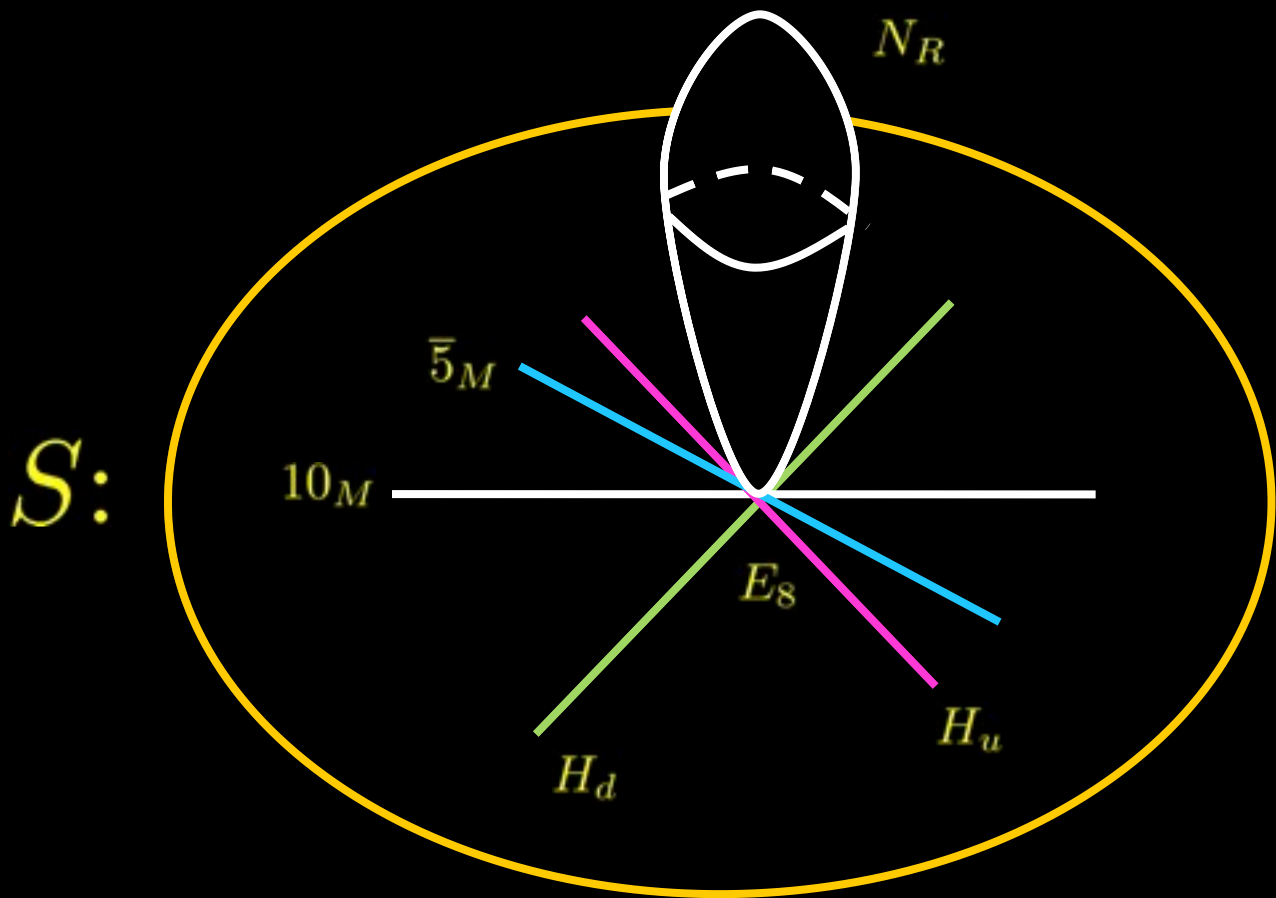
Beasley JJH Vafa II ’08



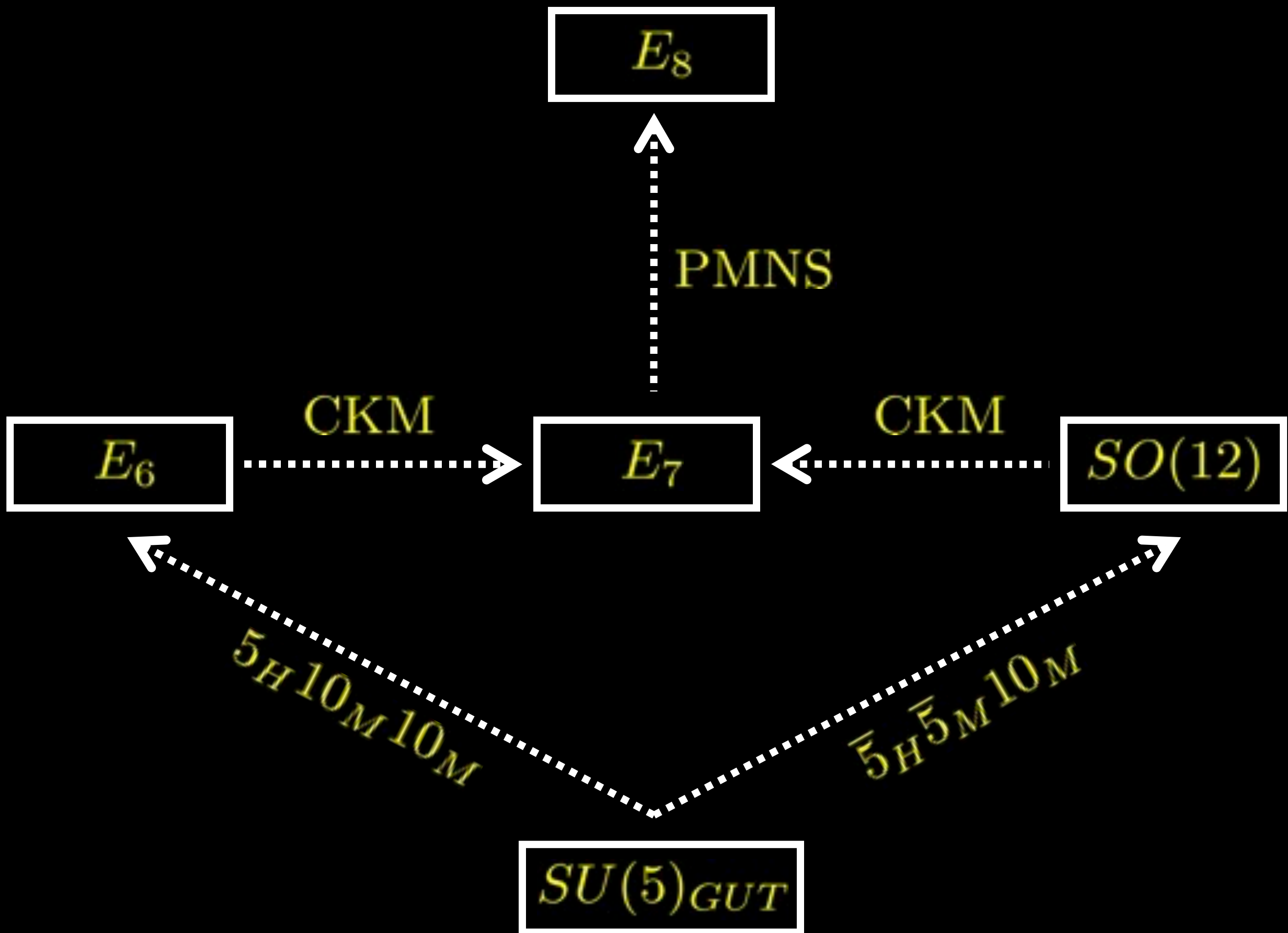
Point Unification



Point Unification



CKM + PMNS $\Rightarrow E_8$



Roadmap

- Flavor

- The Point of E_8

Left-Overs?

