The Kerr/CFT Correspondence

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ETH STRING WORKSHOP

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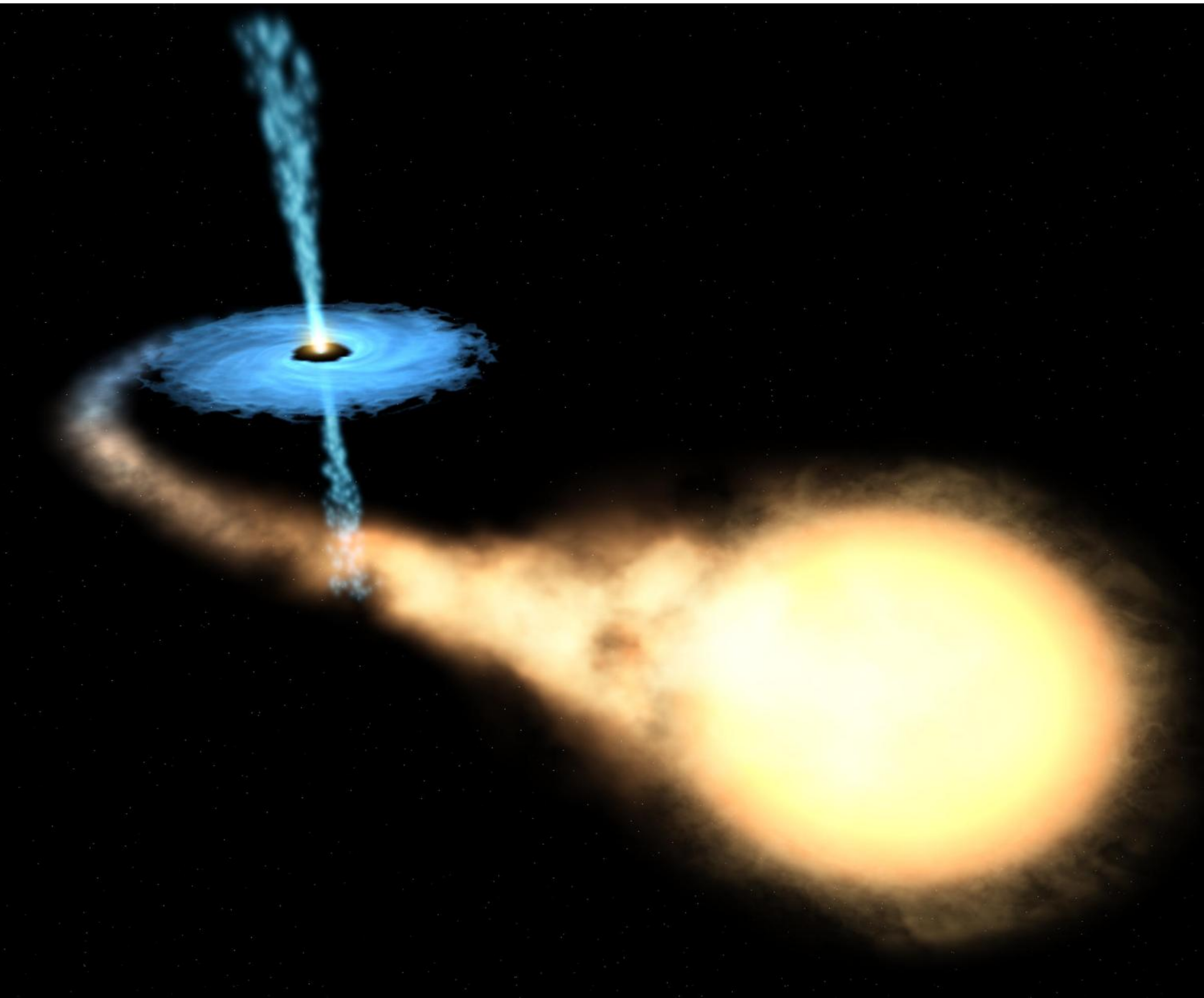
In this talk I will argue & give evidence that observed extreme Kerr black holes such as GRS 1915+105 are dual to 2D CFTs.



An extreme 4D

Kerr  black hole

has angular momentum

$$J = GM^2,$$

Hawking temperature

$$T_H = 0,$$

and Bekenstein-Hawking entropy

$$S_{BH} = \frac{2\pi J}{\hbar}.$$

GRS1915+105 has

$$M \sim 14 M_\odot$$

$$\frac{J}{GM^2} > .98$$

McClintock, Shafee, Narayan, Remillard, Davis & Li (2006)

The basic idea is that $\boxed{\text{Kerr} = \text{CFT}}$ follows in a near-horizon scaling limit in the spirit of Brown & Henneaux from properties of diffeos.

