Hydrodynamics of Holographic Superconductors

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Outline

- Review of the Model
- Hydrodynamics
- Holographic Hydro by Quasinormal Modes
- Summary and Outlook

related work: Kovtun, Herzog, Son [arXiv:0809.4870], Herzog, Pufu [arXiv:0902.0409], Herzog, Yarom [arXiv:0906.4810], Yarom [arXiv:arXiv:0903.1353], Maeda, Nustuume, Okamura [arXiv:0904.1914]

The Model

- Can we realize spontaneous symmetry breaking as function of temperature in AdS/CFT?
- Hartnoll, Herzog, Horowitz: YES, we can! [arXiv: 0803.3295]
  based on Gubser [arXiv:0801.2977]
- Abelian Higgs model in AdS-Blackhole background
- decoupling limit charge q-> Infty
- $$ds^2 = -(\frac{r^2}{L^2} - \frac{M}{r})dt^2 + \frac{dr^2}{\frac{r^2}{L^2} - \frac{M}{r}} + \frac{r^2}{L^2}(dx^2 + dy^2)$$

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - m^2\Psi\bar{\Psi} - (\partial_\mu\Psi - iA_\mu\Psi)(\partial^\mu\bar{\Psi} + iA^\mu\bar{\Psi})$$

The Model

- eoms:

$$\Psi'' + (\frac{f'}{f} + \frac{2}{\rho})\Psi' + \frac{\Phi^2}{f^2}\Psi + \frac{2}{L^2 f}\Psi = 0$$

$$\Phi'' + \frac{2}{\rho}\Phi' - \frac{2\Psi^2}{f}\Phi = 0$$

- boundary conditions at Horizon: $\Phi(\rho_H) = 0 \quad , \quad \Psi(\rho_H)$

  why? bulk current $J_\mu = \psi^2 A_\mu$ finite norm at the Horizon

- values at boundary

$$\Phi = \bar{\mu} - \frac{\bar{n}}{\rho} + O(\frac{1}{\rho^2})$$

$$\Psi = \frac{\psi_1}{\rho} + \frac{\psi_2}{\rho^2} + O(\frac{1}{\rho^2})$$

$$\bar{\mu} = \frac{3L}{4\pi T}\mu\,,$$

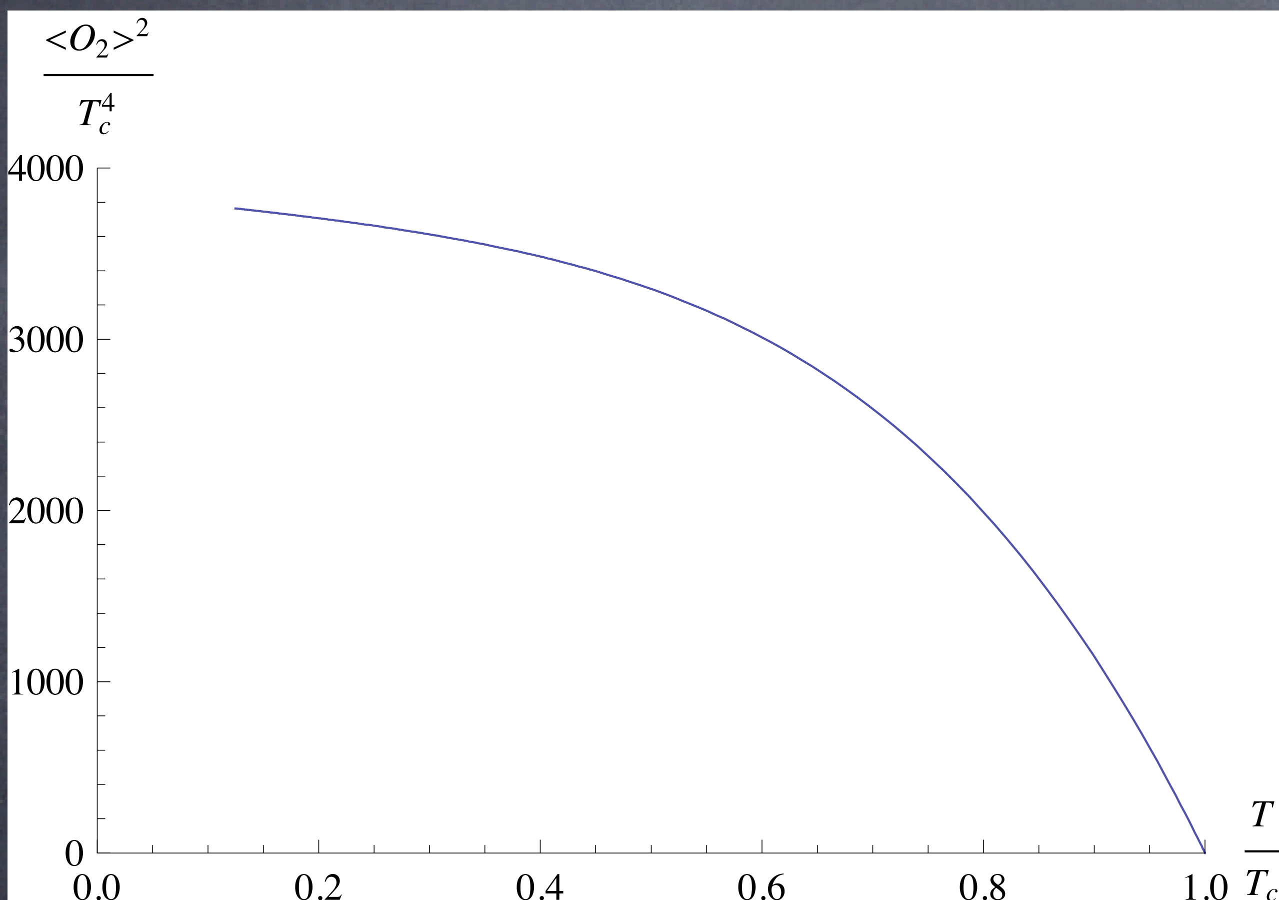
$$\bar{n} = \frac{9L}{16\pi^2 T^2}n\,,$$

