15th European Workshop on String Theory

Zürich, September 9th, 2009

Schrödinger black holes with extremal limits

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based on `arXiv:0907.1892` (with *Aninda Sinha*) - see also `arXiv:0808.1271`

Outline

Introduction and motivations

AdS/CFT and *experimentally testable systems*

Applications to *condensed matter systems*

Non-relativistic *superfluid phases* and *(non)Fermi liquids*?

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Thermodynamics of non-relativistic dual

Extremal black hole and *near horizon* limit

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Phase structure

Schrödinger soliton

A novel *phase transition* at *zero temperature*

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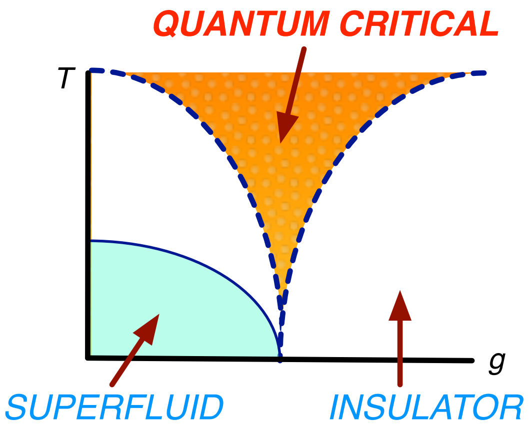

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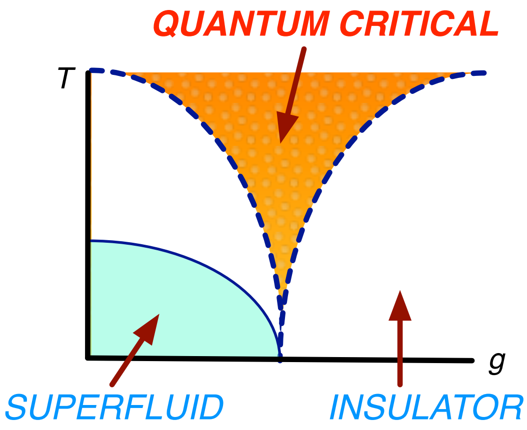

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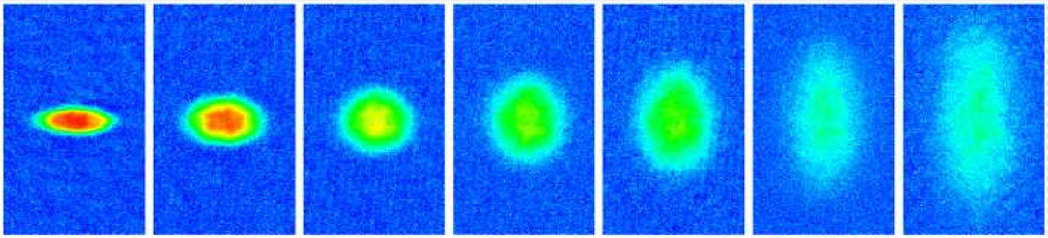

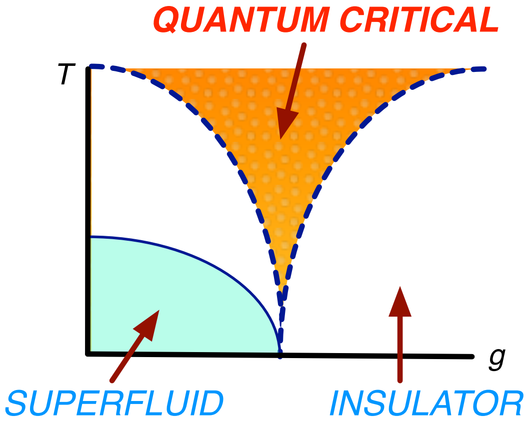

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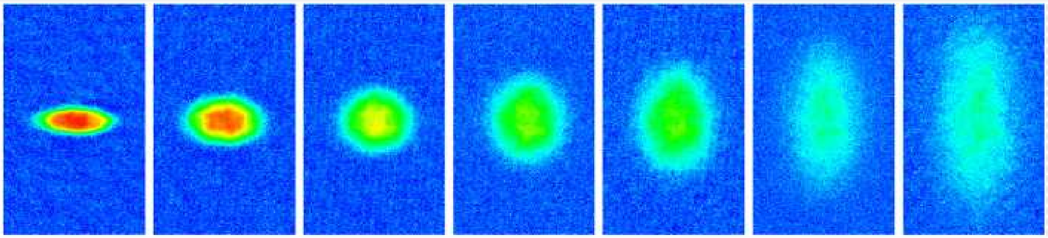

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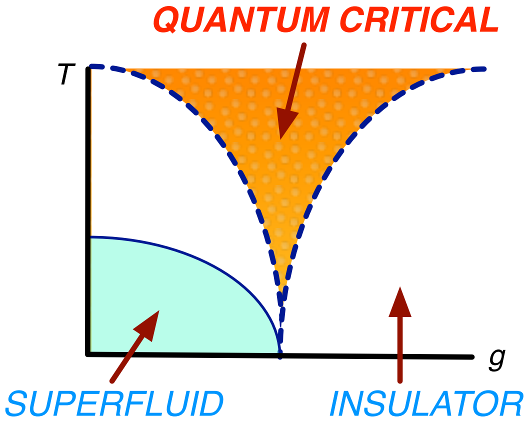

MOST SYSTEMS ARE ***NON-RELATIVISTIC***

Motivations

Study ***non-relativistic systems*** at ***finite temperature / density***

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Derive and study charged black hole solution dual to non-relativistic scale invariant (2+1)-dimensional conformal field theory admitting an extremal limit ($T = 0$)

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[Gubser, Hartnoll-Herzog-Horowitz, ...]