E_8 is a BIG group:

$E_8 \supset SU(5)_{GUT} \times SU(5)_{\perp}$

$$248 \rightarrow (5_G, 10_{\perp}) + (\overline{5}_G, \overline{10}_{\perp}) + (10_G, \overline{5}_{\perp}) + (\overline{10}_G, 5_{\perp}) + adj$$

Monodromy identifies many states, leaving only a few:

Matter 5 & 10 curves

Extra $U(1)$'s such as $U(1)_{PQ}$ and $U(1)_{B-L}$

Classification

Assumptions:

1) Hierarchical CKM and PMNS

2) $\exists\, U(1)_{PQ}$ such that $\mu_{bare} \int d^2\theta H_u H_d$ excluded

3) μ_{eff} from either: $\int d^2\theta S H_u H_d$ or $\int d^4\theta \frac{X^\dagger H_u H_d}{\Lambda_{UV}}$

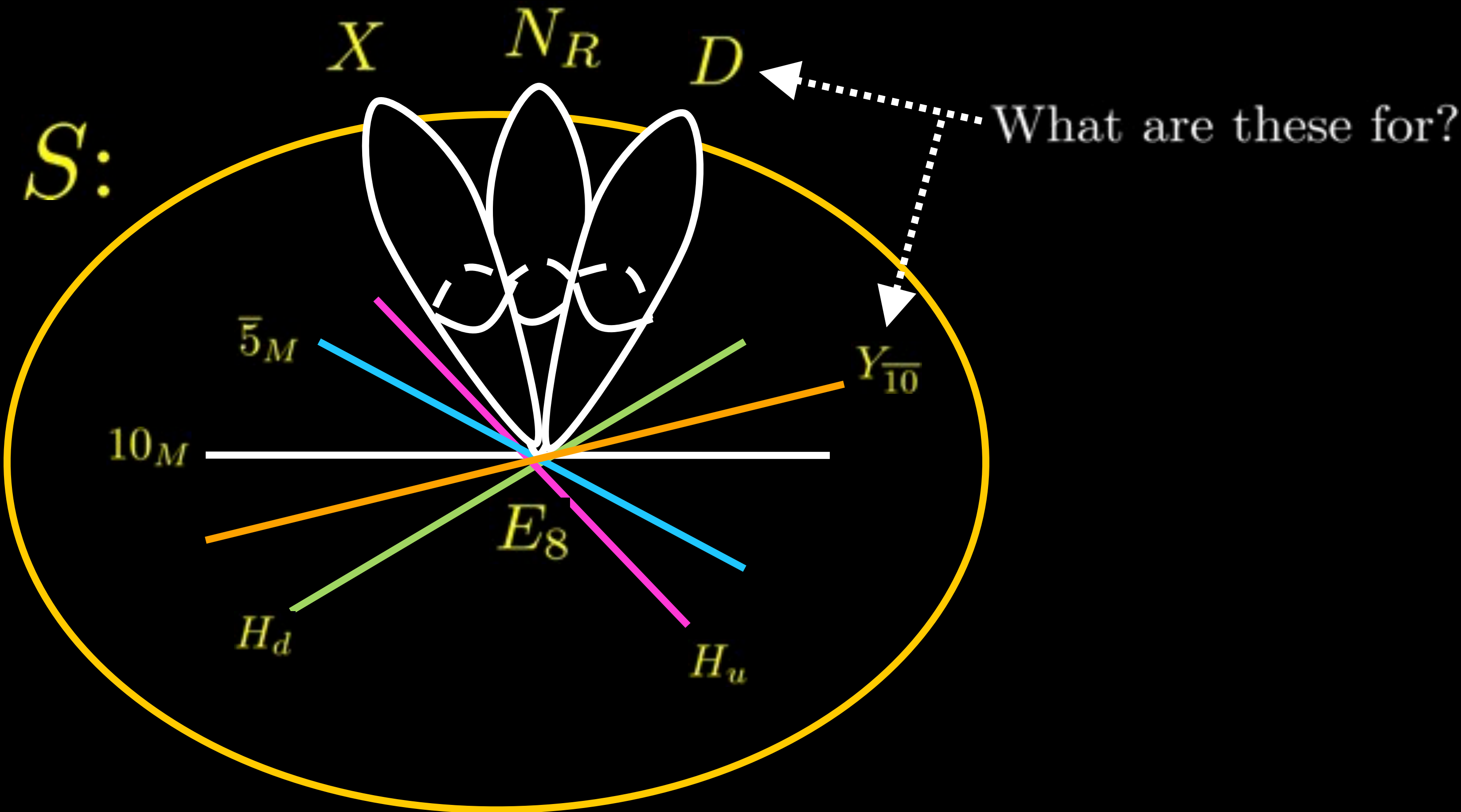
Then:

$\mathfrak{S}_{mono} = \mathbb{Z}_2, \mathbb{Z}_2 \times \mathbb{Z}_2, \mathbb{Z}_3, S_3$ (Dirac ν & $U(1)_{PQ} \times U(1)_{B-L}$)

$\mathfrak{S}_{mono} = \mathbb{Z}_2 \times \mathbb{Z}_2, Dih_4$ (Majorana ν & $U(1)_{PQ}$)

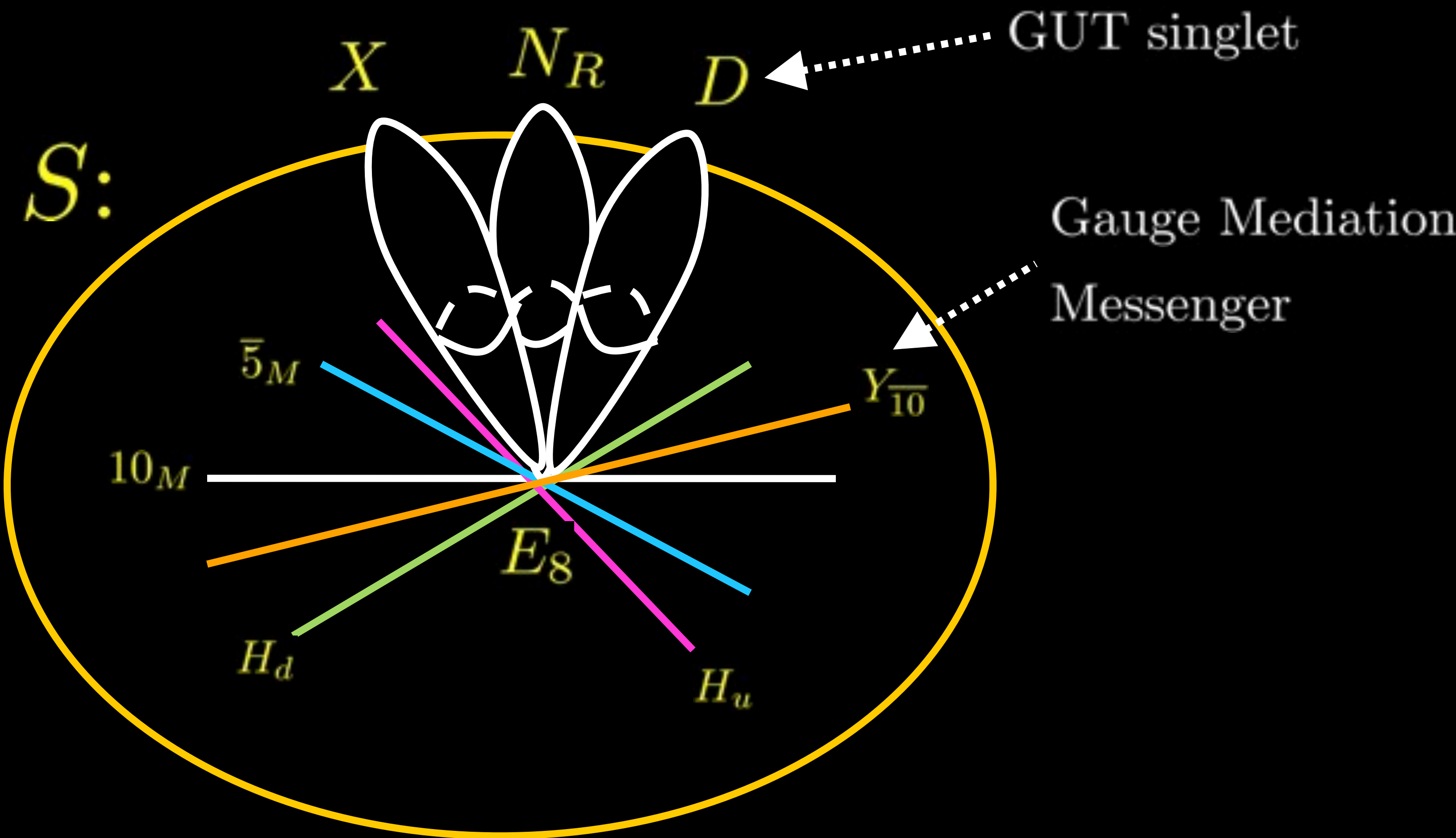
Dih_4 Curves

Two Extra Curves?



Dih_4 Curves

Two Extra Curves?



~~SUSY~~ Mediation

min. gauge mediation review:

$\langle X \rangle = M + \theta^2 F$

$\boxed{M_{\cancel{SUSY}}}$

$\int d^2\theta X Y_R Y'_{\overline{R}}$

$\boxed{\text{Messengers}}$

$m_{soft} \sim \frac{\alpha_{YM}}{4\pi}\frac{F}{M}$

$\boxed{M_{sparticle} \sim 1 \text{ TeV}}$

Messengers already part of the E_8 point

A Surprise: In nearly all cases messengers in $10 \oplus \overline{10}$

Energy Scales

$$\langle X \rangle = M + \theta^2 F$$

$\sqrt{F} =$ ~~SUSY~~ scale

$M =$ Global $U(1)_{PQ}$ symm. breaking scale $= f_{axion}$

μ term from: $\int d^4\theta \frac{X^\dagger H_u H_d}{\Lambda_{UV}} \longrightarrow F \sim 10^{17}\ \mathrm{GeV}^2$

$m_{soft} \sim \frac{\alpha_{YM}}{4\pi} \frac{F}{M} \sim 0.1 - 1\ \mathrm{TeV} \longrightarrow M \sim 10^{12}\ \mathrm{GeV}$