Condensed Matter	Astrophysics
complex molecular mess	complex binary system
$\downarrow$ scaling	$\downarrow$ scaling
critical CFT	critical CFT

low energy = high redshift = near-horizon

NASA has kindly prepared a video of our scaling limit.....

grs1915.mp4

The near-horizon region of extreme Kerr

is ~~like~~ a chiral gravity: all excitations must move counterclockwise at the speed of light!

So in the extreme case we get chiral (half of) 2D CFT on (ϕ, t) cylinder.

Role of string theory?

None (except inspirational). Deriving the universal BH area-entropy law via an exact construction of stringy BH microstates is like deriving the laws of thermodynamics via the periodic table. Boltzman needed only to assume a consistent UV cutoff (molecules) existed. Similarly, deriving the area law should not require detailed microphysics. We will see it doesn't.

Extreme Kerr Review

$J = M^2$ cosmic censorship

$$ds^2 = -\frac{(\hat{r}-M)^2}{\rho^2}\left(d\hat{t} - \sin^2\theta\, d\hat{\phi}\right)^2$$

$$+ \frac{\sin^2\theta}{\rho^2}\left((\hat{r}^2+M^2)\,d\hat{\phi} - M\,d\hat{t}\right)^2$$

$$+ \frac{\rho^2}{(\hat{r}-M)^2}\,d\hat{r}^2 + \rho^2\,d\theta^2$$

$$\rho^2 \equiv \hat{r}^2 + M^2\cos^2\theta$$

Thermodynamic quantities

$$T_H = 0$$

$$\Omega_H = \frac{1}{2M}$$

$$S_{BH} = \frac{\text{Area}}{4} = 2\pi J$$

EXPLAIN THIS!

Near Horizon Extreme Kerr

Bardeen & Horowitz '99

SCALING

Take $t = \frac{\lambda \hat{t}}{2M}$, $r = \frac{\hat{r} - M}{\lambda M}$, $\phi = \hat{\phi} - \frac{\hat{t}}{2M}$

$\lambda \to 0$ w/ (t, r, θ, ϕ) fixed.

$$\Rightarrow ds^2 = 2J\Omega^2 \left[-r^2 dt^2 + \frac{dr^2}{r^2} + d\theta^2 + \Lambda^2 (d\phi + r\,dt)^2 \right]$$

$\Omega^2 \equiv \frac{1+\cos^2\theta}{2}$

$\Lambda \equiv \frac{2\sin\theta}{1+\cos^2\theta}$

ISOMETRY $= SL2R/U(1)$

fixed θ = Warped AdS_3

$\mathbb{Z}$

$\Downarrow$

finite temp. CFT?

ADS IN THE SKY!!!

HOMEWORK

Problem 1. Find consistent NHEK boundary conditions.

Problem 2. Find conserved charges.

Problem 3. Determine asymptotic symmetry group.

Asymptotic
Symmetry
Group

depends on boundary conditions

$$= \frac{\text{allowed diffeos}}{\text{trivial diffeos}}$$

depends on dynamics

One consistent solution

$h_{ab} = g_{ab} - g_{ab}^{NHEK}$

$h_{tt} \sim r^2$, $h_{t\phi} \sim r^0$ $h_{t\theta} \sim \frac{1}{r}$ $h_{tr} \sim \frac{1}{r^2}$

$h_{\phi\phi} \sim r^0$, $h_{\phi r} \sim \frac{1}{r}$, $h_{\phi\theta} \sim \frac{1}{r}$, $h_{\theta r} \sim \frac{1}{r^2}$

$h_{rr} \sim \frac{1}{r^3}$

PLUS $M^2 = J$ ← (Divide & conquer) STUDY EXTREME ONLY

charges are finite of allowed diffeos, not all zero.

SIMILAR TO BMS GROUP.