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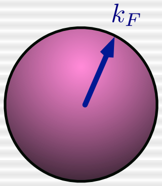
  → ***(non-)Fermi liquids***

  [Liu-McGreevy-Vegh, Faulkner-Liu-McGreevy-Vegh]



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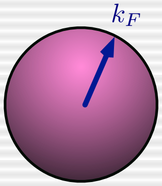

Do these constructions generalize to ***non-relativistic systems*** ?

Non-relativistic AdS/CFT

Symmetry of *non-relativistic "conformal" systems* is ***Schrödinger algebra***

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Sch_d

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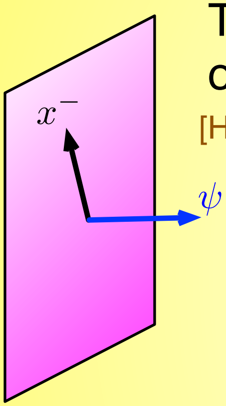

This suggests ***string dual*** obtained via ***TsT transformation*** of relativistic dual along x^- and R-symmetry direction ψ

[Herzog-Rangamani-Ross, Maldacena-Martelli-Tachikawa, Adams-Balasubramanian-McGreevy]

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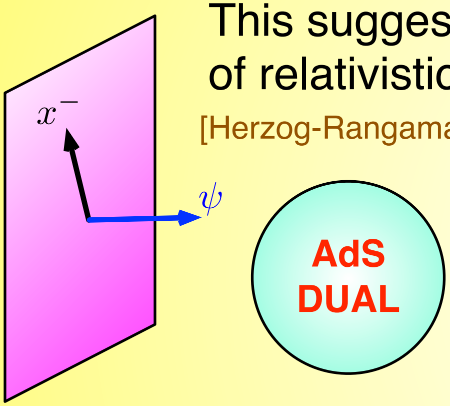

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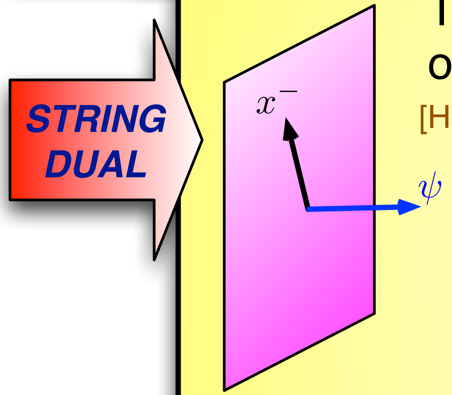

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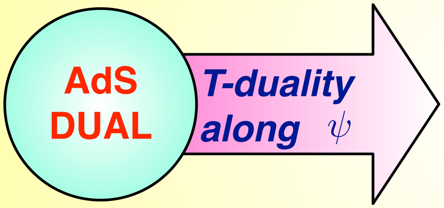

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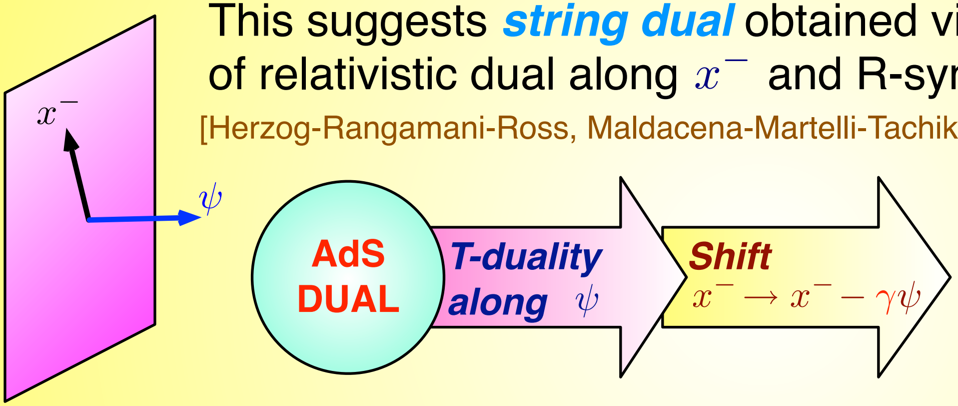

AdS DUAL

T-duality along ψ

Shift $x^- \to x^- - \gamma\psi$

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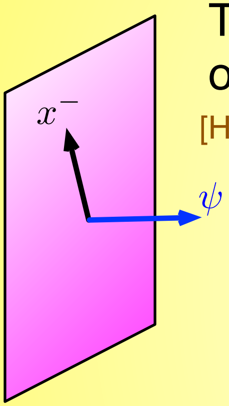

