Refined bound state indices for D-particles

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Motivation

- D–particles as toy models for black holes
- Interest in multi–centered BPS black holes: attractor mechanism, split flows, OSV, entropy enigma, scaling solutions,. . .
- Split attractor flow tree conjecture (Denef, Moore 2007)
- Refinement of indices enumerating BPS microstates in IIA on CY_3
- The chromosomes of D–particles: polar states

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D–particles/black holes

- D–branes wrapped on cycles of CY in type IIA
- One modulus: $H^{2*}(X) = H^0(X) \oplus H^2(X) \oplus H^4(X) \oplus H^6(X)$
- Brane system with charge (p^0, p, q, q_0):
  $\Gamma = p^0 + pH + \frac{q}{\mathcal{H}}H^2 + \frac{q_0}{\mathcal{H}}H^3$, with $\mathcal{H} := \int_X H^3$
- $\langle\Gamma_1, \Gamma_2\rangle := q_{0,1}p_2^0 - q_1 p_2 + p_1 q_2 - p_1^0 q_{0,2}$

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Elliptic genus

- Mixed ensembles of D4–D2–D0 branes with fixed magnetic charge H and variable (q, q_0)
- Modified elliptic genus: BPS indices of (0,4) MSW CFT

$$Z(q,\bar{q},y) = \mathrm{Tr}_R\left(\frac{1}{2}F^2(-1)^F q^{L_0-\frac{c_L}{24}}\bar{q}^{\bar{L}_0-\frac{c_R}{24}}e^{2\pi i y^A q_A}\right)$$

$$Z(q,\bar{q},y) = \sum_\gamma Z_\gamma(q)\,\Theta_\gamma(q,\bar{q},y), \quad F = \frac{H}{2}+f+\gamma$$

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Polar states

- $Z(q, \bar{q}, y)$ is a weak Jacobi form with weight $(-\frac{3}{2}, \frac{1}{2})$
- Farey tail expansion and polar states
- In our setup, these are $D6 - \overline{D6}$ bound states giving rise to a factorization for the index

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Attractor mechanism

- BPS equations for static, spherically symmetric solutions
- Central charge flows to a fixed point ($|Z| \rightarrow$ minimum)
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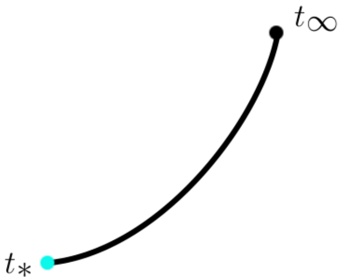
Single flow

In moduli space this looks like:



Attractor points

We distinguish three cases:

- $|Z|$ attains a minimum $\neq 0$: there is a single flow
- $|Z|$ reaches 0 at a regular point in moduli space (crash point): there is no single flow
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Multicenter solutions

- Take a more general (stationary instead of static) ansatz
- The total charge Γ could also split into two charges $\Gamma = \Gamma_1 + \Gamma_2$
- This splitting is marginally stable when $|Z| = |Z_1| + |Z_2|$ or $\arg Z = \arg Z_1 = \arg Z_2$
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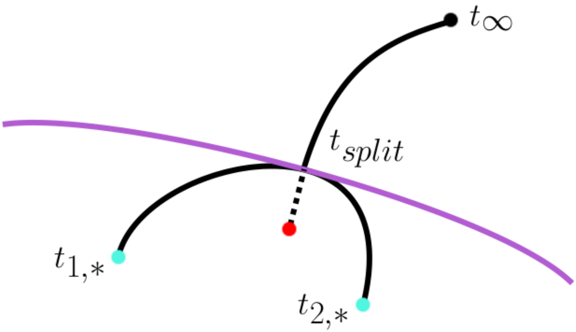

Split attractor flow conjecture

Conjecture by Denef, Moore (2007)

- Weak form: Single or split flows are an existence criterium for BPS states in supergravity and their number is finite.
- Strong form: Single or split flows are an existence criterium for BPS states in the full string theory and their number is finite.

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Bound states of $D6 - \overline{D6}$

- D–particles with low charge $(0, 1, q, q_0)$
  $\rightarrow D6 - \overline{D6}$ bound state
- Index splits up into a sum of split flows

$$\Omega(\Gamma) = \sum_{\Gamma \to \Gamma_1 + \Gamma_2} (-1)^{|\langle \Gamma_1, \Gamma_2 \rangle| - 1} |\langle \Gamma_1, \Gamma_2 \rangle| \, \Omega(\Gamma_1) \, \Omega(\Gamma_2)$$

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$$\Gamma_{D6} = e^{F_1} \left(1 - \beta_1 - \left(\tfrac{1}{2} \chi(C_{\beta_1}) + N_1\right) \omega\right) \left(1 + \frac{c_2(X)}{24}\right)$$

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Refinement (1)

- The tachyon index *jumps* between different $D6-\overline{D6}$ microstates
- Tachyon field: $T \in \Gamma(F_2^* \otimes F_1)$
- In our case $T \in \Gamma(H)$
- Riemann–Roch: $\mathcal{I}(\mathcal{M}_T) = \langle \Gamma_1, \Gamma_2 \rangle$

