Non-supersymmetric extremal
multicenter black holes with superpotentials

Jan Perz
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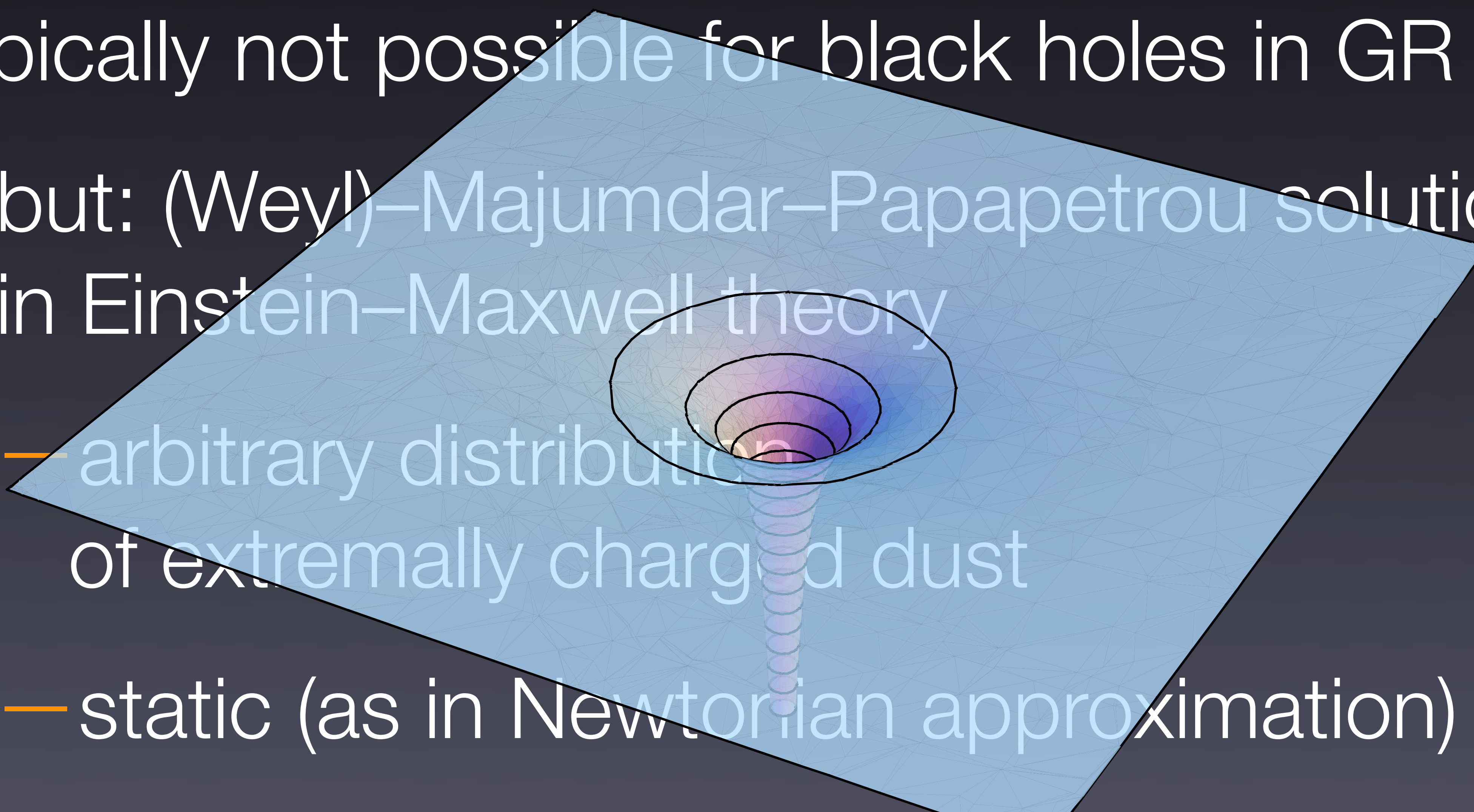
Based on: P. Galli, J. Perz

arXiv:0909.???? [hep-th]

Single-center vs multicenter solutions

- superposition holds for linear systems
- typically not possible for black holes in GR
  - but: (Weyl)–Majumdar–Papapetrou solutions in Einstein–Maxwell theory
    - arbitrary distribution of extremally charged dust
    - static (as in Newtonian approximation)
    - described by harmonic functions