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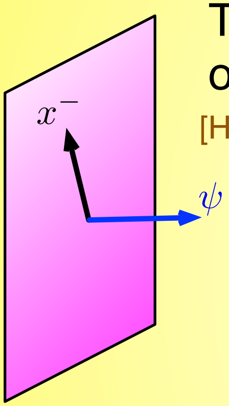

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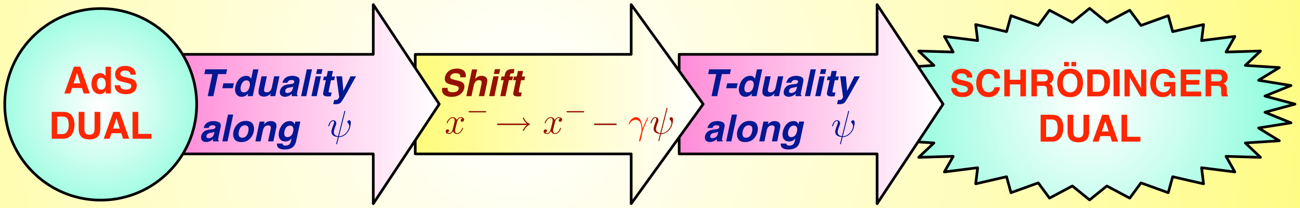

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DIPOLE FIELD THEORY $f g \to f \star g = e^{i\pi\gamma(P_-^f Q_\psi^g - Q_\psi^f P_-^g)} f g$

LIGHT-CONE MOMENTUM *R-CHARGE*

[Lunin-Maldacena]

Non-relativistic AdS/CFT

Obtain ***non-relativistic ground state*** via ***TsT*** of $\mathrm{AdS}_5 \times S^5$

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$\mathrm{AdS}_5 \times S^5$

$$ds^2 = \frac{r^2}{l^2}\left(-4dx^+dx^- + d\vec{y}^2\right) + l^2\left((d\psi + P)^2 + ds^2_{\mathbb{CP}^2}\right)$$

$$F_5 = \ldots$$

Non-relativistic AdS/CFT

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S^5 AS $U(1)$ FIBRATION ON $\mathbb{CP}^2$

Non-relativistic AdS/CFT

Obtain *non-relativistic ground state* via *TsT* of $\mathrm{AdS}_5 \times S^5$

General explicit formulae for *TsT transformation*

Undeformed

$e_{\mu\nu} = g_{\mu\nu} + b_{\mu\nu} \qquad \phi$

$f_p = dc_{p-1} + db \wedge c_{p-3}$

T-duality along φ^1 → **Shift** $\varphi^2 \to \varphi^2 + \gamma\varphi^1$ → **T-duality along** φ^1 →

Deformed

$E_{\mu\nu} = G_{\mu\nu} + B_{\mu\nu} \qquad \Phi$

$\mathcal{F}_p = dC_{p-1} + dB \wedge C_{p-3}$

NS-NS fields

$$E_{\mu\nu} = \mathcal{M}\left\{ e_{\mu\nu} - \gamma \left[\det\begin{pmatrix} e_{12} & e_{1\nu} \\ e_{\mu 2} & e_{\mu\nu} \end{pmatrix} - \det\begin{pmatrix} e_{21} & e_{2\nu} \\ e_{\mu 1} & e_{\mu\nu} \end{pmatrix} \right] + \gamma^2 \det\begin{pmatrix} e_{11} & e_{12} & e_{1\nu} \\ e_{21} & e_{22} & e_{2\nu} \\ e_{\mu 1} & e_{\mu 2} & e_{\mu\nu} \end{pmatrix} \right\}$$

$$e^{2\Phi} = \mathcal{M}\, e^{2\phi}$$

$$\mathcal{M} = \left\{ 1 - \gamma\,(e_{12} - e_{21}) + \gamma^2 \det\begin{pmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{pmatrix} \right\}^{-1}$$

R-R fields

$$\sum_q \mathcal{F}_q \wedge e^B = \sum_q f_q \wedge e^b + \gamma\, \iota_{\varphi^1}\, \iota_{\varphi^2} \left[\sum_q f_q \wedge e^b \right]$$

[E.I.]

Non-relativistic AdS/CFT

Obtain ***non-relativistic ground state*** via ***TsT*** of $\mathrm{AdS}_5 \times S^5$

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[Son, Balasubramanian-McGreevy]

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⇨ ***TsT transformation*** of ***AdS-Schwarzschild black hole***

γ becomes physical parameter ⟺ *chemical potential*

Study *thermodynamics*, *hydrodynamics*, ...