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- Consider for example CY in $\mathbb{WP}^4_{11125}$
- Tachyon field: $T = a_1 x_1 + a_2 x_2 + a_3 x_3$
- T has to vanish on bound $\overline{D0}$'s
- In general, $\mathcal{M}_T$ reduces from $\mathbb{CP}^2$ to $\mathbb{CP}^1$ (for one bound $\overline{D0}$)
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Our Calabi-Yau: The *sextic*

The sextic is a degree 6 hypersurface in $\mathbb{WP}^4_{11112}$:

- Homogeneous coördinates x_1, x_2, x_3, x_4, x_5 with weights $(1, 1, 1, 1, 2)$
- General transversal polynomial of degree 6:

$$P(x) = x_5^3 + x_5 f^{(4)} + f^{(6)}$$

  where the $f^{(i)}$ denotes a homogeneous polynomial of degree i in x_1, x_2, x_3, x_4
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Elliptic genus: factorization

- $\int_X H^3 = 3 \rightarrow$ there are 2 gluing vectors
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Elliptic genus: most polar state

- The most polar state: pure D4–brane
- $q = H \cdot F, \quad q_0 = \frac{F^2}{2} + \frac{c_2(P)}{24} - N, \quad \hat{q}_0 \equiv q_0 - \frac{1}{12} D^{AB} q_A q_B$
- $\rightarrow$ pure D4 charge: $(0, 1, \frac{3}{2}, \frac{9}{4})$
- $\hat{q}_0 = \frac{45}{24}$ so 2 polar states
- Only one split flow into $D6_H$ and $\overline{D6}$ with $\langle \Gamma_1, \Gamma_2 \rangle = -4$
- This gives

$$\begin{aligned}\Omega &= (-1)^{\langle \Gamma_1, \Gamma_2 \rangle - 1} |\langle \Gamma_1, \Gamma_2 \rangle| N_{DT}(0,0) \cdot N_{DT}(0,0) \\ &= -4 \cdot 1 \cdot 1 = -4\end{aligned}$$

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- $\hat{q}_0 = \frac{45}{24}$ so 2 polar states
- Only one split flow into $D6_H$ and $\overline{D6}$ with $\langle \Gamma_1, \Gamma_2 \rangle = -4$
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Elliptic genus: most polar state

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- In the same way we find for pure D4 + 1 $\overline{D0}$ with $\hat{q}_0 = \frac{21}{24}$
- Only one split flow into $D6_H$ and $\overline{D6} - \overline{D0}$ and

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- These polar states completely determine Z_0:
$Z_0(q) =$
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The first non-polar state

- For D4 + 2 $\overline{D0}$ with $\hat{q}_0 = \frac{-3}{24}$
- Still only one split flow into $D6_H$ and $\overline{D6} - 2\overline{D0}$
- The naive index would be

$$\Omega = (-1)^{\langle\Gamma_1,\Gamma_2\rangle-1}|\langle\Gamma_1,\Gamma_2\rangle| N_{DT}(0,0)\cdot N_{DT}(0,2)$$
$$= -2\cdot 1\cdot 20'298 = -40'596$$

- $-40'596 \neq -40'392$!

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Refining: a blind tachyon. . .

- We had: $T = a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_4$
- This field does not 'see' x_5 coordinates of the $\overline{D0}$'s
- Number of constraints of the two $\overline{D0}$'s :

$$\text{rank} \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \end{pmatrix}$$

- Special locus: $x_i = y_i, \quad i = 1, \ldots, 4$ and $x_5 \neq y_5$
  with $f = x_5^3 + f^{(6)} \to 2(\chi - \chi_0)\frac{1}{2}$
  where χ_0 is the Euler characteristic of the locus $x_5 = 0$

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Refining: a hidden blowup. . .

- If both $\overline{D0}$'s coincide, a blowup procedure is needed
- We replace the coincidence locus with the tangent directions ($\mathbb{CP}^2$)
- Number of constraints of the overlapping $\overline{D0}$'s :

$$\text{rank}\begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ X^1 & X^2 & X^3 & X^4 \end{pmatrix}$$

- Special blowup direction: $X^i = \lambda x_i, \quad i = 1, \ldots, 4$
  $\nabla_X f = 3X^5 x_5^2 + 6\lambda f^{(6)} = 3x_5^2(X^5 - 2\lambda x_5) = 0$
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Refining: the result

- The special locus is the sum:
  $2 \cdot \frac{1}{2}(\chi - \chi_0) + \chi_0 = \chi = -204$
- We now have: $\mathcal{N}_{DT}^{(s)}(0,2) = -204$ and
  $\mathcal{N}_{DT}^{(g)}(0,2) = N_{DT}(0,2) - \mathcal{N}_{DT}^{(s)}(0,2) = 20'298 + 204 = 20'502$
- And the index becomes
  $-2 \cdot 20'502 - 3 \cdot (-204) = -40'392$
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Outline

New elliptic genus

- Decantic: degree 10 hypersurface in $\mathbb{WP}^4_{11125}$, $\int_X H^3 = 1$
- Pure D4, $\hat{q}_0 = \frac{35}{24}$: $\Omega = 3$
- Plus one $\overline{D0}$, $\hat{q}_0 = \frac{11}{24}$: naively -576
- Special locus: $x_1 = x_2 = x_3 = 0$, $\chi_0 = 1 = -\mathcal{N}^{(s)}_{DT}(0,1)$
- $\Omega = -2 \cdot (N_{DT}(0,1) - (-1)) - 3 \cdot (-1) = -575$
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Questions

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