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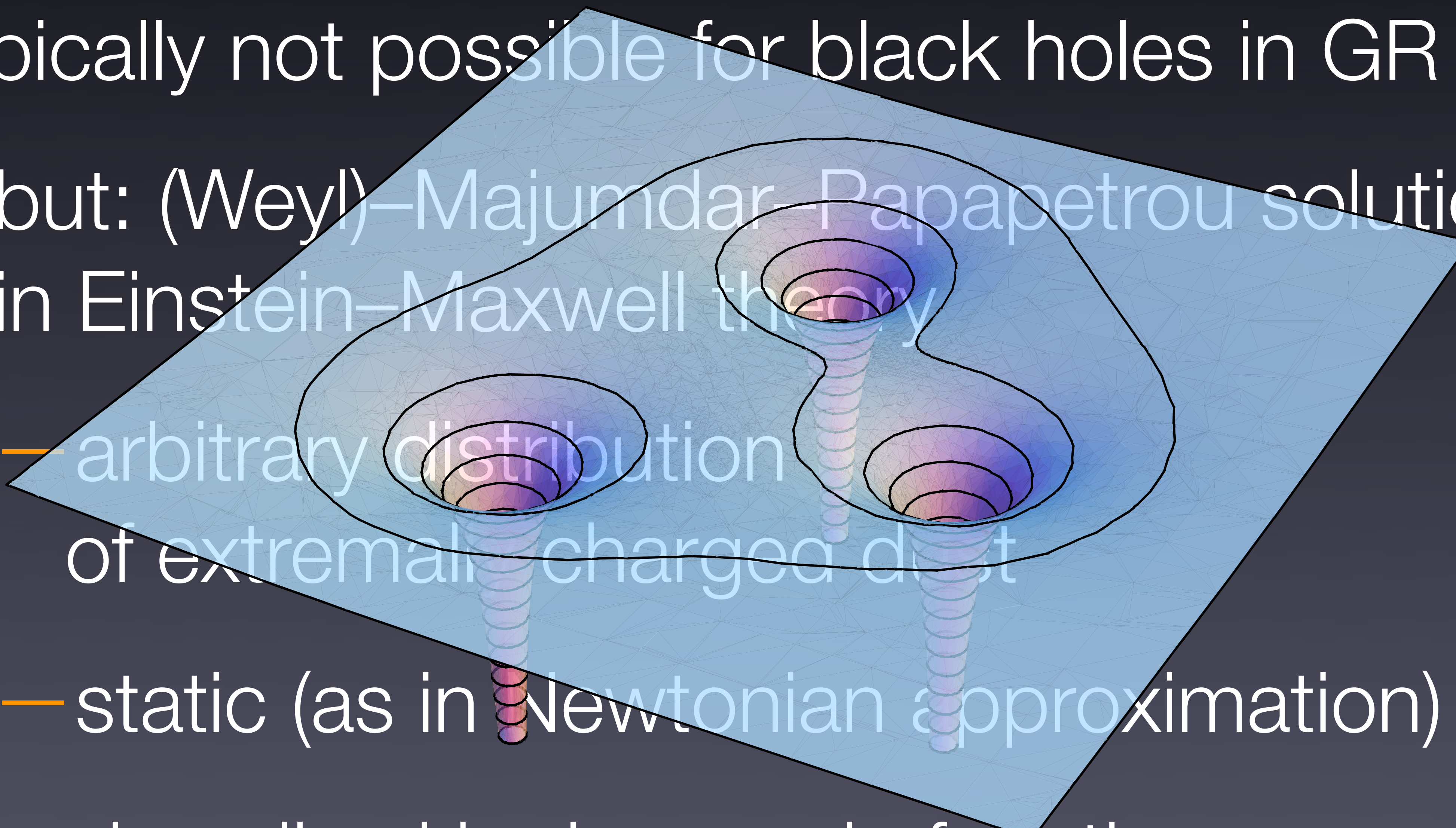

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Supersymmetric black hole composites

- extremal multi-RN solutions are susy [Gibbons, Hull]
- susy (hence extremal) multicenter solutions in 4d $\mathcal{N} = 2$ supergravity with vector multiplets
  - with identical charges [Behrndt, Lüst, Sabra]

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    - relative positions of centers constrained
    - single-center solution may not exist, where a multicenter can

Non-susy extremal composites

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  - reverse orientation of base space in 5D susy solutions
- here: superpotential approach

$\mathcal{N}=2$ *supergravity in 4 dimensions*

- bosonic action with $n_{\rm v}$ vector multiplets

$$I_{\rm 4D} \propto \int \Big(R \star 1 - 2 g_{a\bar{b}}(z) \mathrm{d}z^a \wedge \star \mathrm{d}\bar{z}^{\bar{b}} + \mathrm{Im}\, \mathcal{N}_{IJ}(z) \mathcal{F}^I \wedge \star \mathcal{F}^J + \mathrm{Re}\, \mathcal{N}_{IJ}(z) \mathcal{F}^I \wedge \mathcal{F}^J \Big)$$