[Herzog-Rangamani-Ross, Maldacena-Martelli-Tachikawa, Adams-Balasubramanian-McGreevy]

Charged Schrödinger black hole

Use *TsT transformation* to get ***Schrödinger black hole*** admitting an *extremal limit*

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Charged $Q \neq 0$
AdS-Reissner-Nordström black hole

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$$ds^2 = \frac{r^2}{l^2} \left(-f dt^2 + dx^2 + dy^2 + dz^2\right) + \frac{l^2}{r^2}\frac{dr^2}{f}$$

$$+ l^2 \left((d\psi + P + A)^2 + ds^2_{\mathbb{CP}^2}\right)$$

$$F_5 = (1 + \star) \left[\left(-\frac{4r^3}{l^4} dt \wedge dr - \frac{2Q}{l^4}\,\omega_{\mathbb{CP}^2}\right) \wedge dx \wedge dy \wedge dz\right]$$

$$f(r) = \left(1 - \frac{r_0^2}{r^2}\right)\left(1 + \frac{r_0^2}{r^2} - \frac{Q^2}{r_0^2 r^4}\right) \qquad A = A_t\, dt = \frac{Q}{l^2}\left(\frac{1}{r_0^2} - \frac{1}{r^2}\right) dt$$

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DOUBLE ZERO AT HORIZON

CHARGE Q

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T-duality along ψ

Shift $x^- \to x^- - \gamma\psi$

T-duality along ψ

γ*-deformed*
black hole with
Schrödinger asymptotics

Charged Schrödinger black hole

$$ds^2 = \frac{r^2}{l^2}\mathcal{M}\left(-f\,dt^2 + dx^2 - \gamma^2 r^2 f (dt + dx)^2\right) + \frac{r^2}{l^2}\left(dy^2 + dz^2\right) + \frac{l^2}{r^2}\frac{dr^2}{f}$$

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$$e^{2\Phi} = \mathcal{M}$$

$$B = -\gamma r^2 \mathcal{M}\left(f\,dt + dx\right) \wedge \left(d\psi + P + A\right)$$

$$\boxed{\mathcal{M} = \left(1 + \gamma^2 r^2 (1 - f)\right)^{-1}}$$

$$C_2 = -\gamma l^2 A_t\, \omega_{\mathbb{CP}^2}$$

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[*also* Adams-Brown-DeWolfe-Rosen]

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Black hole ***thermodynamics*** mostly inherited via ***TsT transformation***

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AGREES WITH γ ***- INDEPENDENT*** *HORIZON AREA*

Can also compute ***charges*** $P_i = \partial W/\partial \mu_i$
and ***energy*** $E = W + TS - \mu_i P_i$

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$$\mu_2 = \lim_{r\to\infty} A_u = \frac{1}{2\gamma l}\frac{Q}{l^2 r_0^2}$$

Partition function $Z(T,\mu_i) = e^{-W(T,\mu_i)/T} \simeq e^{-I_{\rm E}}$

$$I_{\rm E} = I_{\rm IIB} + \text{counterterms}$$

GIBBS POTENTIAL

EUCLIDEAN ON-SHELL ACTION

$$W = I_{\rm E}\,T = -\frac{\Delta v V r_0^4}{2\kappa^2}\left(1+\frac{Q^2}{r_0^6}\right)$$

Entropy

$$S = -\left.\frac{\partial W}{\partial T}\right|_{\mu_1,\mu_2} = \frac{\Delta x V\, 2\pi l^2 r_0^3}{\kappa^2}$$

AGREES WITH γ *-* ***INDEPENDENT*** *HORIZON AREA*

Can also compute ***charges*** $P_i = \partial W/\partial\mu_i$
and ***energy*** $E = W + TS - \mu_i P_i$

Equation of state

$$PV = E$$

Thermodynamics

Black hole ***thermodynamics*** mostly inherited via ***TsT transformation***

HOWEVER *Care needed with the identification of* ***CFT time*** [Herzog-Rangamani-Ross]

Introduce coordinates that ***eliminate*** γ from ***asymptotic Schrödinger metric***

$$u = \gamma l\,(t+x) \qquad v = \frac{1}{2\gamma l}\,(t-x)$$

CFT TIME

PERIODIC $v \leftrightarrow$ *DISCRETE NUMBER DENSITY*

Temperature

$$T = \frac{1}{\gamma l}\frac{r_0}{\pi l^2}\left(1-\frac{Q^2}{2r_0^6}\right)$$

Chemical potential $\leftrightarrow \gamma$

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APPROPRIATE FOR A ***NON-RELATIVISTIC 2D CFT***

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from ***asymp***

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$$T = \frac{1}{\gamma l}\,\frac{r}{\pi}$$

ER DENSITY

$l \leftrightarrow Q$

$$\frac{1}{2\gamma l}\,\frac{Q}{l^2 r_0^2}$$

Partition func

GIB

terms

We also computed the
shear viscosity to *entropy density* ratio
finding the universal value

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

even at *zero temperature*

[*compare* Kovtun-Son-Starinets, Iqbal-Liu]

$$W = I_{\rm E}\, T = -\frac{\Delta x V r_0}{2\kappa^2}\left(1 + \frac{Q}{r_0^6}\right)$$

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Extremal black hole and near-horizon limit

Extremal limit $Q = \sqrt{2}\, r_0^3$ corresponding to ***zero temperature***

Admits ***near horizon limit*** [Reall, Kunduri-Lucietti-Reall]

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$$\Big\downarrow \quad r=r_0+\rho$$

Near-horizon limit

$$ds_5^2=-\frac{12\rho^2}{l^2}dt^2+\frac{l^2}{12\rho^2}d\rho^2 +\frac{r_0^2}{l^2}\left(dx^2+dy^2+dz^2\right)$$

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$$ds_5^2 = \frac{r_0^2}{l^2}\left(1 + \gamma^2(r_0^2 - 12\rho^2)\right)^{-1}\Big[- \frac{12\rho^2}{r_0^2}dt^2 + dx^2 - 12\gamma^2\rho^2(dt + dx)^2\Big] + \frac{l^2}{12\rho^2}d\rho^2 + \frac{r_0^2}{l^2}\left(dy^2 + dz^2\right)$$

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SAME AS STANDARD (RELATIVISTIC) DIPOLE THEORY!

