Arnold Sommerfeld Center for Theoretical Physics

Excellence Cluster Universe

Luca Martucci

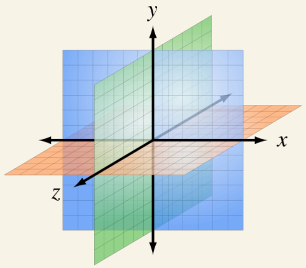
On moduli and effective theory of N=1 warped compactifications

Based on: arXiv:0902.4031

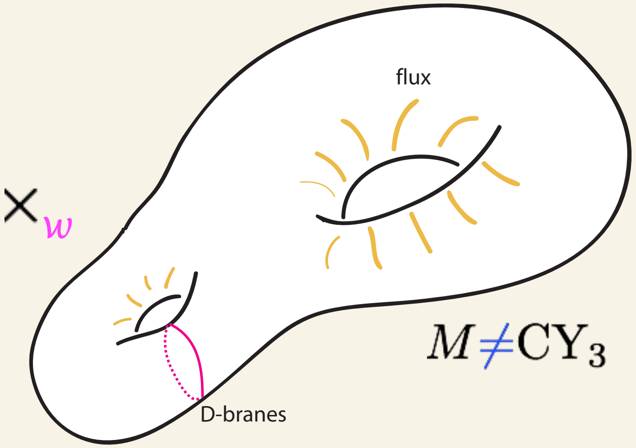
15-th European Workshop on String Theory
Zürich, 7-11 September 2009

Motivation: fluxes and 4D physics

- In type II flux compactifications the internal space is not CY



$\times_w$



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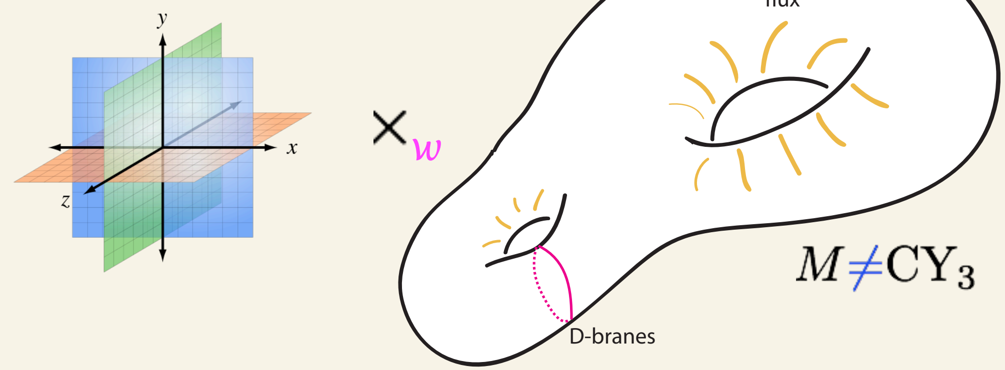

what is the 4D effective physics?

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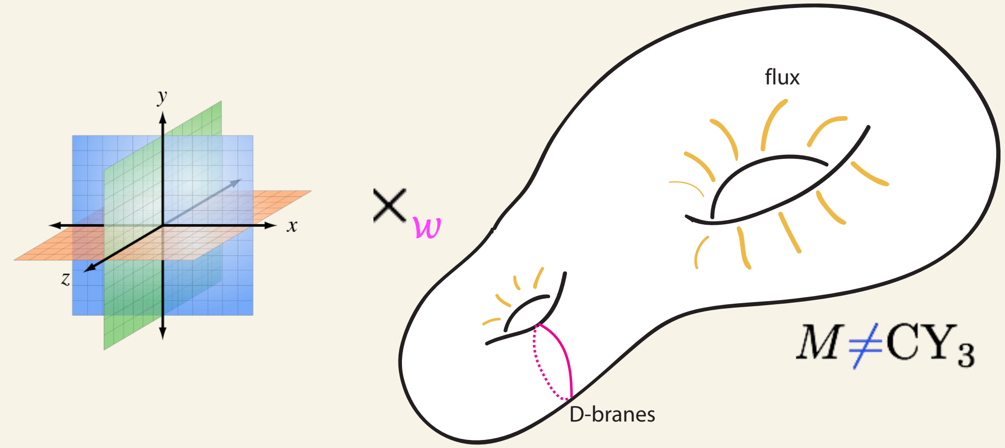

what is the 4D effective physics?