CENTRAL CHARGE

The **ASG** for **NHEK** is generated by

$$\zeta_n = -e^{-in\phi}\partial_\phi + ine^{-in\phi} r\partial_r$$

w/ Lie Bracket algebra = Virasoro

$$i[\zeta_m, \zeta_n]_{L.B.} = (m-n)\zeta_{m+n}$$

The action on fields is generated via Dirac Brackets w/ the conserved charges

Abbot Deser, Barnich Brandt Compere, Brown Henneaux..

$$Q_n[h] = -\frac{1}{32\pi G}\int_{S^2} * dx^\mu \wedge dx^\nu \Big[\zeta_{n\nu} D_\mu h - \zeta_{n\nu} D^\sigma h_{\mu\sigma} + \zeta_n^\sigma D_\nu h_{\mu\sigma} + \frac{h}{2} D_\nu \zeta_{n\mu} - h_{\nu\sigma} D^\sigma \zeta_{n\mu} + \frac{1}{2} h^\sigma{}_\nu (D_\mu \zeta_n^\sigma + D^\sigma \zeta_{n\mu})\Big]$$

THESE ARE FINITE

By construction

$$i\{Q_m, Q_n\}_{D.B} = (m-n)\, Q_{m+n} + i\, Q_n[\mathcal{L}_{\zeta_m} \bar{g}^{NHEK}]$$

$$\Rightarrow \quad \boxed{c_L = 12J}$$

This implies that if we succeed in constructing QG of any kind on NHEK with the given BC it must be a 2D CFT in the sense that the quantum states form Virasoro reps. Bulk locality $\overset{?}{\rightsquigarrow}$ boundary CFT locality, maybe. Physical observation suggests the CFT is unitarity.

So far, we know only the central charge of the CFT. Physical observation → properties of CFT. Knowing the CFT exactly = knowing all the laws of physics. We dont expect that just yet!

THE ANALYSIS IS BOTTOM UP, NOT TOP DOWN!.

TEMPERATURE

A general Kerr black hole corresponds to the thermodynamic ensemble:

$$\rho = e^{-\frac{(\omega - m\Omega_H)}{T_H}}$$

where ω, m are Fourier coefficents.
Define n_R, n_L by

$$e^{-i\omega \hat{t} + im\hat{\phi}} = e^{-in_R t + in_L \phi}$$

$$\Rightarrow \boxed{n_L = m = L_0} = \text{angular momentum}$$

$$n_R = \frac{1}{\lambda}(2\omega - m) = \text{energy above extremality}$$

Define temps T_L, T_R conjugate to n_L, n_R

$$\rho = e^{-\frac{n_R}{T_R} - \frac{n_L}{T_L}}$$

For extremality $T_H \to 0$, n_R

$$\boxed{T_L \to \frac{1}{2\pi}}, \quad T_R \to 0$$

Ensemble of left movers $\quad \rho = e^{-2\pi n_L}$

$\boxed{\text{Entropy}}$

The canonical Cardy formula is

$$S = \frac{\pi^2}{3} c_L T_L$$

Using

$$c_L = 12 J \,, \quad T_L = \frac{1}{2\pi}$$

gives

$$\boxed{S_{micro} = 2\pi J = \frac{\text{Area}}{4} \; !}$$

Summary: Canonical symmetry analysis $\Rightarrow$ extreme Kerr is dual to a $c = 12J$ CFT at $T = \frac{1}{2\pi}$.

Assuming Cardy, unitarity etc, this explains extreme Kerr entropy.

What's new/next?

The analysis can be generalized to higher D, charges, c.c., multiple angular momenta etc. It always works. Agreement much more non-trivial

Lu Soltanpanahi Lu Mei Pope Ogawa Azeyanagi Terashima Murata Nishioka Hartman AS Peng Wu Isono Tai Wen Chow Nakayama Chen Wang Compere Vasquez-Poritz Wu Tian Tachikawa Carolusi Chodsi Matsuo Tsukioka Yoo

Lesson $\exists$ inequivalent consistent BCs w/ different dual CFTs, T_L, c_L but same S_{BH}.

Higher derivative corrections to Wald also matched. Krishnan Kuperstein Azeyanagi Compere Ogawa Terashima Tachikawa

Hints: Kerr Horizon=Surface of Fermi Sea?

Emparan et al Verlinde et al
McGreevy Lu et al …………….

Classical GR / dynamics of NHEK largely unexplored beyond Bardeen & Horowitz.