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Universality of AdS_2 ?

A zero temperature phase transition

Schrödinger soliton obtained by *double Wick rotation* of $Q = 0$ **black hole** (or *TsT* of **AdS soliton** [Horowitz-Myers])

$$t_b \to i x_s \qquad x_b \to i t_s \qquad \gamma_b \to i \gamma_s$$

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$$2 r_s \gamma_s = 1$$

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AT ***T = 0*** :

Extremal
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Match periodicities of τ and x and deformation parameters $\gamma_b = \gamma_s$

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Difference of **Euclidean actions**

$$\Delta I = (I_s - I_b)|_{T=0} = -\frac{\Delta\tau\Delta x V\, l^6}{2\kappa^2\mu_1^2}\left(\mu_1^4 - 3\mu_2^4\right)$$

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⇒ $\mu_1 < 3^{1/4} \mu_2$ **BLACK HOLE**

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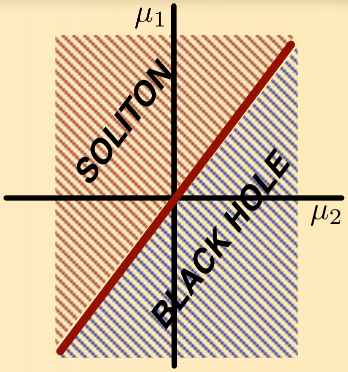


Also expected in *relativistic case*

Conclusions and outlook

In summary...

- ➡ We derived a ***charged black hole*** solution with ***Schrödinger asymptotic symmetry*** dual to a ***non-relativistic*** (2+1)-dimensional CFT at finite temperature / finite chemical potential that admits an ***extremal limit***
- ➡ The ***near-horizon geometry*** exhibits an AdS_2 ***factor*** (or ***not***)
- ➡ We studied ***thermodynamic*** / ***hydrodynamic*** properties of the solution
- ➡ We found a ***zero temperature phase transition*** between black hole and ***Schrödinger soliton***

Conclusions and outlook

In summary...

- ➡ We derived a ***charged black hole*** solution with ***Schrödinger asymptotic symmetry*** dual to a ***non-relativistic*** (2+1)-dimensional CFT at finite temperature / finite chemical potential that admits an ***extremal limit***
- ➡ The ***near-horizon geometry*** exhibits an AdS_2 ***factor*** (or ***not***)
- ➡ We studied ***thermodynamic*** / ***hydrodynamic*** properties of the solution
- ➡ We found a ***zero temperature phase transition*** between black hole and ***Schrödinger soliton***

Outlook

- ➡ Elucidate ***near-horizon geometry*** by studying the ***dual CFT***
- ➡ ***Non-relativistic*** version of (string embedded) ***holographic superconductors***
  [Gubser-Herzog-Pufu-Tesileanu, Gauntlett-Sonner-Wiseman]
- ➡ Emergence of ***Fermi surfaces***
- ➡ ***Phase structure*** in ***Schrödinger global coordinates*** [Blau-Hartong-Rollier, Yamada]
- ➡ $\ldots$

Summary slides follow

Introduction

Can ***string theory*** be used to access ***experimentally testable systems*** ?

The ***gauge/string duality*** is an invaluable tool to study ***strongly coupled*** physical systems

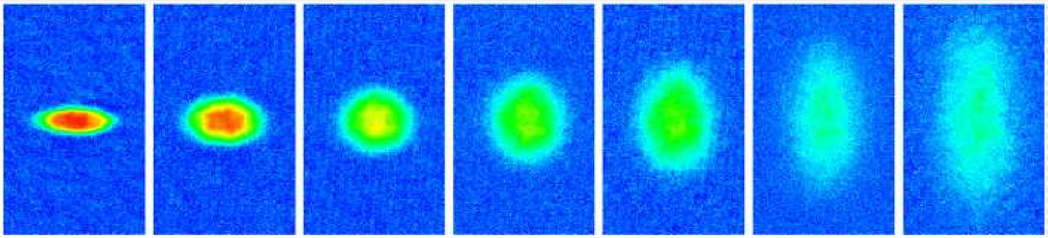
→ Applications to ***strong interactions*** (*confinement*, *χSB*, *quark-gluon plasma*, *...*)

→ Applications to ***condensed matter systems***

Many examples of ***strongly coupled*** ***scale invariant*** systems in condensed matter

Quantum criticality

- ***High -*** T_c ***superconductors***
- ***Fermions at unitarity*** (experimentally realized in ***trapped cold atoms***)