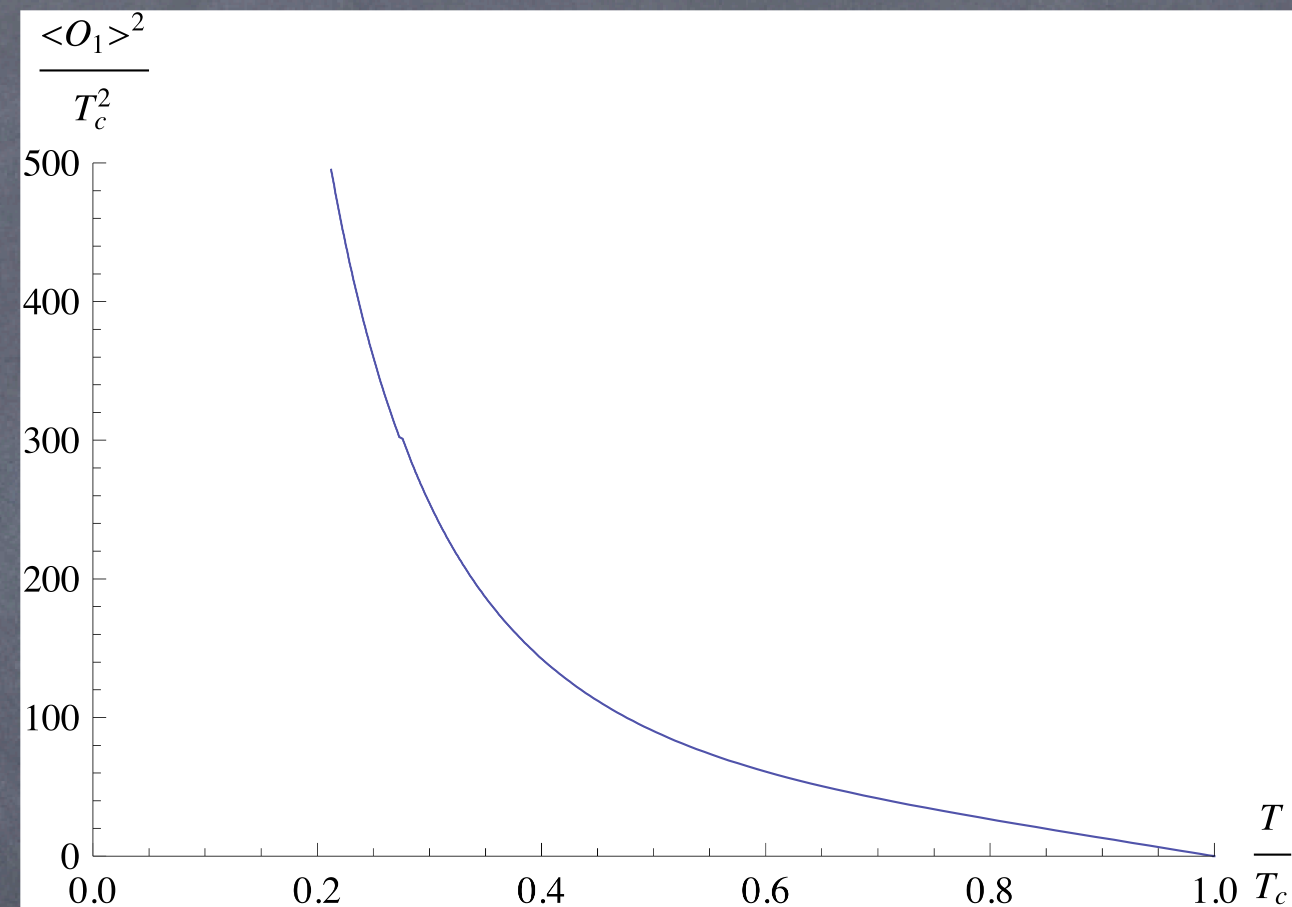
$$\psi_1 = \frac{3}{4\pi T L^2}\langle O_1\rangle\,,$$

$$\psi_2 = \frac{9}{16\pi^2 T^2 L^4}\langle O_2\rangle\,,$$

The Model

- solve eom with either $\psi_2 = 0$ or $\psi_1 = 0$





$$\langle O_i \rangle^2 \propto \left(1 - \frac{T}{T_c}\right)$$

Hydrodynamics

- Hydrodynamics = slow modes $\lim_{k \to 0} \omega(k) = 0$
- conservation law $$\frac{\partial n}{\partial t} + \vec{\nabla} \vec{j} = 0$$
- constitutive relation with external source

$$\vec{j} = -D \vec{\nabla} n + \sigma \vec{E}$$

- taking time derivative and using the continuity eqn

$$\langle j_L \rangle = \frac{i \sigma \omega^2}{\omega + i D \vec{k}^2} A_L$$

$$\sigma = \frac{-i}{\omega} \langle j_L j_L \rangle_{k=0}$$

Hydrodynamics

- broken phase: take Goldstone mode into account (Chaikin, Lubensky)
- generic prediction: appearance of sound modes
- predicts correlator $\langle j_L j_L \rangle = \frac{\hat{\sigma}\omega^2}{\omega^2 - \hat{D}\vec{k}^2}$ $\hat{D} = v_S^2$
- and conductivity

$$\sigma(\omega) = \frac{-i}{\omega + i\epsilon}\hat{\sigma} = -i\mathcal{P}\left(\frac{1}{\omega}\right) + \pi\hat{\sigma}\delta(\omega)$$ $\hat{\sigma} = n_S$