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$$K = -\ln \left[\mathrm{i} \begin{pmatrix} X^I & \partial_I F \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \overline{\begin{pmatrix} X^I \\ \partial_I F \end{pmatrix}} \right]$$

Black holes in 4d $\mathcal{N}=2$ supergravity

- bosonic action with n_v vector multiplets

$$I_{4\mathrm{D}} \propto \int \Big(R \star 1 - 2 g_{a\bar{b}}(z) \mathrm{d}z^a \wedge \star \mathrm{d}\bar{z}^{\bar{b}} + \mathrm{Im}\,\mathcal{N}_{IJ}(z) \mathcal{F}^I \wedge \star \mathcal{F}^J + \mathrm{Re}\,\mathcal{N}_{IJ}(z) \mathcal{F}^I \wedge \mathcal{F}^J \Big)$$

- static, spherically symmetric ansatz (1 center)

$$\mathrm{d}s^2 = -\mathrm{e}^{2U(\tau)} \mathrm{d}t^2 + \mathrm{e}^{-2U(\tau)} \delta_{ij} \mathrm{d}x^i \mathrm{d}x^j \qquad \tau = \frac{1}{|\mathbf{x}|}$$

- charged solution

$$p^I \propto \int_{S^2_\infty} \mathcal{F}^I \qquad q_I \propto \int_{S^2_\infty} \frac{\partial \mathcal{L}}{\partial \mathcal{F}^I} \qquad \begin{pmatrix} p^I \\ q_I \end{pmatrix} =: Q$$

Black hole potential (single-center)

$$I_{\mathrm{4D}} \propto \int \Big(R \star 1 - 2 g_{a\bar{b}}(z) \mathrm{d}z^a \wedge \star \mathrm{d}\bar{z}^{\bar{b}} + \mathrm{Im}\, \mathcal{N}_{IJ}(z) \mathcal{F}^I \wedge \star \mathcal{F}^J + \mathrm{Re}\, \mathcal{N}_{IJ}(z) \mathcal{F}^I \wedge \mathcal{F}^J \Big)$$

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$$I_{\text{eff}} \propto \int \mathrm{d}\tau \left(\dot{U}^2 - 2g_{a\bar{b}}(z)\mathrm{d}z^a \wedge \star \mathrm{d}\bar{z}^{\bar{b}} \right. \qquad \dot{} = \frac{\mathrm{d}}{\mathrm{d}\tau}$$

$$\left. + \operatorname{Im} \mathcal{N}_{IJ}(z) \mathcal{F}^I \wedge \star \mathcal{F}^J + \operatorname{Re} \mathcal{N}_{IJ}(z) \mathcal{F}^I \wedge \mathcal{F}^J \right)$$

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Black hole potential (single-center)

- action with effective potential [Ferrara, Gibbons, Kallosh]

$$I_{\text{eff}} \propto \int d\tau \left(\dot{U}^2 + g_{a\bar{b}} \dot{z}^a \dot{\bar{z}}^{\bar{b}} + e^{2U} V_{\text{BH}} \right) \qquad \dot{} = \frac{d}{d\tau}$$

$$V_{\text{BH}} = |Z|^2 + 4 g^{a\bar{b}} \partial_a |Z| \partial_{\bar{b}} |Z|$$

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$$V_{\text{BH}} = |Z|^2 + 4g^{a\bar{b}}\,\partial_a|Z|\partial_{\bar{b}}|Z|$$

- rewriting not unique [Ceresole, Dall'Agata]

$$V_{\text{BH}} = Q^{\mathrm{T}}\mathcal{M}Q = Q^{\mathrm{T}}S^{\mathrm{T}}\mathcal{M}SQ \qquad S^{\mathrm{T}}\mathcal{M}S = \mathcal{M}$$

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- 'superpotential' W not necessarily equal to $|Z|$

$$V_{\text{BH}} = W^2 + 4g^{a\bar{b}}\,\partial_a W \partial_{\bar{b}} W$$

Flow equations

- effective Lagrangian as a sum of squares

$$\mathcal{L}_{\text{eff}} \propto \dot{U}^2 + g_{a\bar{b}}\dot{z}^a\dot{\bar{z}}^{\bar{b}} + \mathrm{e}^{2U}\left(W^2 + 4g^{a\bar{b}}\partial_a W \partial_{\bar{b}} W\right)$$

- first-order gradient flow, equivalent to EOM

- when : non-supersymmetric

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- when $W \neq |Z|$: non-supersymmetric

