The superstring in $AdS_4 \times CP^3$

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Based on:

- 0811.1566 with J. Gomis and D. Sorokin
- 0903.5407 with P.A. Grassi and D. Sorokin

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Motivations

- Multiple M2-branes
  - BLG,ABJM-models

ABJM:

- $D = 3$, $\mathcal{N} = 6$ CS matter theory w. gauge group $U(N)_k \times U(N)_{-k}$
- In 't Hooft-limit:

$$N, k \to \infty \qquad \text{with} \qquad \lambda \sim \frac{N}{k} \qquad \text{fixed}$$

Dual to type IIA string theory on $AdS_4 \times CP^3$

- Interesting realization of AdS4/CFT3-correspondence
- To investigate the string side an action for the string is needed

Superstring in $AdS_5 \times S^5$

- Recall the $AdS_5 \times S^5$ case
- $AdS_5 \times S^5$ is maximally supersymmetric preserving 32 SUSYs
- Full $(10|32)$ dim. supergeometry can be described as the supercoset:

[Kallosh Rahmfeld Rajaraman]

[Metsaev Tseytlin]

$$\frac{PSU(2,2|4)}{SO(4,1) \times SO(5)}$$

- Superstring action can be formulated as a sigma model on this supercoset [Metsaev Tseytlin]
- Integrability follows from supercoset structure [Bena Polchinski Roiban]
- Can we repeat this story for $AdS_4 \times CP^3$?
  - Almost. . .

The $AdS_4 \times CP^3$ supercoset model

- One major difference:
  - $AdS_4 \times CP^3$ not maximally supersymmetric preserving only 24 SUSYs
    $\rightarrow$ Supergeometry cannot be realized as a supercoset
- We can come close however. Take

$$\frac{OSp(6|4)}{SO(3,1) \times U(3)}$$

  Bosonic part $AdS_4 \times CP^3$ but only 24 fermionic directions (unbroken SUSYs)
- The string has only 16 physical fermionic d.o.f. so this is okay
- String can be described as sigma model on this supercoset [Arutyunov Frolov] [Stefański]
- Integrability follows from the supercoset structure just as in the $AdS_5 \times S^5$ case

Shortcomings of the supercoset model

- Contains only 24 of the 32 fermions of the IIA background
  - The eight fermions corresponding to broken SUSYs missing
  - *i.e.* kappa-symmetry partially gauge fixed to remove 8 of 16 unphysical fermions
- More serious problem:

  For string moving only in AdS_4 the model describes 12 physical fermions instead of 16!

  $\rightarrow$ Gauge-fixing is inconsistent for these configurations
- We would like an action that can describe any string configuration

  $\rightarrow$ Need the full (Green-Schwarz) superstring action w. all 32 fermions

The Green-Schwarz action in a general background

- The GS superstring action in a general background is given by

[Grisaru Howe Mezincescu Nilsson Townsend]

$$S = -\int d^2\xi \sqrt{-\det(\mathcal{E}_i{}^A \mathcal{E}_j{}^B \eta_{AB})} + \int B$$

where $i, j = 1, 2,\ A, B = 0, \ldots, 9$.

- Involves the (pull-back of) bosonic supervielbeins

$$\mathcal{E}_i{}^A = \partial_i z^{\mathcal{M}} E_{\mathcal{M}}{}^A(x, \theta) \qquad z = (x, \theta)$$

and the NS-NS two-form field $B(x, \theta)$

- Need to know the full supergeometry, *i.e.* supervielbeins $\mathcal{E}^A(x, \theta)$ and $B(x, \theta)$ explicitly as functions of x and 32 θ
- D-brane actions can then be written as well

$AdS_4 \times CP^3$ supergeometry

- How to find the supergeometry?
    1. Determine order by order in θ? Very tedious!
    2. Obtain it by dimensional reduction from $D = 11$

  The bosonic story:

- The 11 dim. geometry is the near horizon geometry of an M2: $AdS_4 \times S^7$ (mod $\mathbb{Z}_k$)
- S^7 can be realized as an S^1 bundle over CP^3
    - Locally $S^7 \sim CP^3 \times S^1$
- Reducing $AdS_4 \times S^7$ on the circle fiber gives precisely $AdS_4 \times CP^3$

[Nilsson Pope]

[Sorokin Tkach Volkov]

- This is precisely what happens in the 't Hooft limit:

$$k \to \infty \qquad \Rightarrow \qquad S^7/\mathbb{Z}_k \to CP^3$$

The superspace story:

- $AdS_4 \times S^7$ is maximally supersymmetric (32 SUSYs)
- Can be realized as the supercoset [Kallosh Rahmfeld Rajaraman]

$$\frac{OSp(8|4)}{SO(3,1) \times SO(7)}$$

$\rightarrow$ Supervielbeins and form fields explicitly known as functions of x and 32 θ

- Idea:
  - Mod out by $\mathbb{Z}_k$
  - Perform Kaluza-Klein reduction in superspace [Howe Sezgin]
  - Gives explicit expressions for the $AdS_4 \times CP^3$ supervielbeins, NS-NS and RR fields

$\rightarrow$ Green-Schwarz string and D-brane actions

Slightly more complicated...