- Furthermore fluxes generically generate a non-trivial warping:

$$\mathrm{d}s_{10}^2 = e^{2A}\,\mathrm{d}s_4^2 + \mathrm{d}s_6^2 \qquad \text{with} \qquad \nabla^2 A \simeq (\text{fluxes})^2 + \sum \tau_i \delta_i^{(\text{loc})}$$

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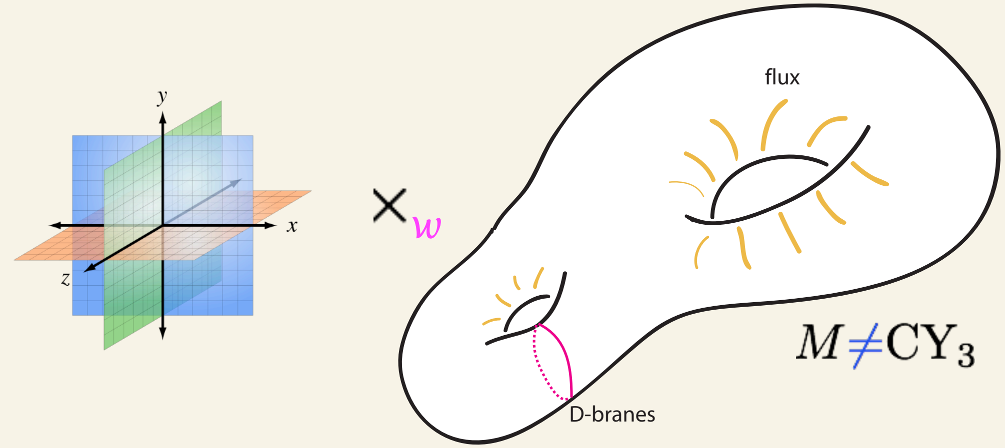

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- Neglecting back-reaction: $M \simeq \mathrm{CY}_3$, $e^A \simeq 1$

→ 4D effective theory:
- * (fluxless) CY spectrum
- * flux induced potential

(using standard CY tools)

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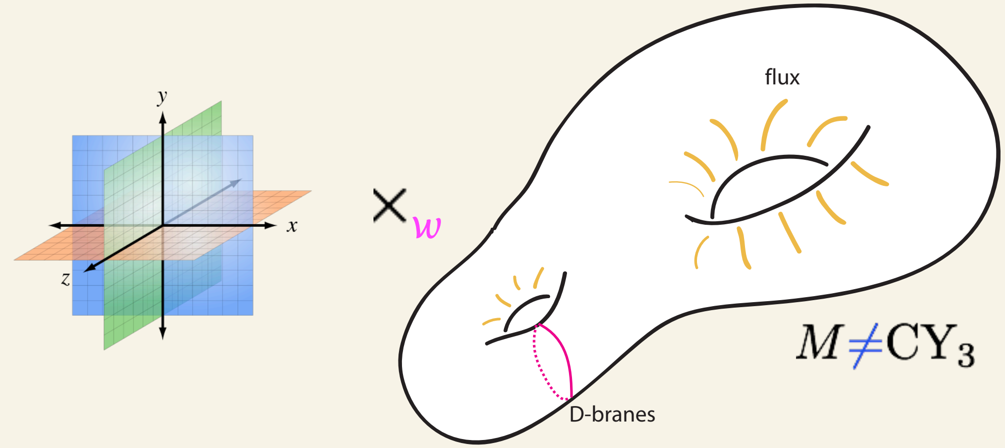

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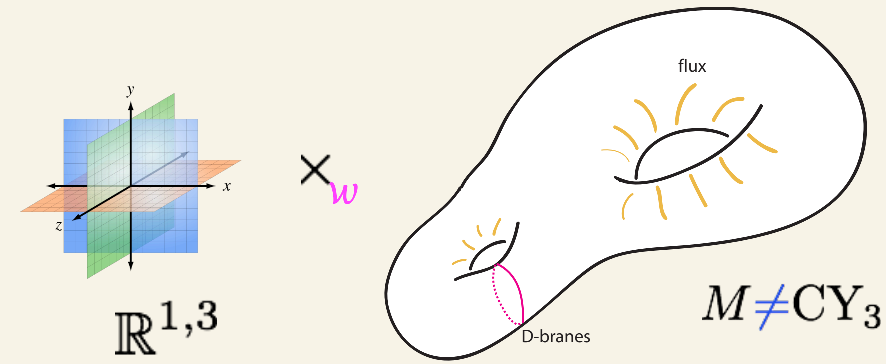
What can we say about 4D effective theory of fully back-reacted vacua?