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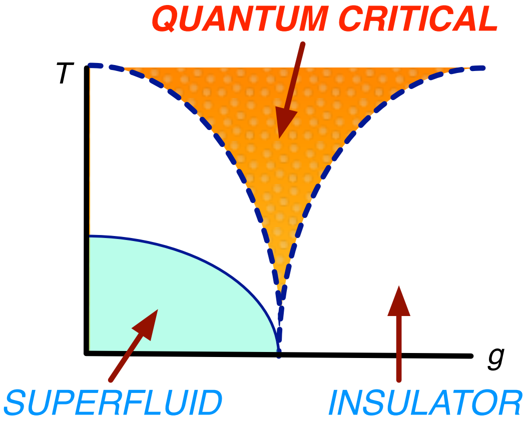


MOST SYSTEMS ARE
NON-RELATIVISTIC

Motivations

Study ***non-relativistic systems*** at ***finite temperature / density***

AdS/CFT → ***Black holes*** with ***non-relativistic asymptotic symmetry***

IN THIS TALK → *Derive and study charged black hole solution dual to non-relativistic scale invariant (2+1)-dimensional conformal field theory admitting an extremal limit ($T = 0$)*

WHY?

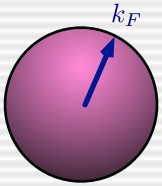
- Adding ***U(1) charge*** may allow spontaneous breaking by a ***condensate*** → ***superfluid phase***

  [Gubser, Hartnoll-Herzog-Horowitz, ...]

- ***Near-horizon geometry*** of ***charged extremal black holes*** exhibits an AdS_2 ***factor*** that has been shown to be related to the emergence of ***sharp Fermi surfaces*** in the dual theory

  → ***(non-)Fermi liquids***

  [Liu-McGreevy-Vegh, Faulkner-Liu-McGreevy-Vegh]



Do these constructions generalize to ***non-relativistic systems*** ?

Non-relativistic AdS/CFT

Symmetry of *non-relativistic "conformal" systems* is ***Schrödinger algebra***

Sch_d

GALILEAN SYMMETRIES *ROTATIONS* M_{ij} *TRANSLATIONS* H P_i *BOOSTS* K_i

DILATATIONS $x_i \to \lambda x_i \quad t \to \lambda^2 t$ ← *TIME AND SPACE SCALE* ***DIFFERENTLY***

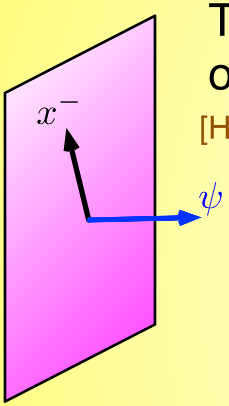
"SPECIAL CONFORMAL" $x_i \to \frac{x_i}{1+\mu t} \quad t \to \frac{t}{1+\mu t}$ *+ CENTRAL ELEMENT*

How does it arise from *relativistic* theory?

- Start with *conformal group* in one higher dimension
- Choose *light-cone* coordinates $x^{\pm} = \frac{1}{2}(t \pm x)$

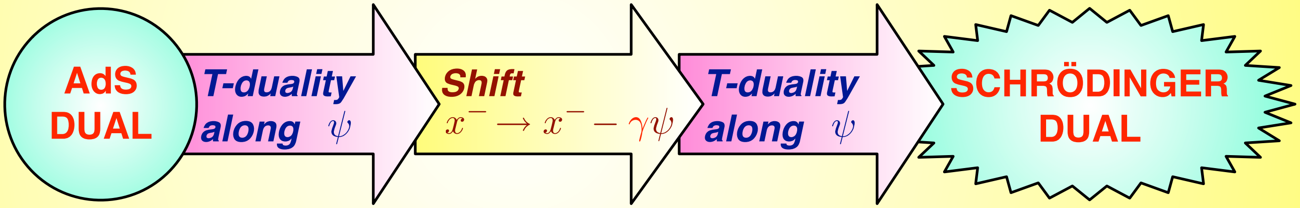
Sch_d ⟺ subalgebra of generators that ***commute with*** P_-

STRING DUAL



This suggests ***string dual*** obtained via ***TsT transformation*** of relativistic dual along x^- and R-symmetry direction ψ

[Herzog-Rangamani-Ross, Maldacena-Martelli-Tachikawa, Adams-Balasubramanian-McGreevy]



AdS DUAL → ***T-duality along*** ψ → ***Shift*** $x^- \to x^- - \gamma\psi$ → ***T-duality along*** ψ → **SCHRÖDINGER DUAL**

DIPOLE FIELD THEORY

$$fg \to f \star g = e^{i\pi\gamma(P_-^f Q_\psi^g - Q_\psi^f P_-^g)} fg$$

LIGHT-CONE MOMENTUM *R-CHARGE*

[Lunin-Maldacena]

Non-relativistic AdS/CFT

Obtain ***non-relativistic ground state*** via ***TsT*** of $\mathrm{AdS}_5 \times S^5$