What are possible b.c.s?
What is causal structure/geodesics?
Positive energy theorem, uniqueness?
Instabilities?
How do we describe the boundary?
⋮

Recent progress

Amsel, Horowitz, Marolf, Roberts I, II
Dias, Reall Santos
Balasubramanian, deBoer, Sheik-Jabbari, Simon

Seems to confirm uniquess of NHEK up to diffeos (which may be nontrivial)

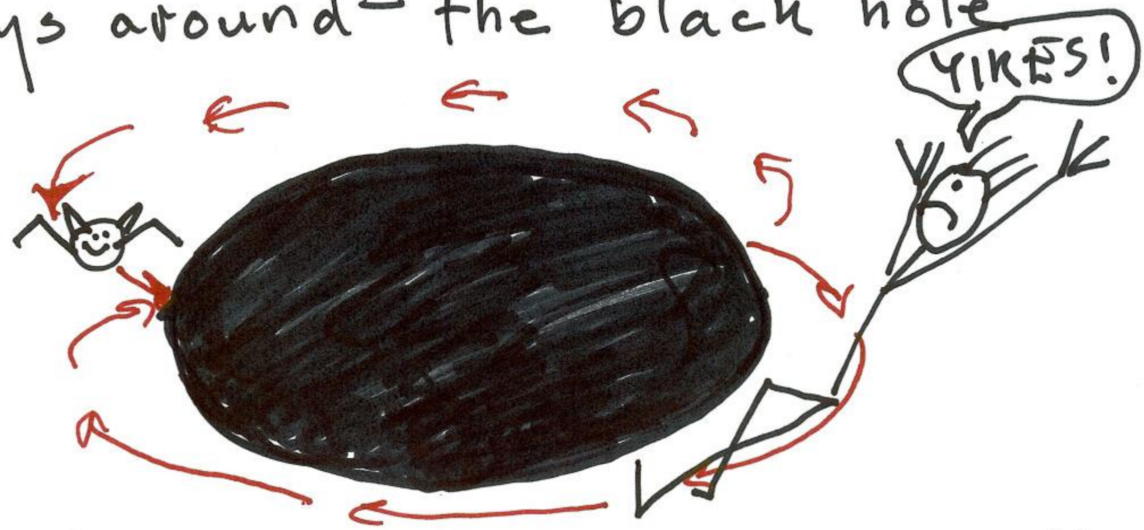
EVENTUALLY KERR/CFT MAY EXPLAIN CURRENTLY MYSTERIOUS OBSERVED BLACK HOLE BEHAVIORS.

BEYOND EXTREMALITY

``BLACK HOLE SUPERRADIANCE FROM KERR/CFT'"

I. Bredberg, T. Hartman, W. Song & A. S.

For $M^2 > J$ things can move both ways around the black hole



so we expect a non-chiral CFT, with $\bar{L}_n$ enhancing $SL2R_R$ of NHEK.

$C_R=12J!$

Matsuo Tsukioka Yoo

Castro Larsen

This is a hard problem, due to AdS_2-type IR divergences. We have taken the smallest step away from extremality we could find....

Scattering

Superradiant

Do ~~CFT~~ and gravity agree?

c.f. Maldacena & AS '96

Kerr scattering cross sections for massless scalar were computed in Starobinsky & Churilov '73, Teukolsky & Press '74. In the near-superradiant limit

$T_R \to 0$, $\tilde{n}_R = \frac{n_R}{2\pi T_R}$ fixed, their formula is

$$\sigma_{abs} = \frac{|\Gamma(\frac{1}{2}+\beta+im)|^2 (m 8\pi M T_H)^{2\beta}}{2\pi\,\Gamma(2\beta)^2\,\Gamma(2\beta+1)^2}$$

Normalization from near-far matching

$$\times\ e^{-\pi \tilde{n}_R} |\Gamma(\tfrac{1}{2}+\beta+i\tilde{n}_R)|^2$$

$\langle \mathcal{O}_\beta(0)\,\mathcal{O}_\beta(x^+)\rangle_{CFT}$

$$\times\ \Gamma\,|(\tfrac{1}{2}+\beta+im)|^2$$

$\langle \mathcal{O}_\beta(0)\,\mathcal{O}_\beta(x^-)\rangle_{CFT}$

$$\beta^2 = K_\ell - 2m^2 + \tfrac{1}{4}$$

← separation constants

Matches perfectly CFT calculation with $\dim(\mathcal{O}) = \frac{1}{2} + \beta$, computed from highest weight solns of wave eq.

ALSO WORKS FOR KERR-NEWMAN, HIGHER SPIN!

In fact the match between bulk gravity and boundary CFT is better than expected. We don't understand why it works so well! But we seem to be on the right track.

Summary

1) Extreme Kerr entropy is matched by $c = 12J$ chiral CFT.
2) Scattering cross sections reproduced by non-chiral CFT.
3) Many aspects of Kerr/CFT remain mysterious.