Plan of the talk

* Type II (generalized complex) flux vacua
* Moduli, twisted cohomologies and 4D fields
* Kähler potential

Type II (generalized complex) flux vacua

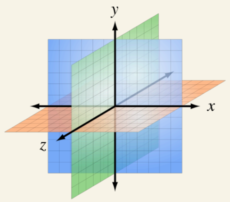
Fluxes and $\mathcal{N}=1$ SUSY



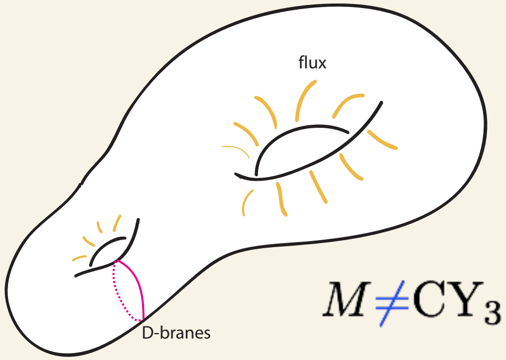
Fluxes and $\mathcal{N}=1$ SUSY

- NS sector:

  - metric $\quad ds^2_{10} = e^{2A} ds^2_4 + ds^2_6$
  - dilaton $\quad \phi$
  - 3-form $\quad H \qquad (H = dB$ locally)



$\mathbb{R}^{1,3} \times_w$



$M \neq \mathrm{CY}_3$

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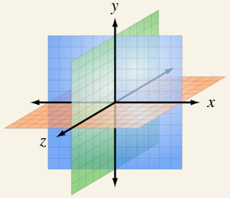

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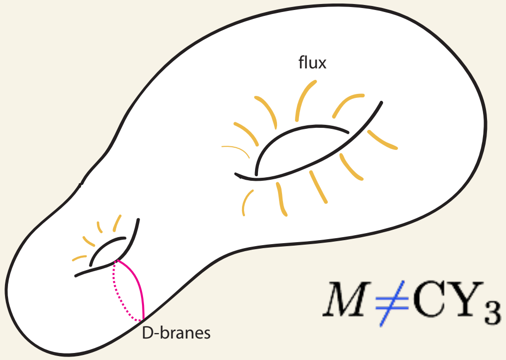

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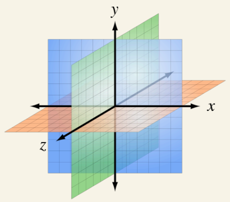
- RR sector: $\quad F = \sum F_k \qquad \mathrm{d}_H F = -j \rightarrow \sim \delta^{\mathrm{loc}} \wedge e^{-\mathcal{F}}$

Fluxes and $\mathcal{N}=1$ SUSY



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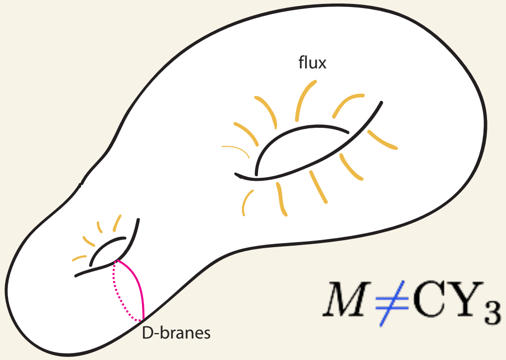

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$\mathrm{d}_H := \mathrm{d} + H\wedge$ ($\mathrm{d}_H^2 = 0$)

- RR sector: $F = \sum F_k$

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$\sim \delta^{\mathrm{loc}} \wedge e^{-\mathcal{F}}$

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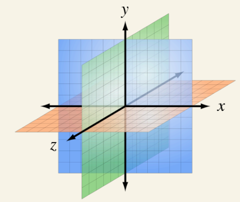

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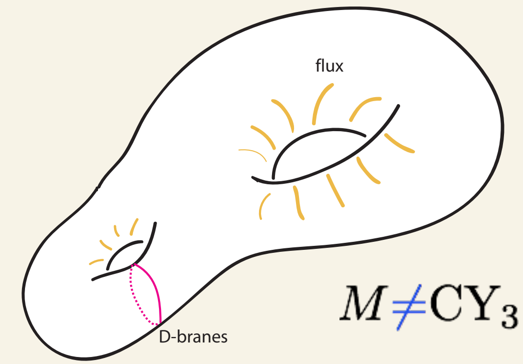