Geometrical perspective

- IIA string theory compactified on a CY 3-fold X

$$D_{abc} = \int_X D_a \wedge D_b \wedge D_c \qquad D_a : \text{basis of } H^2(X, \mathbb{Z})$$

- scalars: in the normalized period vector

- charges: branes wrapping even cycles of

- central charge

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$$\Gamma = p^0 + p^a D_a + q_a D^a + q_0 \mathrm{d}V \in H^{2*}(X, \mathbb{Z})$$

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Integration of flow equations

- yet another rewriting (susy case) [Denef]

$$\mathcal{L} \propto \mathrm{e}^{2U} \left| 2\,\mathrm{Im}\left(\left(\partial_\tau + \mathrm{i}\,\mathrm{Im}(\partial_a K \dot{z}^a) + \mathrm{i}\dot{\alpha}\right) \left(\mathrm{e}^{-U} \mathrm{e}^{-\mathrm{i}\alpha} \Omega\right) \right) + \Gamma \right|^2$$

$$\alpha = \arg Z$$

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- solutions for scalars implicit, but can be inverted explicitly, also for multiple centers

[Bates & Denef]

Multicenter generalization [Denef]

- metric $\mathrm{d}s^2 = -\mathrm{e}^{2U}(\mathrm{d}t + \omega_i \mathrm{d}x^i)^2 + \mathrm{e}^{-2U}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$
- multicenter harmonic function

$$H = \sum_{n=1}^{N} \Gamma_n \tau_n - 2\,\mathrm{Im}[\mathrm{e}^{-\mathrm{i}\alpha}\Omega]_{\tau=0} \qquad \tau_n = \frac{1}{|\mathbf{x}-\mathbf{x}_n|}$$

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- constraints on positions

$$\sum_{m=1}^{N} \frac{\langle \Gamma_n, \Gamma_m \rangle}{|\mathbf{x}_n - \mathbf{x}_m|} = 2\,\mathrm{Im}[\mathrm{e}^{-\mathrm{i}\alpha} Z(\Gamma_n)]_{\tau=0}$$

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- angular momentum

$$\mathbf{J} = \frac{1}{2} \sum_{m<n} \langle \Gamma_m, \Gamma_n \rangle \frac{\mathbf{x}_m - \mathbf{x}_n}{|\mathbf{x}_m - \mathbf{x}_n|}$$

Extension to non-susy solutions

- Denef's formalism involves a change of basis

$$\Gamma = p^0 \cdot 1 + p^a D_a + q_a D^a + q_0 \mathrm{d}V$$

- analogously, in our generalization:

- non-susy solutions

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$$\Gamma = \mathrm{i}\bar{Z}\Omega - \mathrm{i}g^{\bar{a}b}\bar{\mathcal{D}}_{\bar{a}}\bar{Z}\mathcal{D}_b\Omega + \mathrm{i}g^{a\bar{b}}\mathcal{D}_a Z\bar{\mathcal{D}}_{\bar{b}}\bar{\Omega} - \mathrm{i}Z\bar{\Omega}$$

- analogously, in our generalization:

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Extension to non-susy solutions

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$$\Gamma = 2\,\mathrm{Im}\left[\bar{Z}(\Gamma)\Omega - g^{\bar{a}b}\bar{\mathcal{D}}_{\bar{a}}\bar{Z}(\Gamma)\mathcal{D}_b\Omega\right]$$

- analogously, in our generalization:

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Extension to non-susy solutions

- Denef's formalism involves a change of basis

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$$2\,\mathrm{Im}(\mathrm{e}^{-U}\mathrm{e}^{-\mathrm{i}\tilde{\alpha}}\Omega) = -\tilde{H} \qquad \tilde{\alpha} = \arg Z(\tilde{\Gamma})$$

$$\tilde{H}(\mathbf{x}) = \sum_{n=1}^{N}\tilde{\Gamma}_n\tau_n - 2\,\mathrm{Im}[\mathrm{e}^{-\mathrm{i}\tilde{\alpha}}\Omega]_{\tau=0} \qquad \tilde{\Gamma}_n = \Gamma(S_nQ_n)$$

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- formalism works unchanged for constant S (a subclass of superpotentials)
  - mutually local ($\langle \Gamma_m, \Gamma_n \rangle = 0$) electric or magnetic configurations
- constraints on charges (rather than positions)

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- *stu*: solution agrees with known/conjectured

[Kallosh, Sivanandam, Soroush]

BPS constituent model of non-susy bh

- ADM mass formula for a non-susy *stu* bh:

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- in our context: supersymmetry of each center unaffected by the nontrivial matrices S_i (nontrivial S_i necessary for consistency with nontrivial S of a non-supersymmetric single-center black hole)

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  - can they yield all possible solutions?