- including dissipation:

$$\omega = \pm v_s k - i\Gamma_s k^2$$

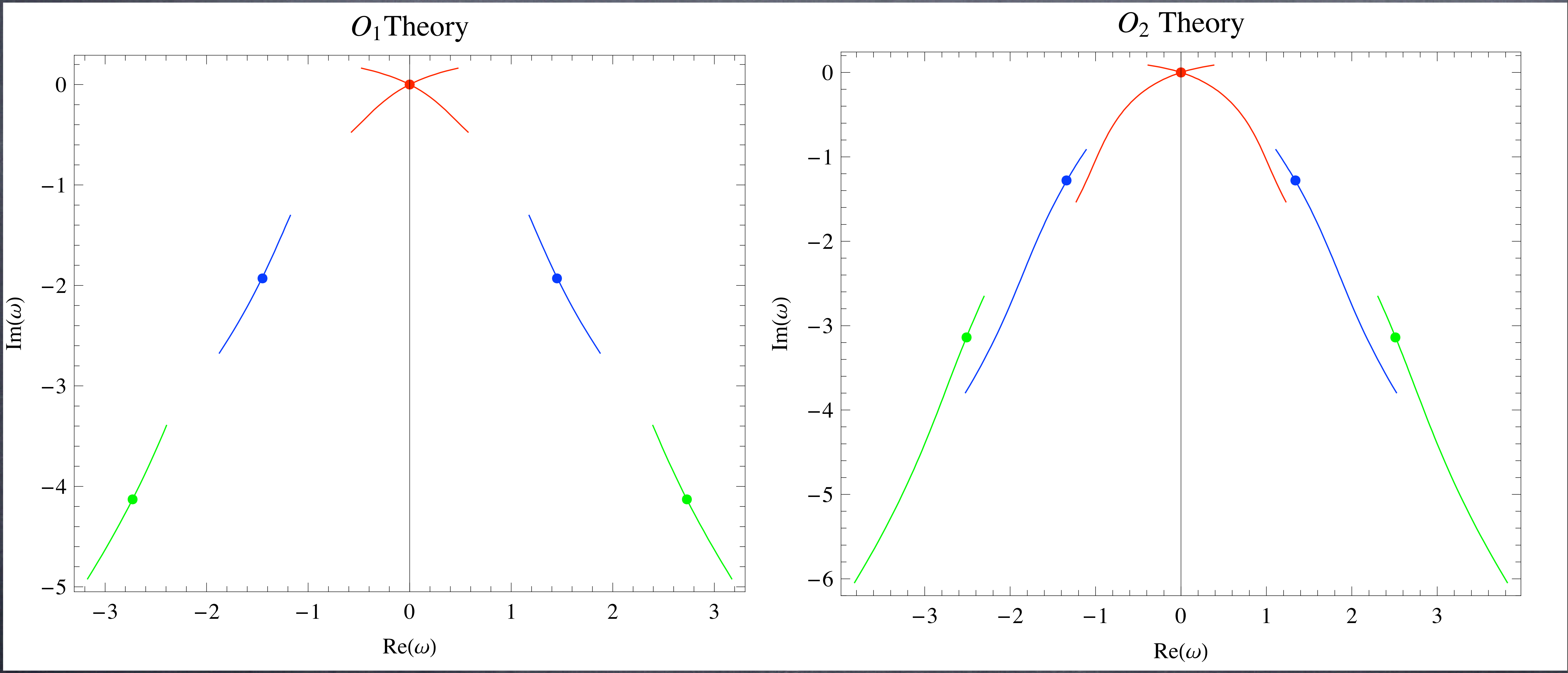
Hydro and QNMs

- poles of retarded Green functions = Quasinormal Modes
- "Eigenmodes"
  - Horizon: infalling $\Psi_H = (\rho - 1)^{-i\omega/3}(1 + O(\rho - 1))$
  - Boundary: Pole of holographic GF
  - complex scalar field $\Psi_B = \frac{A}{\rho} + \frac{B}{\rho^2} + O\left(\frac{1}{\rho^3}\right)$
  - theory I $\langle O_1 \bar{O}_1 \rangle = \frac{A}{B}$ theory II $\langle O_2 \bar{O}_2 \rangle = \frac{B}{A}$
  - complex frequencies

$$\Psi \propto e^{-i\omega_R t} e^{-\omega_I t}$$

Hydro and QNMs

- Unbroken phase: superconducting Instability



- Vector channel: Diffusion mode $D = \frac{3}{4\pi T}$

Hydro and QNMs

- Broken phase: Second sound and Pseudodiffusion
- $$
\begin{aligned}
0 &= f\eta'' + \left(f' + \frac{2f}{\rho}\right)\eta' + \left(\frac{\phi^2}{f} + \frac{2}{L^2} + \frac{\omega^2}{f} - \frac{k^2}{\rho^2}\right)\eta - \frac{2i\omega\phi}{f}\sigma - \frac{i\omega\psi}{f}a_t - \frac{ik\psi}{r^2}a_x\,,\\
0 &= f\sigma'' + \left(f' + \frac{2f}{\rho}\right)\sigma' + \left(\frac{\phi^2}{f} + \frac{2}{L^2} + \frac{\omega^2}{f} - \frac{k^2}{\rho^2}\right)\sigma + \frac{2\phi\psi}{f}a_t + \frac{2i\omega\phi}{f}\eta\,,\\
0 &= f{a_t}'' + \frac{2f}{\rho}{a_t}' - \left(\frac{k^2}{\rho^2} + 2\psi^2\right)a_t - \frac{\omega k}{\rho^2}a_x - 2i\omega\psi\,\eta - 4\psi\phi\,\sigma\,,\\
0 &= f{a_x}'' + f'{a_x}' + \left(\frac{\omega^2}{f} - 2\psi^2\right)a_x + \frac{\omega k}{f}a_t + 2ik\psi\,\eta\,.
\end{aligned}
$$
- constraint: $\frac{\omega}{f}{a_t}' + \frac{k}{\rho^2}{a_x}' = 2i\left(\psi'\,\eta - \psi\,\eta'\right)$
- local ward identity: $\partial_\mu\langle j^\mu\rangle = 2\langle O_i\rangle\eta_0^i$