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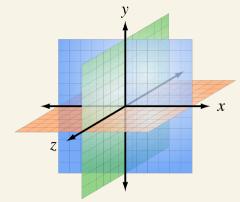
$F = \mathrm{d}_H C$, with $C = \sum_k C_{k-1}$

Fluxes and $\mathcal{N} = 1$ SUSY

- Killing spinors:

$$\epsilon_1 = \zeta \otimes \eta_1 + \text{c.c.}$$
$$\epsilon_2 = \zeta \otimes \eta_2 + \text{c.c.}$$



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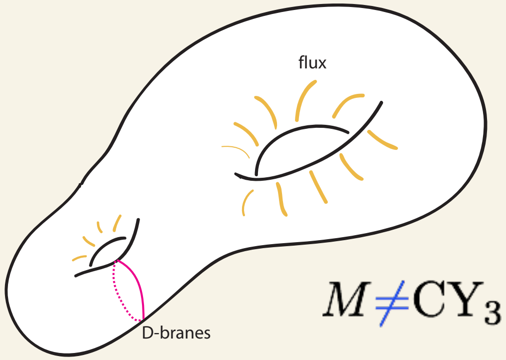

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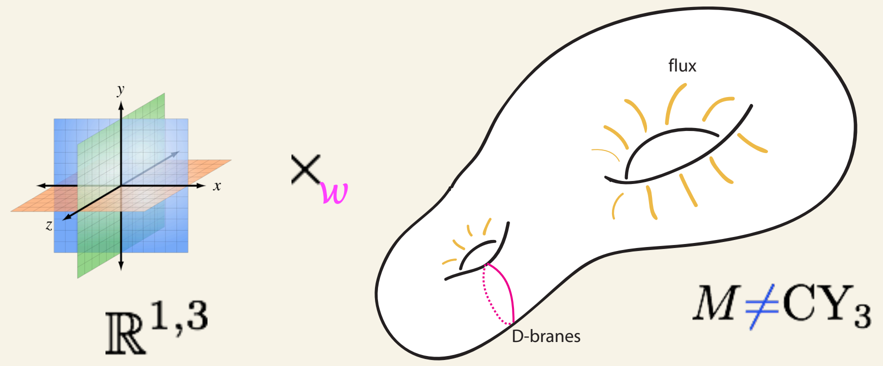

- Polyforms:

$$\mathcal{Z} \simeq e^B e^{3A-\phi} \eta_1 \otimes \eta_2^T \quad , \quad T \simeq e^B e^{-\phi} \eta_1 \otimes \eta_2^\dagger$$

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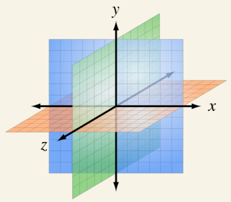

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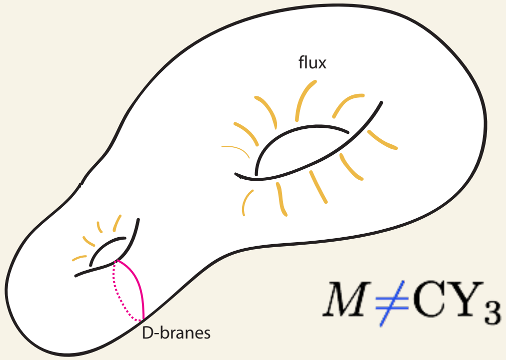

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IIA $$\mathcal{Z} = \mathcal{Z}_0 + \mathcal{Z}_2 + \mathcal{Z}_4 + \mathcal{Z}_6 \quad , \quad T = T_1 + T_3 + T_5$$

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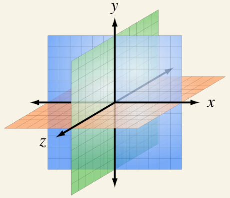

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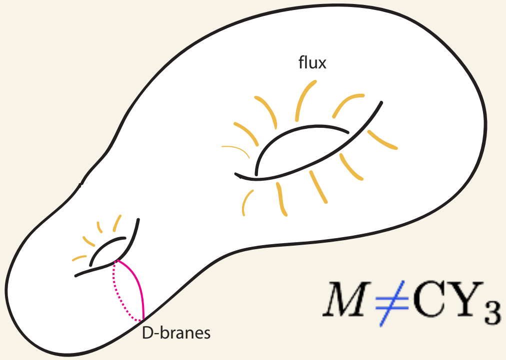