- To perform KK-reduction the vielbeins should have the form

$$E_{\widehat{M}}{}^{\widehat{A}} = \begin{pmatrix} E_M{}^A & A_M \\ 0 & \Phi \end{pmatrix} \qquad \widehat{A} = (A, 11)$$

- Possible to find a realization of $AdS_4 \times S^7$ supervielbeins where this is almost true
- Except $E_{11}{}^a \neq 0$ with a an AdS_4 index
- Perform a Lorentz rotation in the 5-plane spanned by AdS_4 and the 11th (S^1) direction to fix this
- Perform the KK-reduction

  $\rightarrow$ Explicit (slightly complicated) form of $AdS_4 \times CP^3$ supervielbeins

Kappa-symmetry

- Fermions transform as

$$\delta_\kappa z^{\mathcal{M}} E_{\mathcal{M}}{}^{\alpha} = \frac{1}{2}(1+\Gamma)^{\alpha}{}_{\beta}\kappa^{\beta}$$

where $\frac{1}{2}(1+\Gamma)$ projects onto the 16 unphysical fermions

$$\Gamma \propto \varepsilon^{ij}\mathcal{E}_i{}^{A}\mathcal{E}_j{}^{B}\Gamma_{AB}\Gamma_{11}$$

- Split bosonic indices $A=(a,a')$ into AdS_4: $a=0,\ldots,3$ and CP^3: $a'=1,\ldots,6$
- Projection matrix that projects onto the 8 broken SUSY directions:

$$\mathcal{P}_8 = \frac{1}{8}(2+J) \qquad J = -iJ_{a'b'}\Gamma^{a'b'}\Gamma^7 \qquad J_{a'b'} \quad \text{Kähler form on } CP^3$$

Note: Involves only Γ-matrices corresponding to the CP^3-directions

Gauge fixing kappa-symmetry

- Making the (partial) gauge-fixing

$$\mathcal{P}_8\theta = 0$$

Leaves only the 24 fermions corresponding to SUSYs $\rightarrow$ $OSp(6|4)$ supercoset model

- However: For string moving in AdS_4 only we have ${\mathcal{E}_i}^{a'} = 0$ so

$$\Gamma \sim \varepsilon^{ij}{\mathcal{E}_i}^a{\mathcal{E}_j}^b\Gamma_{ab}\Gamma_{11}$$

But

$$\{\Gamma_a, \Gamma_{a'}\} = 0 \qquad \Rightarrow \qquad [\Gamma, \mathcal{P}_8] = 0$$

$\Rightarrow$ Only 4 of the 8 broken fermions can be gauged away!

- Explains why the supercoset model is missing 4 physical d.o.f. for these configurations

Other gauges

- Having the full GS action makes other gauge choices possible
- Can be used to simplify string/D-brane actions

  Example:

$$(1 + \Gamma^0 \Gamma^1 \Gamma^2)\theta = 0$$

  - Consistent for string moving only on AdS_4
  - Removes 4 broken and 12 unbroken fermions

    $\longrightarrow$ Action with up to 8th order in fermions
  - T-duality $\longrightarrow$ Action with terms up to 4th order in fermions
- D-branes as well
- *e.g.* D2-brane on the boundary of AdS_4

Integrability?

- ABJM-model is integrable
- Great advantage of a supercoset description of the string:
  - Generic construction of Lax connection [Bena Polchinski Roiban]
    $\rightarrow$ (classical) integrability
- Supercoset description applies in $AdS_4 \times CP^3$ except when the string moves only in AdS_4
- One would expect integrability also for these configurations
- How to prove it?
  - No supercoset description available
    $\rightarrow$ New tools are needed

Conclusion

- $AdS_4 \times CP^3$ supergeometry completely determined
  $\rightarrow$ GS string and D-brane actions
  - Useful for studying various aspects of the AdS4/CFT3-correspondence
  - *e.g.* semiclassical quantization around classical string solutions
- String described by supercoset model except when it moves entirely in AdS_4
  - Integrability for these configurations?