Hydro and QNMs

- How to compute QNMs of coupled system
- four l.i. solutions (one is pure gauge)

$$\eta^{IV} = i\lambda\psi\,, \quad \sigma^{IV} = 0\,, \quad a_t^{IV} = \lambda\omega\,, \quad a_x^{IV} = -\lambda k\,.$$

- rescale scalar fields $\tilde{\eta}(\rho) = \rho\eta(\rho) \quad , \quad \tilde{\sigma}(\rho) = \rho\sigma(\rho)$
- general solution is now

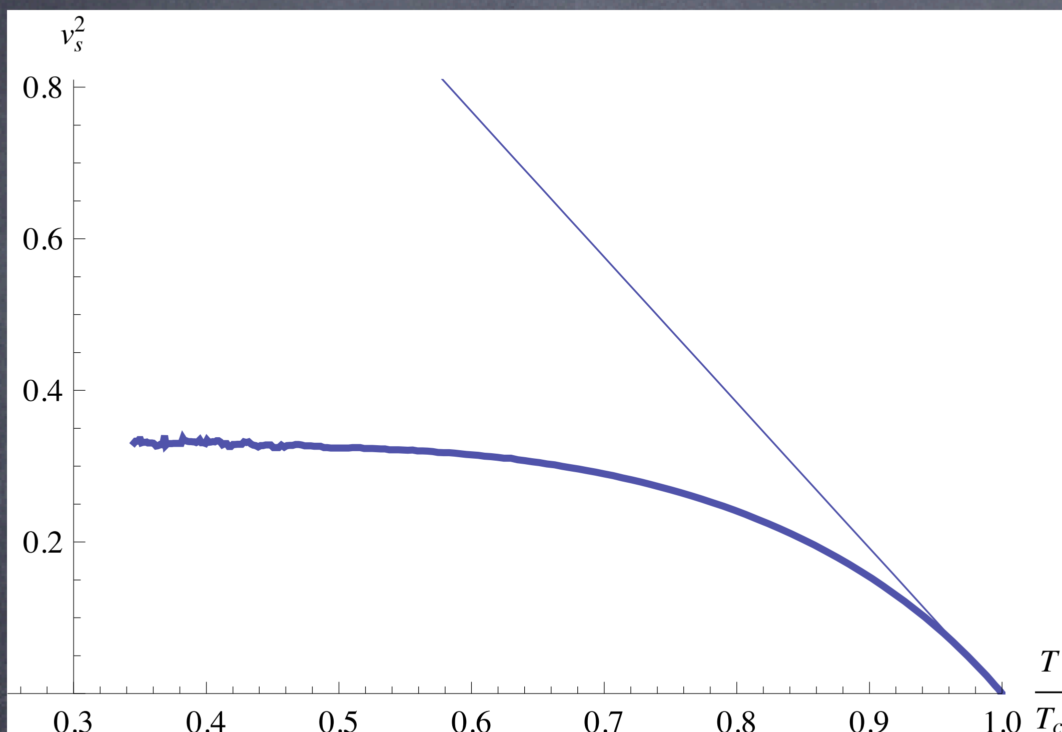
$$\varphi_i = \alpha_1 {\varphi_i}^I + \alpha_2 {\varphi_i}^{II} + \alpha_3 {\varphi_i}^{III} + \alpha_4 {\varphi_i}^{IV}$$

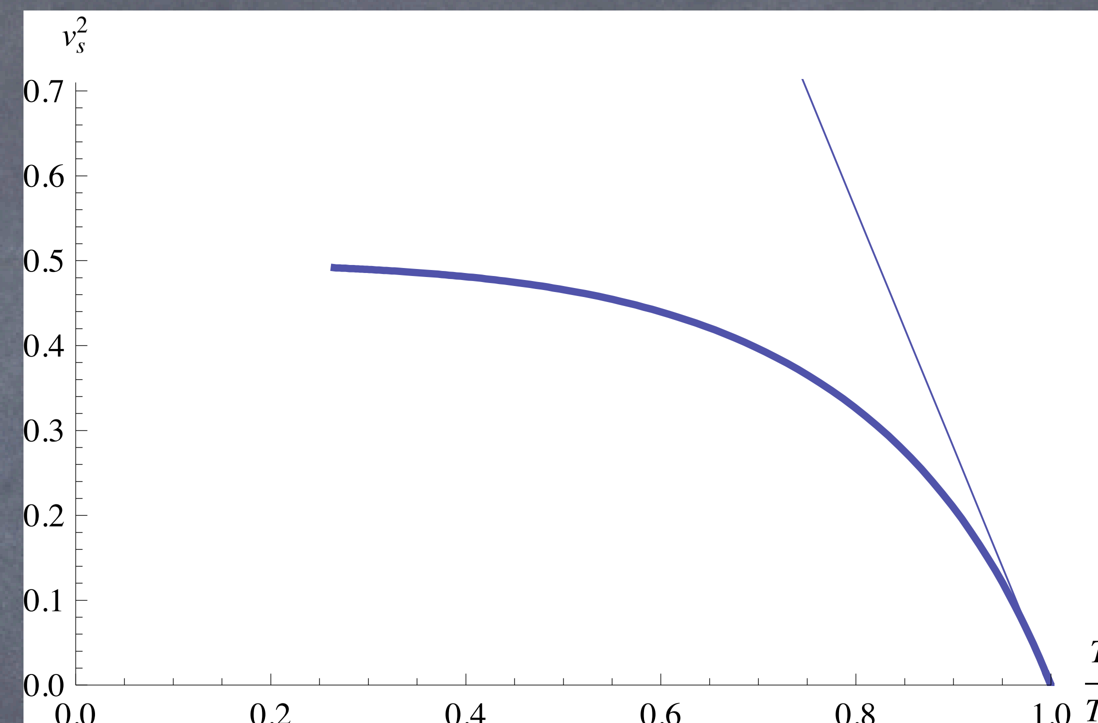
- QNM = no-source term -> zero determinant