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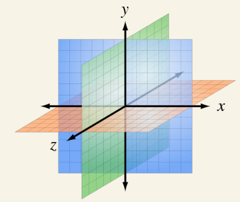

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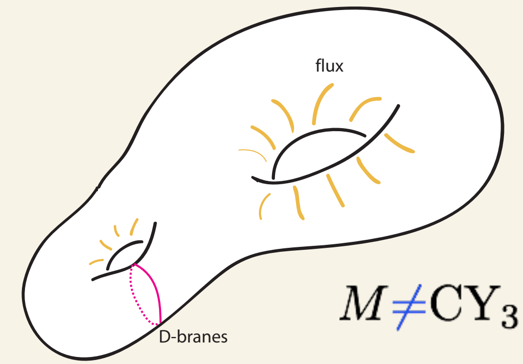

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$\mathcal{Z}$ and T are O(6,6) pure spinors!

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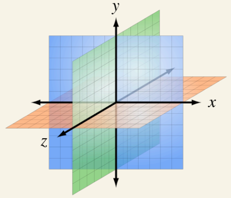

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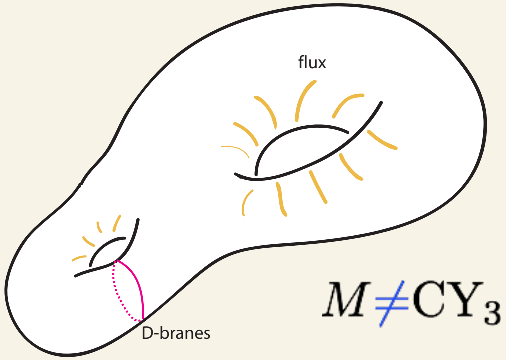

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→ they contain complete information about NS sector and SUSY

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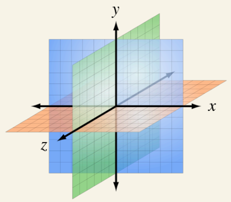

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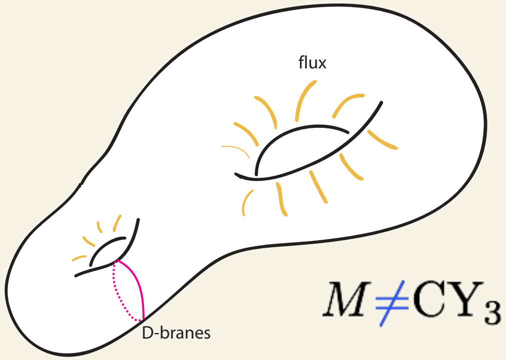

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- SUSY conditions

Graña, Minasian, Petrini & Tomasiello `05

$$d_H \mathcal{Z} = 0 \quad , \quad d_H(e^{2A}\text{Im}T) = 0 \quad , \quad d_H(e^{4A}\text{Re}T) = e^{4A} * F$$

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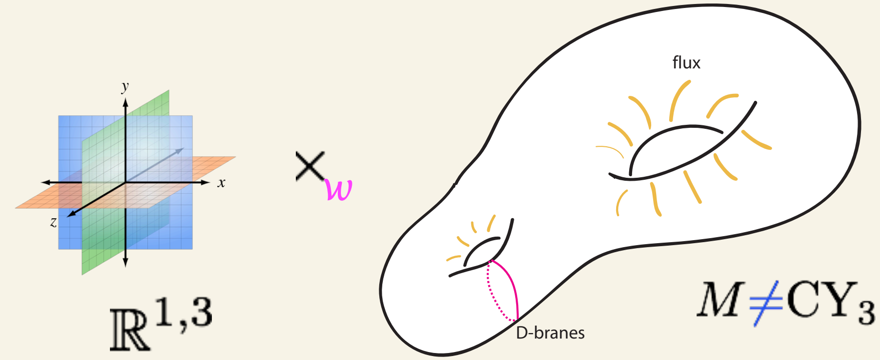

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precise interpretation in terms of:

- generalized calibrations — *L.M. & Smyth `05*
- F- and D- flatness — *Koerber & L.M. `07*

SUSY and GC geometry

$$\mathrm{d}_H \mathcal{Z} = 0$$

(F-flatness)

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→ integrable generalized complex structure

Hitchin `02

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e.g. $\mathcal{Z} \sim e^{i\omega}$ symplectic (IIA)

$\mathcal{Z} \sim \Omega$ complex (IIB)

SUSY and GC geometry

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- Induced polyform decomposition