$\mathrm{AdS}_5 \times S^5$

$$ds^2 = \frac{r^2}{l^2}\left(-4dx^+\boxed{dx^-} + d\vec{y}^2\right) + l^2\left(\left(\boxed{d\psi} + P\right)^2 + ds^2_{\mathbb{CP}^2}\right)$$

$$F_5 = \ldots$$

TsT

Schrödinger space-time

$$ds^2 = \frac{r^2}{l^2}\left(-2dudv - \frac{r^2}{l^2}du^2 + d\vec{y}^2\right) + l^2\left((d\psi + P)^2 + ds^2_{\mathbb{CP}^2}\right)$$

$$F_5 = \ldots \qquad B = -\frac{r^2}{l}du \wedge (d\psi + P)$$

Coordinates that ***eliminate*** γ $\qquad u = 2\gamma l x^+ \qquad v = x^-/(\gamma l)$

GALILEAN SCALING SYMMETRY $\quad \vec{y} \to \lambda\vec{y} \quad u \to \lambda^2 u \quad v \to v \quad r \to \lambda^{-1} r$

[Son, Balasubramanian-McGreevy]

COMPACT v ⟺ ***DISCRETE MASS / PARTICLE NUMBER***

Pathologies of DLCQ ameliorated by going to
finite temperature / ***finite density***

⇒ ***TsT transformation*** of ***AdS-Schwarzschild black hole***
γ becomes physical parameter ⟺ *chemical potential*
Study *thermodynamics*, *hydrodynamics*, ...

[Herzog-Rangamani-Ross, Maldacena-Martelli-Tachikawa, Adams-Balasubramanian-McGreevy]

Charged Schrödinger black hole

Use ***TsT transformation*** to get ***Schrödinger black hole*** admitting an ***extremal limit***

Charged $Q \neq 0$
AdS-Reissner-Nordström black hole

Charged AdS black hole

$$x^- = \frac{1}{2}(t - x)$$

$$ds^2 = \frac{r^2}{l^2}\left(\boxed{-f\,dt^2 + dx^2} + dy^2 + dz^2\right) + \frac{l^2}{r^2}\frac{dr^2}{f} + l^2\left(\boxed{(d\psi} + P + A)^2 + ds^2_{\mathbb{CP}^2}\right)$$

$$F_5 = (1+\star)\left[\left(-\frac{4r^3}{l^4}dt \wedge dr - \frac{2Q}{l^4}\,\omega_{\mathbb{CP}^2}\right) \wedge dx \wedge dy \wedge dz\right]$$

DOUBLE ZERO AT HORIZON

CHARGE Q

$$f(r) = \left(1 - \frac{r_0^2}{r^2}\right)\left(1 + \frac{r_0^2}{r^2} - \frac{Q^2}{r_0^2 r^4}\right)$$

$$A = A_t\,dt = \frac{Q}{l^2}\left(\frac{1}{r_0^2} - \frac{1}{r^2}\right)dt$$

Charged Schrödinger black hole

Use ***TsT transformation*** to get ***Schrödinger black hole*** admitting an ***extremal limit***

Charged $Q \neq 0$
AdS-Reissner-
Nordström black hole

T-duality along ψ

Shift $x^- \to x^- - \gamma\psi$

T-duality along ψ

γ*-deformed*
black hole with
Schrödinger asymptotics

Charged Schrödinger black hole

$$ds^2 = \frac{r^2}{l^2}\mathcal{M}\left(-f\,dt^2 + dx^2 - \gamma^2 r^2 f(dt+dx)^2\right) + \frac{r^2}{l^2}\left(dy^2+dz^2\right) + \frac{l^2}{r^2}\frac{dr^2}{f}$$
$$+\, l^2\left(\mathcal{M}\,(d\psi + P + A)^2 + ds^2_{\mathbb{CP}^2}\right)$$

$$F_5 = (1+\star)\left[\left(-\frac{4r^3}{l^4}dt\wedge dr - \frac{2Q}{l^4}\,\omega_{\mathbb{CP}^2}\right)\wedge dx\wedge dy\wedge dz\right] - H\wedge C_2$$

$$e^{2\Phi} = \mathcal{M}$$

$$B = -\gamma r^2 \mathcal{M}\,(f\,dt + dx)\wedge(d\psi + P + A)$$

$$\boxed{\mathcal{M} = \left(1+\gamma^2 r^2(1-f)\right)^{-1}}$$

$$C_2 = -\gamma l^2 A_t\,\omega_{\mathbb{CP}^2}$$

$$f(r) = \left(1-\frac{r_0^2}{r^2}\right)\left(1+\frac{r_0^2}{r^2}-\frac{Q^2}{r_0^2 r^4}\right) \qquad A = A_t\,dt = \frac{Q}{l^2}\left(\frac{1}{r_0^2}-\frac{1}{r^2}\right)dt$$

[*also* Adams-Brown-DeWolfe-Rosen]

Thermodynamics

Black hole ***thermodynamics*** mostly inherited via ***TsT transformation***

HOWEVER *Care needed with the identification of* ***CFT time*** [Herzog-Rangamani-Ross]

Introduce coordinates that ***eliminate*** γ from ***asymptotic Schrödinger metric***

$$u = \gamma l \, (t + x) \qquad v = \frac{1}{2\gamma l} \, (t - x)$$

CFT TIME

PERIODIC $v \leftrightarrow$ *DISCRETE NUMBER DENSITY*

Temperature

$$T = \frac{1}{\gamma l} \frac{r_0}{\pi l^2} \left(1 - \frac{Q^2}{2 r_0^6} \right)$$