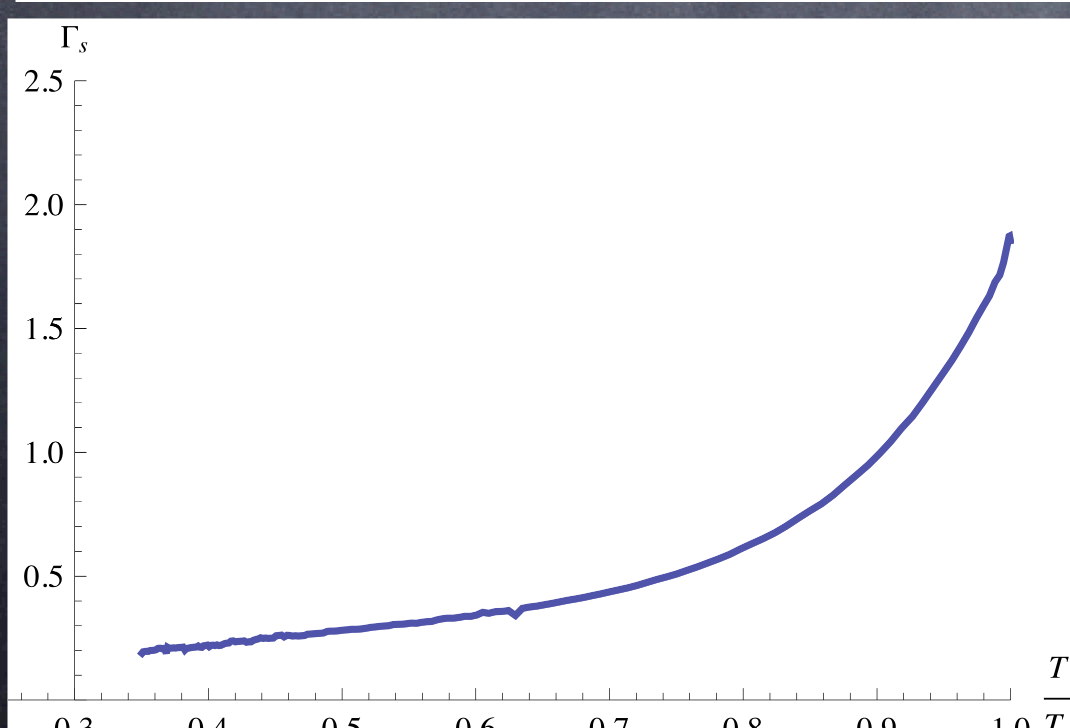
$$0 = \left| \begin{array}{cccc} {\varphi_\eta}^I & {\varphi_\eta}^{II} & {\varphi_\eta}^{III} & {\varphi_\eta}^{IV} \\ {\varphi_\sigma}^I & {\varphi_\sigma}^{II} & {\varphi_\sigma}^{III} & {\varphi_\sigma}^{IV} \\ \varphi_t^I & \varphi_t^{II} & \varphi_t^{III} & \varphi_t^{IV} \\ \varphi_x^I & \varphi_x^{II} & \varphi_x^{III} & \varphi_x^{IV} \end{array} \right|_{\rho=\Lambda}$$