Gualtieri `04

$$\bigoplus_{n=0}^{6} \Lambda^n T^*_M = \bigoplus_{k=-3}^{3} U_k$$

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$$\begin{aligned}
\mathcal{Z} &\in U_3 \\
& \quad U_2 \\
& \quad U_1 \\
T &\in U_0 \\
& \quad U_{-1} \\
& \quad U_{-2} \\
\bar{\mathcal{Z}} &\in U_{-3}
\end{aligned}$$

SUSY and GC geometry

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$\mathcal{Z} \in U_3$

U_2

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U_{-1}

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$\bar{\mathcal{Z}} \in U_{-3}$

e.g. $\mathcal{Z} \sim \Omega$ → complex case

$\Lambda^{3,0}$

$\Lambda^{2,0} \quad \Lambda^{3,1}$

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→ $\mathrm{d}_H = \partial_H + \bar{\partial}_H$ with

$$\bar{\partial}_H : U_k \to U_{k-1}$$
$$\partial_H : U_k \to U_{k+1}$$

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with

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- Generalized Hodge decomposition (assuming $\partial_H \bar{\partial}_H$-lemma)

Cavalcanti `05

$$\mathrm{H}^\bullet_{\mathrm{d}_H}(M) \simeq \mathrm{H}^3_{\bar{\partial}_H}(M) \oplus \mathrm{H}^2_{\bar{\partial}_H}(M) \oplus \ldots \oplus \mathrm{H}^{-3}_{\bar{\partial}_H}(M)$$

Moduli, twisted cohomologies and 4D fields

Moduli and polyforms

Moduli and polyforms

- The full closed string information is stored in

Koerber & L.M. `07

see also: Graña, Louis & Waldram `05; Benmachiche and Grimm `06

$$\mathcal{Z} \qquad , \qquad \mathcal{T} := \mathrm{Re}\,\mathcal{T} - iC$$

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RR degrees of freedom

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$\mathcal{Z}$ → `half' of NS degrees of freedom

$\mathrm{Re}\,\mathcal{T}$ → information encoded in $\mathcal{T}$ (second `half' of NS degrees of freedom)

C → RR degrees of freedom

- The $\mathcal{Z}$ and $\mathcal{T}$ moduli are associated to twisted cohomology classes of:

$$\mathrm{d}_H \qquad , \qquad \bar{\partial}_H$$

Moduli and 4D fields

- $$\mathcal{M}_{\mathcal{Z}} \simeq \{z^I \in \mathrm{H}^{\mathrm{ev/od}}_{\mathrm{d}_H}(M;\mathbb{R}) : \mathrm{d}\mathcal{W}(z) = 0\}$$

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moduli space of

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- z^I and t^a will be 4D chiral fields of 4D superconformal theory

Weyl-chiral weights: z^I: $(3,1)$, t^a: $(0,0)$

see e.g.: Kallosh, Kofman, Linde & Van Proeyen `00

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bosonic components of linear multiplets dual to t^a

- Linear-chiral functional dependence $l_a = l_a(z,\bar{z};t+\bar{t})$

explicit form depends on microscopical details

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Grañã & Polchinski;
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  $\mathrm{H}^{0,0}_+$ → removed axion-dilaton; $\mathrm{H}^{1,1}_-$ → ϕ^a; $\mathrm{H}^{2,2}_+$ → ρ^A

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  $\mathrm{H}^{1,1}_- \to \phi^a$, $\mathrm{H}^{2,2}_+ \to \rho^A$; $\mathrm{H}^{0,0}_+ \to$ removed axion-dilaton

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  $\mathrm{H}^{3,3}_+ \to$ removed axion-dilaton; $\mathrm{H}^{1,1}_+ \to v_A$, $\mathrm{H}^{2,2}_- \to l_a$

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- $\mathcal{K}$ does not seem topological! However

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topologically well defined & in agreement with 4D interpretation

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- Freezing the z^I-moduli, knowing $l_a(t+\bar{t})$ one can obtain by integration $\mathcal{K}(t+\bar{t})$

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* redefining $\rho \to \rho + \text{Vol}^{\text{w}}_0/2$ ⟶ unwarped Kähler potential — Grimm & Louis `04

Conclusions

- Under some assumptions (e.g. $\partial_H\bar{\partial}_H$-lemma), the 4D spectrum has been identified with H-twisted cohomologies

- The 4D couplings of probe D-branes (space-filling, instantons, DW's and strings) depend only on the cohomology classes

- The Kähler potential determined only implicitly. However, 4D chiral-linear duality can help in reconstructing it.