PQ Deformed GMSB

JJH, Vafa ‘08

String Theory $\Rightarrow U(1)_{PQ}$ gauge boson

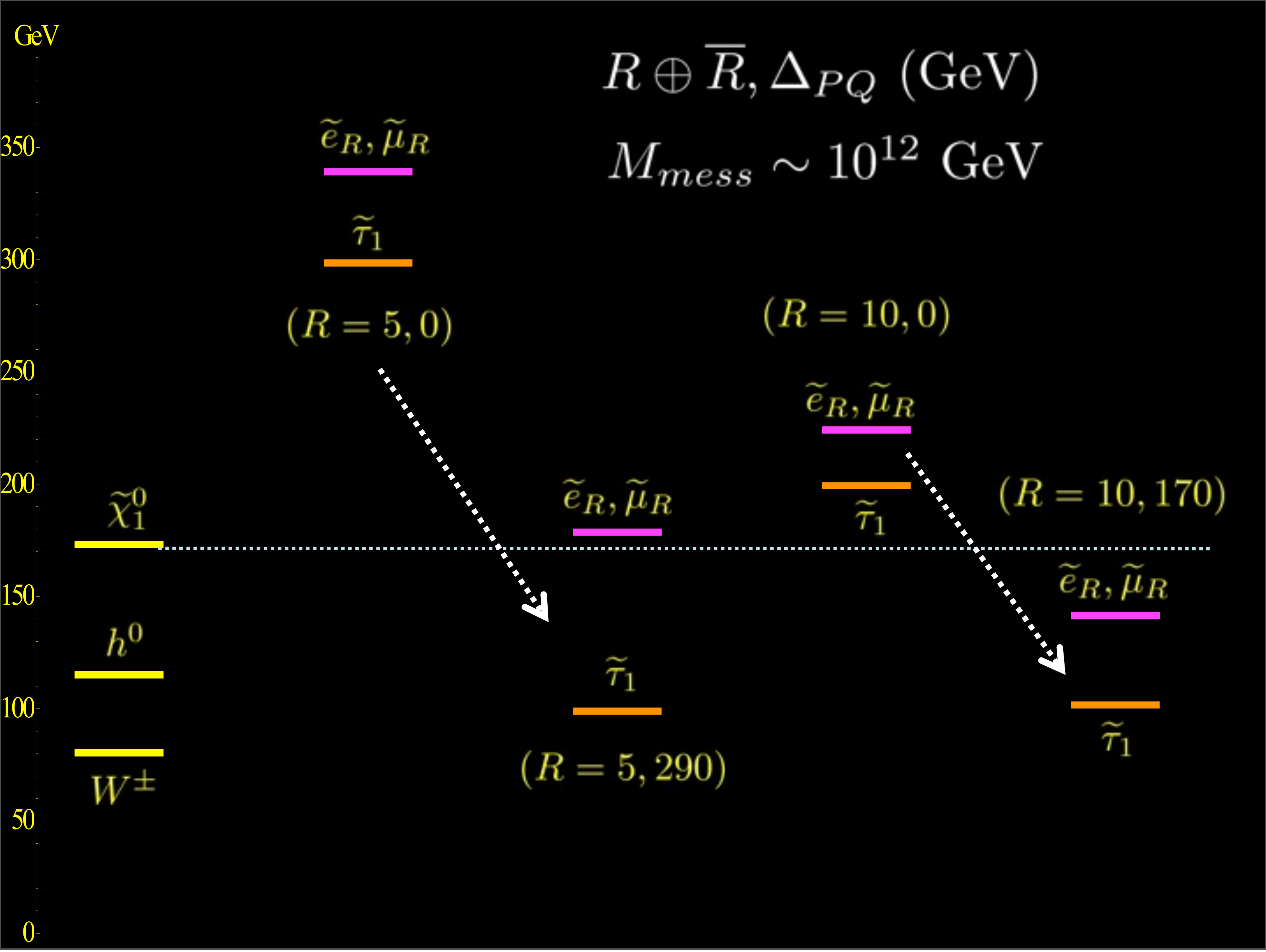
Heavy $U(1)_{PQ}$ exchange $\Rightarrow$

(see e.g. Arkani-Hamed, Dine, Martin ’97)

@ UV: $m^2_{soft} = m^2_{mGMSB} - q\Delta^2_{PQ}$

For most common $10 \oplus \overline{10}$ scenario
$\widetilde{\tau}_1$ is typically the quasi-stable NLSP

$\Rightarrow$ charged track at LHC


GeV
$R \oplus \overline{R}, \Delta_{PQ}$ (GeV)
$M_{mess} \sim 10^{12}$ GeV
$\tilde{e}_R, \tilde{\mu}_R$
$\tilde{\tau}_1$
$(R = 5, 0)$
$(R = 10, 0)$
$\tilde{e}_R, \tilde{\mu}_R$
$\tilde{\tau}_1$
$\tilde{\chi}_1^0$
$(R = 10, 170)$
$\tilde{e}_R, \tilde{\mu}_R$
$\tilde{e}_R, \tilde{\mu}_R$
h^0
$\tilde{\tau}_1$
$\tilde{\tau}_1$
$W^{\pm}$
$(R = 5, 290)$
350
300
250
200
150
100
50
0

Conclusions

- Bottom Up GUTs and F-theory
- Flavor $\Rightarrow E_8$
- $10 \oplus \overline{10}$ Messengers Most Common
- Global Models?