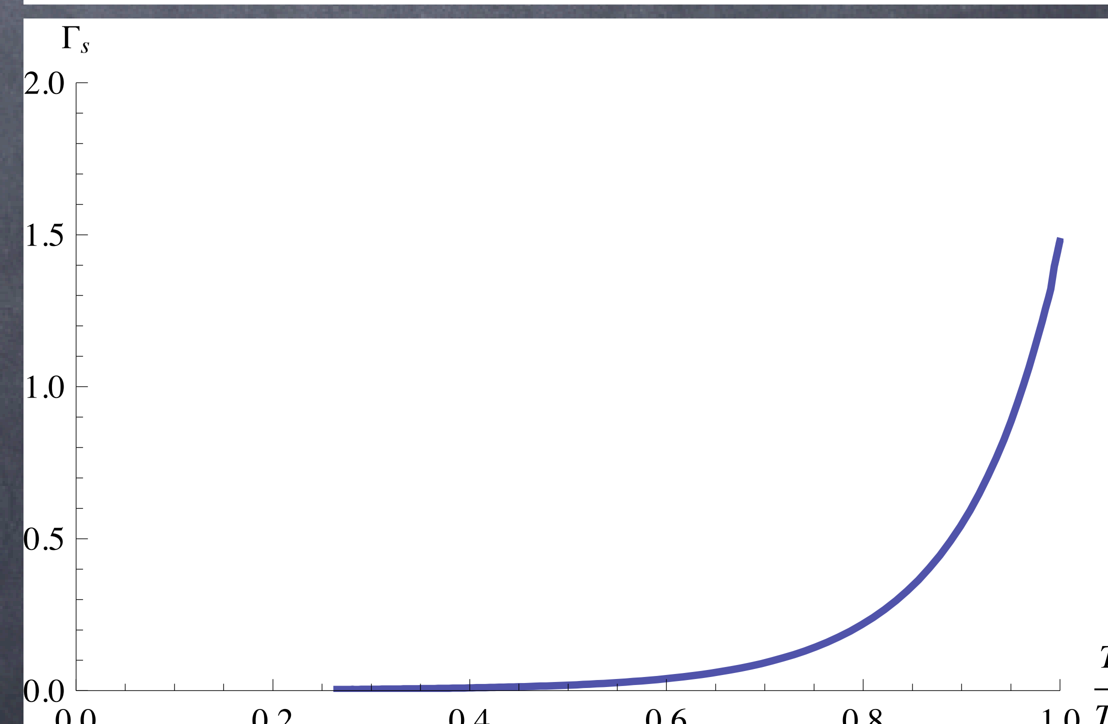
Hydro and QNMs

- Dispersion relation: $\omega = v_s k - i\Gamma_s k^2$



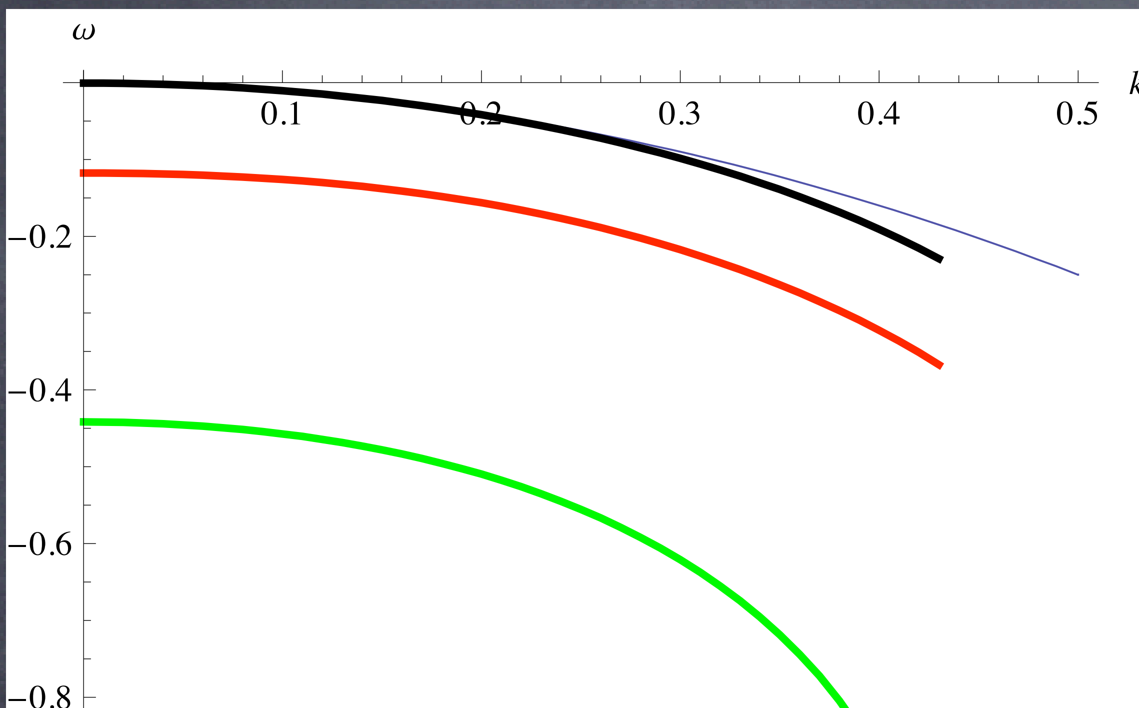


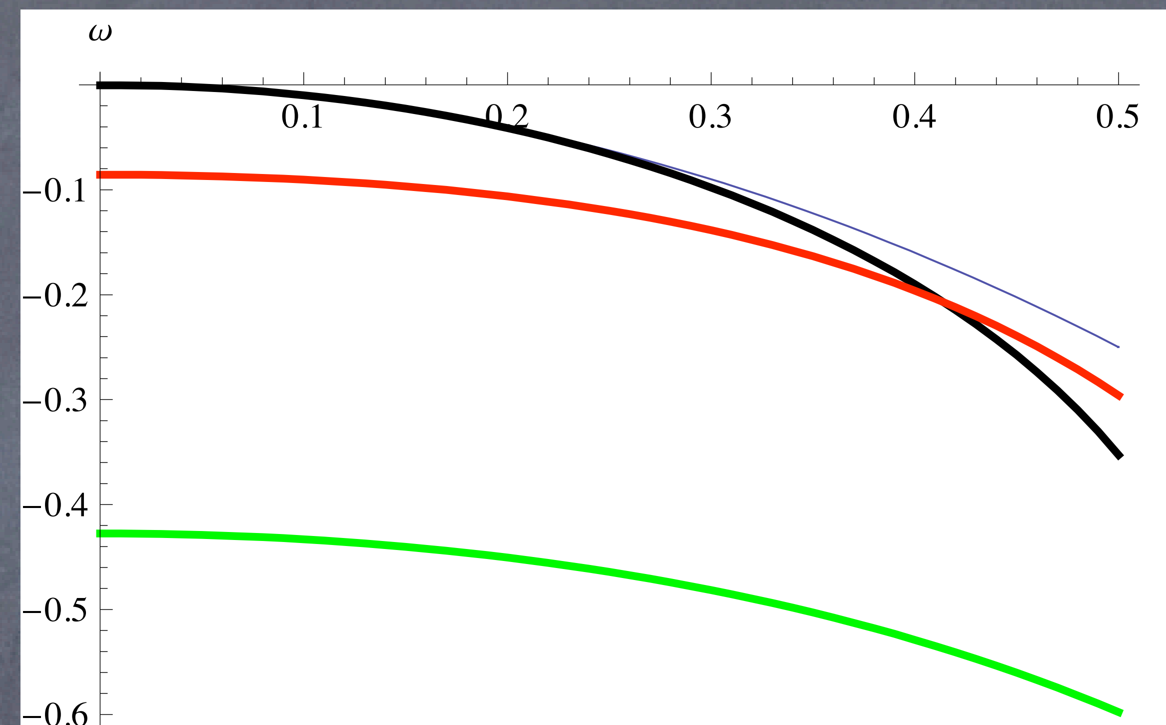


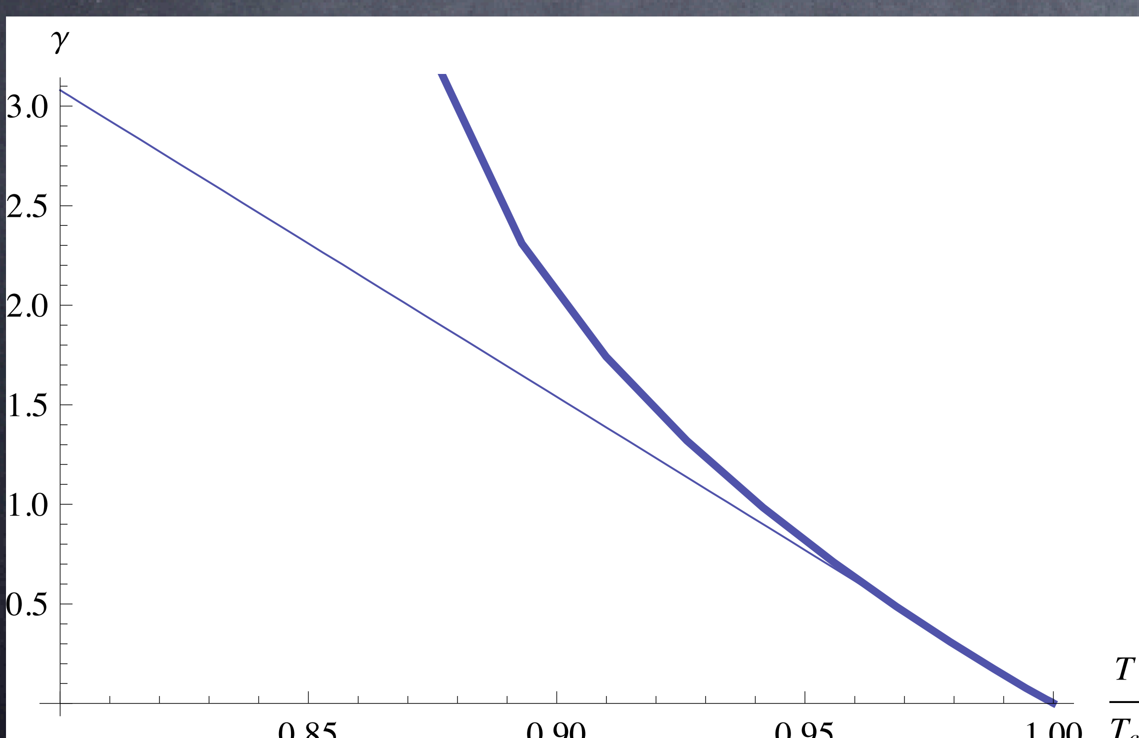


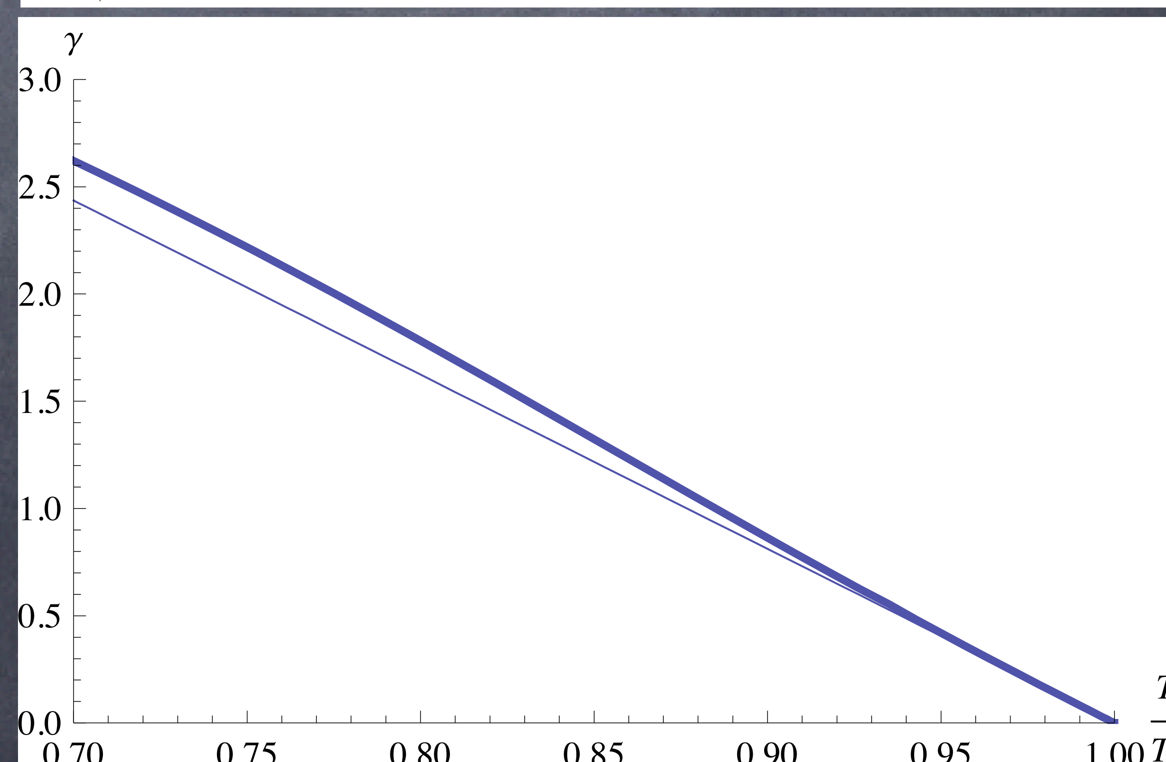
Hydro and QNMs