Chemical potential $\leftrightarrow \gamma$

$$\mu_1 = \frac{1}{2\gamma^2 l^2}$$

Chemical potential $\leftrightarrow Q$

$$\mu_2 = \lim_{r \to \infty} A_u = \frac{1}{2\gamma l} \frac{Q}{l^2 r_0^2}$$

Partition function $Z(T, \mu_i) = e^{-W(T,\mu_i)/T} \simeq e^{-I_{\rm E}}$

$$I_{\rm E} = I_{\rm IIB} + \text{counterterms}$$

GIBBS POTENTIAL

EUCLIDEAN ON-SHELL ACTION

$$W = I_{\rm E} \, T = - \frac{\Delta v V r_0^4}{2\kappa^2} \left(1 + \frac{Q^2}{r_0^6} \right)$$

Entropy

$$S = - \frac{\partial W}{\partial T} \bigg|_{\mu_1, \mu_2} = \frac{\Delta x V \, 2\pi l^2 r_0^3}{\kappa^2}$$

AGREES WITH γ ***- INDEPENDENT*** *HORIZON AREA*

Can also compute ***charges*** $P_i = \partial W / \partial \mu_i$ and ***energy*** $E = W + TS - \mu_i P_i$

Equation of state

$$PV = E$$

APPROPRIATE FOR A ***NON-RELATIVISTIC 2D CFT***

Extremal black hole and near-horizon limit

Extremal limit $Q = \sqrt{2}\, r_0^3$ corresponding to ***zero temperature***

Admits ***near horizon limit*** [Reall, Kunduri-Lucietti-Reall] ***Does an*** AdS_2 ***arise?***

Relativistic

Non-relativistic

Charged AdS black hole

$r = r_0 + \rho$

TsT (x^-, ψ)

Charged Schrödinger black hole

Near-horizon limit

$$ds_5^2 = -\frac{12\rho^2}{l^2}dt^2 + \frac{l^2}{12\rho^2}d\rho^2 + \frac{r_0^2}{l^2}\left(dx^2 + dy^2 + dz^2\right)$$

$\Rightarrow$ $\mathrm{AdS}_2 \times \mathbb{R}^3$

TsT (x^-, ψ)

$$ds_5^2 = \frac{r_0^2}{l^2}\left(1+\gamma^2(r_0^2 - 12\rho^2)\right)^{-1}\left[-\frac{12\rho^2}{r_0^2}dt^2 + dx^2 - 12\gamma^2\rho^2(dt+dx)^2\right] + \frac{l^2}{12\rho^2}d\rho^2 + \frac{r_0^2}{l^2}\left(dy^2+dz^2\right)$$

NO AdS_2 ***FACTOR***

"NON-RELATIVISTIC NEAR-HORIZON"

SCALING LIMIT

$t \to t/\lambda \quad \rho \to \lambda\rho$

$\lambda \to 0$

TsT (x, ψ)

$$ds_5^2 = -\frac{12\rho^2}{l^2}dt^2 + \frac{l^2}{12\rho^2}d\rho^2 + \frac{r_0^2}{l^2}\left(\left(1+\gamma^2 r_0^2\right)^{-1}dx^2 + dy^2 + dz^2\right)$$

$\Rightarrow$ $\mathrm{AdS}_2 \times \mathbb{R}^3$

SAME AS STANDARD (RELATIVISTIC) DIPOLE THEORY!

Universality

of AdS_2 ?

A zero temperature phase transition

Schrödinger soliton obtained by *double Wick rotation* of $Q=0$ **black hole** (or *TsT* of **AdS soliton** [Horowitz-Myers])

$$t_b \to i x_s \qquad x_b \to i t_s \qquad \gamma_b \to i \gamma_s$$

PERIODICALLY IDENTIFIED FOR REGULARITY

Regularity at $r = r_s$ imposes an *additional condition*

Non-relativistic soliton only exists for **special values of** r_s

$$2 r_s \gamma_s = 1$$

Phase transition

AT **T = 0** :

Extremal Schrödinger black hole ⟷ **PHASE TRANSITION?** ⟷ *Schrödinger soliton*

Match periodicities of τ and x and deformation parameters $\gamma_b = \gamma_s$

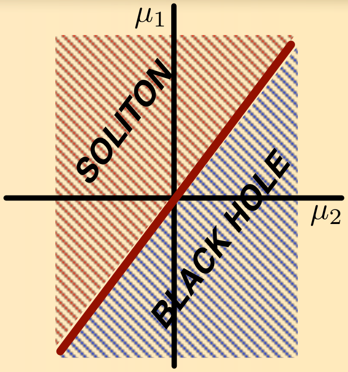
ONLY POSSIBLE FOR $2 r_s \gamma_s = 1$

Difference of **Euclidean actions**

$$\Delta I = (I_s - I_b)|_{T=0} = -\frac{\Delta\tau\Delta x V\, l^6}{2\kappa^2\mu_1^2}\left(\mu_1^4 - 3\mu_2^4\right)$$

⇨ $\mu_1 > 3^{1/4}\mu_2$ **SOLITON**

⇨ $\mu_1 < 3^{1/4}\mu_2$ **BLACK HOLE**



Also expected in *relativistic case*