- Pseudo Diffusion $\omega = -iDk^2 - i\gamma$



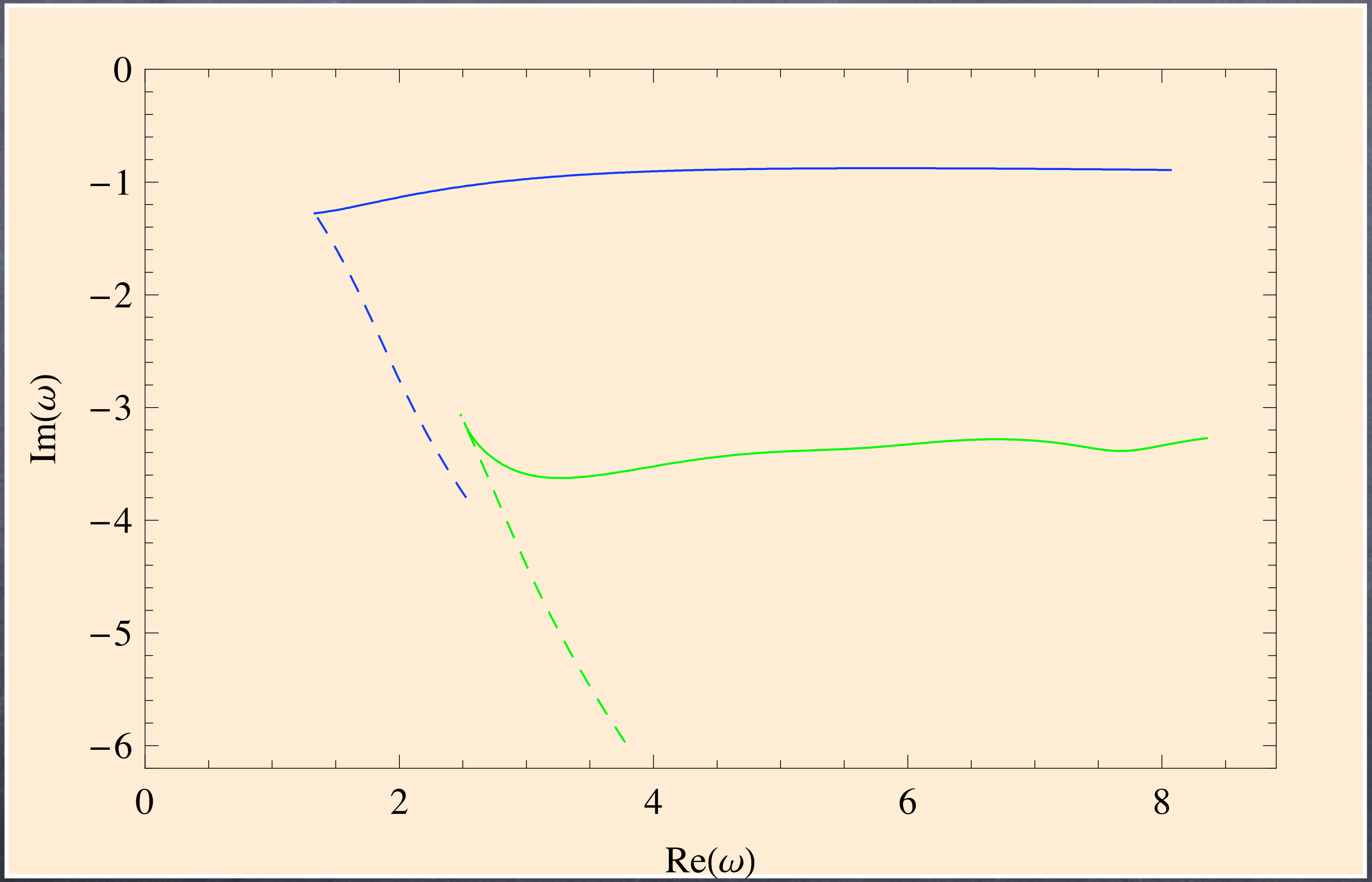






Hydro and QNMs

- Higher Quasinormal modes



Summary and Outlook

- Relevant modes of the phase transition
  - unbroken phase: 1 Diffusion mode
  - critical point: 2 massless scalar modes + Diffusion
  - broken phase: 2 modes of sound, Pseudo Diffusion, dynamical scaling z=2
- Outlook:
  - study hydro QNMs in the backreacted model
  - (much) more complicated 11 coupled diff eqns
  - two different mechanism of spontaneous symmetry breaking?
    (2 different QNMs cross the real axes for large and small charges)
  - include fermionic operator